A Novel Multiuser Beamforming System With Reduced Complexity and Beam Optimizations

Kuo-Chen Ho and Shang-Ho Tsai, *Senior Member, IEEE*

***Abstract*— Analog beamforming is widely used in the incoming 5G/B5G communications systems to leverage against power consumption by significantly reducing the number of RF chains. In this paper, we propose a new multiuser beamforming scheme that allows the number of RF chains to be smaller than the number of users. In addition, we propose the beamforming algorithms to minimize the transmit power and weighted symbol error rates for the proposed scheme. For both high- and low-SNR regimes, the proposed beamforming designs provide simple closed-form solutions. The simulation results show that with half the number of RF chains and, thus, lower hardware cost, the proposed scheme outperforms the conventional zero-forcing beamforming systems in several interested parameter settings. Meanwhile, the performance degradation due to the quantization effect of the proposed system is less pronounced than that of the conventional beamforming systems.**

***Index Terms*— Analog beamforming, hybrid beamforming, precoding, MIMO, constant envelope precoding, mmWave.**

I. INTRODUCTION

MILLIMETER (mmWave) technology is one of the key features in the incoming 5G/B5G communications [1], [2]. However, channel measurements show that signals degrade seriously in the mmWave band [3]–[5]. This makes the channel model totally different from the traditional communications, *e.g.,* 3G and 4G, in which the carrier frequency is under 6GHz and the path loss is not that serious. Hence channel model and communication techniques should be developed for this new technology. The extended Saleh Valenzuela model with clustering is often used in literature [6] and standards [7]. The channel is sparse and composed of a few numbers of propagation paths in the spatial domain [8].

To overcome the challenges of serious degradation and sparsity in the mmWave channels, several techniques have been developed including channel estimation [9]–[12] and beamforming designs [13]–[16], [18]–[26]. A comprehensive survey of mmWave communications for future mobile networks can be found in [27]. Unlike the traditional MIMO precoding/beamforming that is often fully-digital and each antenna element requires a dedicated RF chain, in mmWave systems the short wavelength enables packing a large number of antenna elements feasible and thus such systems rely heavily on analog or RF processing to leverage against high hardware cost and power consumption issues [28], [29]. This brings the hybrid analog and digital architectures in [13]–[16], [18]–[22] and most research works assume that in the analog part, only the phase is adjustable via phase shifters (PS) in each antenna element. In addition, it is assumed that the number of RF chains should be larger or equal to the number of data streams. More specifically, the authors in [13] and [14] developed asymptotically optimal schemes assuming perfect CSI (channel state information) is available and no quantization effect is considered. In [15] and [18], suboptimal solutions using beam steering vectors were proposed for single-user systems. The achievable rate was shown to depend on the Frobenius norm of the difference of the optimal and hybrid beamformer in [14]. Multiuser transmission schemes with limited feedback were considered in [17]. In [19], a 2-bit phase resolution in the analog side with a low-dimensional baseband zero-forcing beamforming was studied. Nonorthogonal beam designs were considered for multiuser transmission in [20]. An iterative matrix decomposition based hybrid beamforming was proposed in [21]. A low-complexity directional hybrid beamforming problem under a finite alphabet constraint was studied in [22].

Another category of beamforming schemes attempts to reduce the hardware cost and does not assume that the number of RF chains is larger or equal to the number of data streams. These schemes use one RF chain to support multiple data streams. In addition, constant envelope constraint is applied in each antenna element and each antenna element only contains a PS to adjust the phase. This scheme is called the constant envelope precoding (CEP) [23]–[26]. More specifically, a simple and efficient CEP algorithm whose complexity is linear in the number of antennas for single-user MISO channels was proposed in [23]. CEP schemes for multiple users were considered in [24]–[26], where [24] investigated a CEP scheme that finds a near-optimal constant envelope (CE) signal for transmission, [25] considered digital PSs and developed a trellis-based CEP scheme, and [26] used cross-entropy optimization for solving the nonlinear least squares problem of CEP. The idea that CEP can support multiple data streams by only one RF chain via frequent switching of the

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The authors are with the Department of Electrical Engineering, National Chiao Tung University, Hsinchu 30010, Taiwan (e-mail: shanghot@alumni.usc.edu).

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beamforming coefficients is interesting. However, in this case, the data rate is limited by the switching speed of the PS.

In addition, the application of large arrays in mmWave systems increases the overhead for the feedback of CSI to the transmitter. Joint Spatial Division and Multiplexing (JSDM) algorithm has been proposed to deal with this issue. Reference [30] developed greedy algorithms to solve the problem of user grouping for maximizing spatial multiplexing and maximizing total received power. Furthermore, low-resolution DACs can also be considered as a good solution to reduce the complexity of large-scale antenna arrays [31].

From the discussion above, a system that can support multiple or multiuser data streams using a few RF chains is of desired. Also, avoiding frequent switching of the beamforming coefficients is also an important design goal. Several vendors have announced gain and phase adjustable architectures. Comparing to the pure shifter architectures, new architectures need extra PA and attenuator in each antenna. Hence the cost and complexity increase. Nevertheless, the extra components are realizable and affordable for technology nowadays. For instance, in the design document of the Anokiwave, 5 bits are used for phase control and 5 bits are used for gain control, see [32, p. 1].

This architecture has been used to reduce the sidelobe in [33]. Based on the gain and phase adjustable antenna array, one has more freedom to design the analog beamformer in achieving various design criterions. These motivate us to develop novel schemes for achieving these goals using the new gain and phase adjustable antenna architecture.

In this study, we propose a new scheme that can support multiuser transmission with few RF chains. More specifically, we first introduce a system that can support two users with only one RF chain. Then this system is extended to support $2K$ users with K RF chains. To avoid frequent switching of the beamforming coefficients, we suggest encoding the data of the two users into the same symbol. Another benefit of this suggestion is the simplicity of the encoding and decoding process. The beamforming designs are formulated into four optimization problems. The first problem minimizes the transmit power for a given symbol error rate (SER) in this two-user system. The second problem minimizes the weighted SER for a given transmit power in this two-user system. The third and fourth problems extends the results to a $2K$-user system from the first and second problems respectively. It is worth mentioning that the result of Problem 1 is key in solving Problems 2 to 4. By solving the problems, we obtain closed-form expressions of the optimal beamformers in both high and low SNR regimes.

Simulation results show that with fewer RF chains and thus lower hardware cost, the proposed multiuser scheme outperforms the conventional fully-digital zero-forcing beamforming (ZFBF) scheme in several interested parameter settings. The performance improvement of the proposed scheme is more pronounced when the number of users increases. Meanwhile, the performance degradation due to the quantization effect of the proposed scheme is less pronounced than the ZFBF scheme.

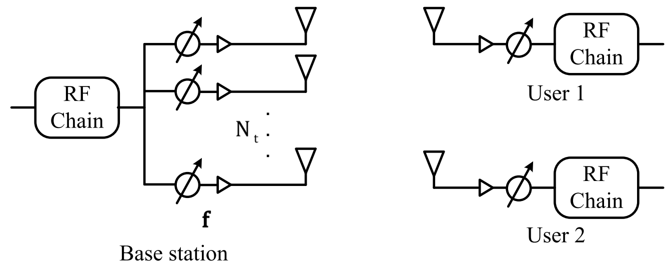

Fig. 1. The proposed scheme that uses a single RF chain to support 2 users.

The outline of the remaining parts of this manuscript is organized as follow: In Section II, we present the system model, the joint constellation design and problem formulations for the two-user case. In Section III, we solve the problems and obtain optimal beamforming solutions for the proposed two-user system. In Section IV, the problem is reformulated and extended to a $2K$-user case. The corresponding solution is also provided in this section. In Section V, we demonstrate the simulation results of the proposed system and algorithm. Conclusion remarks are provided in Section VI.

II. System and Problem Formulation

A. System Model

A block diagram of the proposed system of a two-user case is shown in Fig. 1, in which the base station has N_{t} antennas and $N_{\mathrm{RF}} = 1$ RF chain. Each user has one antenna. The system serves two users with only one RF chain. Since there is only one RF chain and thus only one data stream, to support two users, the transmit symbol s and beamforming vector $\mathbf{f}$ should be artificially designed. The transmit vector is given by

$$\mathbf{x} = \mathbf{f}s, \tag{1}$$

where $s \in \mathbb{C}^{1\times 1}$ and $\mathbb{E}[ss^*] = 1$. The total power constraint is enforced by normalizing $\mathbf{f}$ such that $\|\mathbf{f}\|^2 = P_{\mathrm{T}}$. The received signal of the uth user can be expressed by

$$r_u = \mathbf{h}_u^T \mathbf{f} s + n_u, \tag{2}$$

where $\mathbf{h}_u^T \in \mathbb{C}^{1\times N_{\mathrm{t}}}$ is the channel between the BS and the uth user, and $n_u \sim \mathcal{CN}(0, \sigma^2)$ is the Gaussian noise. The receive weight $w_u \in \mathbb{C}^{1\times 1}$ is used to process the received signal r_u:

$$y_u = w_u \mathbf{h}_u^T \mathbf{f} s + w_u n_u, \tag{3}$$

and w_u can be shown to be $(\mathbf{h}_u^T\mathbf{f})^*/|\mathbf{h}_u^T\mathbf{f}|$.

B. Joint Constellation Design for Individual Users

To support two users using only one RF chain, the transmit signal should be artificially designed. Let us explain this in this subsection. Consider the transmitted vector for the two users. Assume that $\mathbf{f}_1$ and $\mathbf{f}_2$ are the beamforming vectors for User 1 and User 2 respectively, and s_1 and s_2 are the data symbols for these two users. The transmit vector can be rewritten as

$$\mathbf{x} = \mathbf{f}_1 s_1 + \mathbf{f}_2 s_2 = \mathbf{f}s. \tag{4}$$

Because $\dim(\text{range}([\mathbf{f}_1\ \mathbf{f}_2])) = 2$ and $\dim(\text{range}(\mathbf{f})) = 1$, frequent beam switching is required if s_1 and s_2 are arbitrary constellation points. Frequent beam switching is power hungry. Meanwhile, the transmission speed is limited by the switching ability. To avoid frequent beam switching, we adopt a combined PAM strategy. That is, referring to (4), the two users transmit the signals s_1 and s_2 using PAM in the x-axis and the y-axis, respectively. The combined signal s in (4) is a QAM signal, and frequent switching can be avoided.

C. Problem Formulation

Two interesting problems are discussed here. One is to minimize the transmit power for given target symbol error rates (SERs) of individual users, and the other is to minimize a weighted symbol error rate (SER) for a given transmit power. The reason to consider the weighted SER is to more generally reflect different error rate priority of individual users. Many applications use error rate as the performance metric to design systems. Taking 5G NR (new radio) for instance, the uRLLC (Ultra Reliable Low Latency Communications) asks the system to achieve an error rate of 10^{-5} in a very short latency.

Let the signal s in (4) be a M-QAM signal, at the user sides, each user detects a $\sqrt{M}$-PAM constellation. The beamformer introduces gain values of g_1 and g_2 to User 1 and User 2 respectively. The SERs for these two users can be written as [34]

$$P_{e_u} = 2\left(1 - \frac{1}{\sqrt{M}}\right) \mathrm{Q}\left(\frac{d_{\min} g_u}{2\sigma}\right), \tag{5}$$

where $g_u = |w_u \mathbf{h}_u^T \mathbf{f}|$, $u = 1$ and 2, and $d_{\min}$ is the minimum distance of the constellation. Because w_u satisfies $|w_u| = 1$, this leads to $g_u = |\mathbf{h}_u^T \mathbf{f}|$.

Problem 1. Minimizing transmit power for given target SERs. The first problem is to minimize the average transmit power for fixed target SERs. Once the target SERs are fixed, the gains g_1 and g_2 are fixed since the SERs are related with the SNRs in Eq. (5). Thus the problem is equivalent to designing the beamformer to minimize the transmit power for fixed beamforming gains. The problem can be formulated as

$$\begin{aligned}(\tilde{\mathbf{f}}, \tilde{\phi_1}, \tilde{\phi_2}) = \underset{\mathbf{f},\phi_1,\phi_2}{\arg\min}\ & \text{Transmit power}, \\ \text{s.t.}\ & \begin{bmatrix}\mathbf{h}_1^T \\ \mathbf{h}_2^T\end{bmatrix} \mathbf{f} = \begin{bmatrix} g_1 e^{j\phi_1} \\ g_2 e^{j\phi_2}\end{bmatrix},\end{aligned} \tag{6}$$

where there exists freedom to adjust the phases ϕ_1 and ϕ_2 to reduce the transmit power when g_1 and g_2 are fixed. This will become clear later.

Problem 2. Minimizing weighted SER for a given transmit power. The second problem is to minimize the weighted SER for a given transmit power, which can be formulated as

$$\begin{aligned}(\tilde{\mathbf{f}}, \tilde{g_1}, \tilde{g_2}, \tilde{\phi_1}, \tilde{\phi_2}) = \underset{\mathbf{f},g_1,g_2,\phi_1,\phi_2}{\arg\min}\ & c_1 P_{e_1} + c_2 P_{e_2}, \\ \text{s.t.}\ & \begin{bmatrix}\mathbf{h}_1^T \\ \mathbf{h}_2^T\end{bmatrix} \mathbf{f} = \begin{bmatrix} g_1 e^{j\phi_1} \\ g_2 e^{j\phi_2}\end{bmatrix}, \\ & \text{Transmit power} = P_{\mathrm{T}},\end{aligned} \tag{7}$$

where c_1 and c_2 are the positive weights for User 1 and User 2. Again, there exists freedom in ϕ_1 and ϕ_2 to decrease the weighted SER when the transmit power is fixed.

III. Proposed Solutions

The solutions for the two problems are introduced in the following two subsections.

A. Minimizing Transmit Power for Given Target SERs

From Eq. (1), the average transmit power can be written as

$$\mathbb{E}\left\{\|\mathbf{f}s\|^2\right\} = \mathrm{tr}(\mathbf{f}\mathbb{E}\left\{|s|^2\right\}\mathbf{f}^H) = \mathbb{E}\left\{|s|^2\right\}\|\mathbf{f}\|^2. \tag{8}$$

Since $\mathbb{E}\{|s|^2\}$ is normalized to a fixed value for different constellation sizes, minimizing the average transmit power is equivalent to minimizing $\|\mathbf{f}\|^2$. Therefore Problem 1 in (6) can be reformulated as

$$\begin{aligned}(\tilde{\mathbf{f}}, \tilde{\phi_1}, \tilde{\phi_2}) = \underset{\mathbf{f},\phi_1,\phi_2}{\arg\min}\ & \|\mathbf{f}\|^2, \\ \text{s.t.}\ & \begin{bmatrix}\mathbf{h}_1^T \\ \mathbf{h}_2^T\end{bmatrix} \mathbf{f} = \begin{bmatrix} g_1 e^{j\phi_1} \\ g_2 e^{j\phi_2}\end{bmatrix}.\end{aligned} \tag{9}$$

Defining $\mathbf{H} = \begin{bmatrix}\mathbf{h}_1^T \\ \mathbf{h}_2^T\end{bmatrix}$, the solution to (9) is provided in the following proposition.

Proposition 1: Given the target SERs for users and thus the corresponding beamforming gains g_1 and g_2 are fixed, the optimal beamformer that minimizes the transmit power is

$$\mathbf{f} = \mathbf{H}^H(\mathbf{H}\mathbf{H}^H)^{-1}\begin{bmatrix} g_1 e^{j\phi_1} \\ g_2 e^{j\phi_2}\end{bmatrix}, \tag{10}$$

where the phases satisfy

$$\phi_1 - \phi_2 = \pi - \mathrm{phase}(\mathbf{q}_2^H \mathbf{q}_1), \tag{11}$$

and $\mathbf{q}_1$ and $\mathbf{q}_2$ are the first and second columns of $\mathbf{H}^H(\mathbf{H}\mathbf{H}^H)^{-1}$.

Proof: See Appendix A. ■

B. Minimizing Weighted SER for Given Transmit Power

Now let us see the second problem in (7). From Eq. (5) and since the term $2\left(1 - \frac{1}{\sqrt{M}}\right)$ is a constant, the problem is equivalent to

$$\begin{aligned}&(\tilde{\mathbf{f}}, \tilde{g}_1, \tilde{g}_2, \tilde{\phi_1}, \tilde{\phi_2}) \\ &= \underset{\mathbf{f},g_1,g_2,\phi_1,\phi_2}{\arg\min}\ c_1 \mathrm{Q}\left(\frac{\mathrm{d}_{\min} g_1}{2\sigma}\right) + c_2 \mathrm{Q}\left(\frac{\mathrm{d}_{\min} g_2}{2\sigma}\right), \\ &\qquad \text{s.t.}\ \begin{bmatrix}\mathbf{h}_1^T \\ \mathbf{h}_2^T\end{bmatrix} \mathbf{f} = \begin{bmatrix} g_1 e^{j\phi_1} \\ g_2 e^{j\phi_2}\end{bmatrix}, \\ &\qquad \text{Transmit power} = P_{\mathrm{T}}.\end{aligned} \tag{12}$$

The following lemma helps in solving the problem:

Lemma 1: The optimal phase setting of Problem 2 in (12) is

$$\phi_1 - \phi_2 = \pi - \mathrm{phase}(\mathbf{q}_2^H \mathbf{q}_1),\ \textit{for}\ \forall g_1, g_2 > 0. \tag{13}$$

Proof: See Appendix B. ■

From Lemma 1, the phases satisfying $\phi_1 - \phi_2 = \pi - \mathrm{phase}(\mathbf{q}_2^H \mathbf{q}_1)$ is the optimal phase solution for Problem 2.

By substituting this result into (12) and noticing that the transmit power depends on $\|\mathbf{f}\|^2$ in Eq. (58), the problem can be reformulated as

$$(\tilde{\mathbf{f}}^{\perp}, \tilde{g}_1, \tilde{g}_2) = \underset{\mathbf{f}^{\perp}, g_1, g_2}{\arg\min}\ c_1\,\mathrm{Q}\left(\frac{\mathrm{d}_{\min} g_1}{2\sigma}\right) + c_2\,\mathrm{Q}\left(\frac{\mathrm{d}_{\min} g_2}{2\sigma}\right),$$
$$\text{s.t. } g_1^2\|\mathbf{q}_1\|^2 + g_2^2\|\mathbf{q}_2\|^2 - 2g_1g_2|\mathbf{q}_2^H\mathbf{q}_1| + \|\mathbf{f}^{\perp}\|^2 = P_{\mathrm{T}}. \tag{14}$$

The solution of gains to (14) meets a specific relationship introduced in the following proposition.

Proposition 2: The optimal solution of g_1 and g_2 in (14) satisfies

$$g_1^2 - g_2^2 = -\frac{2}{\gamma^2}\ln\left(\left(\frac{c_2}{c_1}\right)\frac{g_1\|\mathbf{q}_1\|^2 - g_2|\mathbf{q}_2^H\mathbf{q}_1|}{g_2\|\mathbf{q}_2\|^2 - g_1|\mathbf{q}_2^H\mathbf{q}_1|}\right), \tag{15}$$

where $\gamma = \frac{\mathrm{d}_{\min}}{2\sigma}$.

Proof: See Appendix C. ∎

We would like to find the relationship between g_1 and g_2 from the above equations so that the number of variables can be reduced to one. Substituting this variable into Eq. (65), one can obtain the values of g_1 and g_2 which minimize the weighted SER. It seems difficult to separate g_1 and g_2 in Eqs. (15) and (67). Fortunately, in high and low SNR regimes, we can derive simple closed-form expressions for the relationship between g_1 and g_2. These results are introduced as follows:

1) In High SNR Regime:

Proposition 3: Given a transmit power P_{T}, as the value of SNR approaches ∞, the optimal beamformer that minimizes the weighted SER defined in (14) can be expressed as

$$\mathbf{f} = \mathbf{H}^H(\mathbf{H}\mathbf{H}^H)^{-1}\begin{bmatrix} g_1e^{j\phi_1} \\ g_2e^{j\phi_2}\end{bmatrix}, \tag{16}$$

where the phases satisfy

$$\phi_1 - \phi_2 = \pi - \mathrm{phase}(\mathbf{q}_2^H\mathbf{q}_1), \tag{17}$$

and the gains satisfy

$$g_1 = g_2 = \sqrt{\frac{P_{\mathrm{T}}}{\|\mathbf{q}_1\|^2 + \|\mathbf{q}_2\|^2 - 2|\mathbf{q}_2^H\mathbf{q}_1|}}. \tag{18}$$

Proof: See Appendix D. ∎

2) In Low SNR Regime:

Proposition 4: Given a transmit power P_{T}, as the value of SNR approaches zero, the optimal beamformer that minimizes the weighted SER defined in (14) can be expressed as

$$\mathbf{f} = \mathbf{H}^H(\mathbf{H}\mathbf{H}^H)^{-1}\begin{bmatrix} g_1e^{j\phi_1} \\ g_2e^{j\phi_2}\end{bmatrix}, \tag{19}$$

where the phases satisfy

$$\phi_1 - \phi_2 = \pi - \mathrm{phase}(\mathbf{q}_2^H\mathbf{q}_1), \tag{20}$$

and gains satisfy

$$g_2 = \sqrt{\frac{P_{\mathrm{T}}}{B}} \quad \textit{and } g_1 = \sqrt{\frac{P_{\mathrm{T}}}{C}}, \tag{21}$$

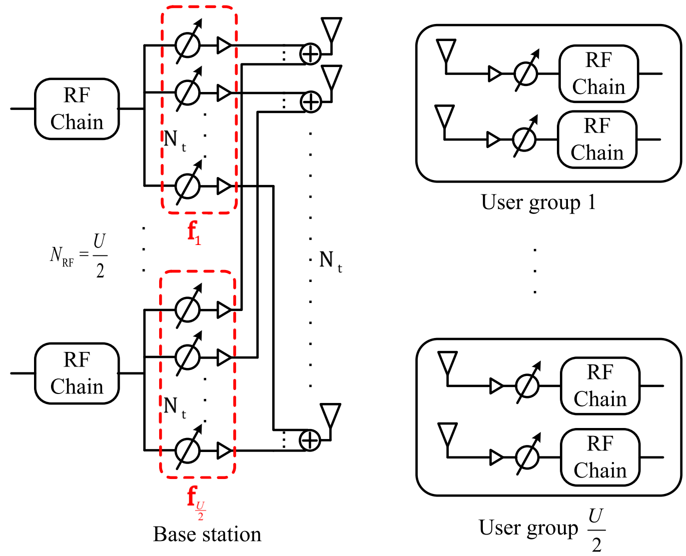

Fig. 2. A block diagram of the proposed system with multiple RF chains.

where

$$B = \left(\frac{c_1\|\mathbf{q}_2\|^2 + c_2|\mathbf{q}_2^H\mathbf{q}_1|}{c_2\|\mathbf{q}_1\|^2 + c_1|\mathbf{q}_2^H\mathbf{q}_1|}\right)^2\|\mathbf{q}_1\|^2 + \|\mathbf{q}_2\|^2 - 2\left(\frac{c_1\|\mathbf{q}_2\|^2 + c_2|\mathbf{q}_2^H\mathbf{q}_1|}{c_2\|\mathbf{q}_1\|^2 + c_1|\mathbf{q}_2^H\mathbf{q}_1|}\right)|\mathbf{q}_2^H\mathbf{q}_1|, \tag{22}$$

and

$$C = \|\mathbf{q}_1\|^2 + \left(\frac{c_2\|\mathbf{q}_1\|^2 + c_1|\mathbf{q}_2^H\mathbf{q}_1|}{c_1\|\mathbf{q}_2\|^2 + c_2|\mathbf{q}_2^H\mathbf{q}_1|}\right)^2\|\mathbf{q}_2\|^2 - 2\left(\frac{c_2\|\mathbf{q}_1\|^2 + c_1|\mathbf{q}_2^H\mathbf{q}_1|}{c_1\|\mathbf{q}_2\|^2 + c_2|\mathbf{q}_2^H\mathbf{q}_1|}\right)|\mathbf{q}_2^H\mathbf{q}_1|. \tag{23}$$

Proof: See Appendix E. ∎

IV. Extension to Multiple RF Chains

In this section, we extend the results to multiple RF chains according to Secs. II and III.

A. System Model for Multiple RF Chains

A block diagram for the proposed system with $2K$ users is shown in Fig. 2, where the base station has N_{t} antennas and N_{RF} RF chains. The system is assumed to transmit N_{s} data streams to $U = 2K$ users. In the proposed system, $N_{\mathrm{t}} \geq N_{\mathrm{s}} = N_{\mathrm{RF}} = \frac{U}{2}$. Each user is equipped with an antenna. Every two users are clustered as a group like that discussed in Secs. II and III. The two users in a group are served by one RF chain and one data stream in the BS as described in Sec. II-B.

Let the beamforming vector for the kth group be $\mathbf{f}_k$. The beamforming matrix for all groups is $\mathbf{F} = [\mathbf{f}_1\ \mathbf{f}_2 \cdots \mathbf{f}_{N_{\mathrm{s}}}] \in \mathbb{C}^{N_{\mathrm{t}} \times N_{\mathrm{s}}}$. The transmit vector for all users can be expressed as

$$\mathbf{x} = \mathbf{F}\mathbf{s}, \tag{24}$$

where $\mathbf{s} = [s_1\ s_2 \cdots s_{N_{\mathrm{s}}}]^T \in \mathbb{C}^{N_{\mathrm{s}} \times 1}$ is the transmit symbol vector, such that $\mathbb{E}[\mathbf{s}\mathbf{s}^H] = \frac{1}{N_{\mathrm{s}}}\mathbf{I}_{N_{\mathrm{s}}}$. If the transmit power for each data stream is P_{T}, the total power constraint normalizes $\mathbf{F}$

such that $\|\mathbf{F}\|_F^2 = N_\mathrm{s} P_\mathrm{T} = \frac{U}{2} P_\mathrm{T}$. At the uth user, the weight w_u is used to process the received signal:

$$y_u = w_u \mathbf{h}_u^T \sum_{n=1}^{N_\mathrm{s}} \mathbf{f}_n s_n + w_u n_u, \tag{25}$$

where $\mathbf{h}_u^T \in \mathbb{C}^{1\times N_\mathrm{t}}$ is the channel vector between the BS and the uth user, and $n_u \sim \mathcal{CN}(0,\sigma^2)$ is the received noise. We propose methods to maintain orthogonality and determine the optimal gain and phase for the groups of users to optimize the performance in the following subsections. To represent the kth group, the superscript (k) is added. For instance, $g_1^{(k)}$ denotes the gain for User 1 in the kth group.

B. Problem Formulation for Multiple RF Chains

Problem 3. Minimizing transmit power for given target SERs (multiple RF chains). For an even number of U, the problem can be formulated as

$$\begin{aligned}&\left(\tilde{\mathbf{G}}, \tilde{\mathbf{F}}, \tilde{\phi}_u^{(k)} \quad u = 1 \text{ and } 2, \quad k = 1,2,\cdots,\frac{U}{2}\right)\\ &\quad = \arg\min \text{ Total transmit power},\\ &\qquad \text{s.t. } \mathbf{HF} = \mathbf{G}.\end{aligned} \tag{26}$$

where $\mathbf{H} = \left[\mathbf{h}_1\ \mathbf{h}_2\ \cdots\ \mathbf{h}_U\right]^T$ and $\mathbf{G}$ is the beamforming gain matrix satisfies

$$\|[\mathbf{G}]_{l,:}\|_0 = 1, \quad l = 1,2,\cdots,U \tag{27}$$

$$\|[\mathbf{G}]_{:,k}\|_0 = 2, \quad k = 1,2,\cdots,\tfrac{U}{2} \tag{28}$$

and $[\mathbf{G}]_{l,k}$ is equal to

$$\begin{cases} 0, & \|[\mathbf{G}]_{l,k}\|_0 = 0, \\ g_1^{(k)} e^{j\phi_1^{(k)}}, & \|[\mathbf{G}]_{l,k}\|_0 = 1 \ \&\ \|[\mathbf{G}]_{l_1,k}\|_0 = 0 \ if\ l_1 < l, \\ g_2^{(k)} e^{j\phi_2^{(k)}}, & otherwise. \end{cases} \tag{29}$$

Eqs. (26) to (29) imply that this problem includes how to group the users and adjust the phases $\phi_u^{(k)}$ to minimize transmit power while gains $g_u^{(k)}$ is given.

Problem 4. Minimizing weighted SER for a given average transmit power (multiple RF chains). For an even number of U, the problem can be written as

$$\begin{aligned}&\left(\tilde{\mathbf{G}}, \tilde{\mathbf{F}}, \tilde{g}_u^{(k)}, \tilde{\phi}_u^{(k)} \quad u = 1 \text{ and } 2, \quad k = 1,2,\cdots,\frac{U}{2}\right)\\ &\quad = \arg\min \sum_{k=1}^{\frac{U}{2}} \sum_{u=1}^{2} c_u^{(k)} \,\mathrm{Q}\left(\frac{\mathrm{d}_{\min} g_u^{(k)}}{2\sigma}\right),\\ &\qquad \text{s.t. } \mathbf{HF} = \mathbf{G},\\ &\qquad\quad \text{Total transmit power} = P_\mathrm{T}.\end{aligned} \tag{30}$$

where $\mathbf{H} = \left[\mathbf{h}_1\ \mathbf{h}_2\ \cdots\ \mathbf{h}_U\right]^T$ and $\mathbf{G}$ is the beamforming gain matrix satisfies Eqs. (27) to (29). Eqs. (27) to (30) imply that this problem includes how to group the users and adjust the gains and phases to minimize the weighted SER. More specifically, Eq. (27) indicates that each user is assigned to only one group, Eq. (28) implies that there are two users in each group and Eq. (29) are the gains and phases that minimize the weighted SER. For example, assume there are four users clustered into two groups. Users 1 and 3 are in Group 1, and Users 2 and 4 are in Group 2. Then the phase and gain matrix should be

$$\mathbf{G} = \begin{bmatrix} g_1^{(1)} e^{j\phi_1^{(1)}} & 0 \\ 0 & g_1^{(2)} e^{j\phi_1^{(2)}} \\ g_2^{(1)} e^{j\phi_2^{(1)}} & 0 \\ 0 & g_2^{(2)} e^{j\phi_2^{(2)}} \end{bmatrix}, \tag{31}$$

and the columns of the pseudo-inverse of $\mathbf{H}$ is

$$\mathbf{H}^H(\mathbf{H}\mathbf{H}^H)^{-1} = \left[\mathbf{q}_1^{(1)}\ \mathbf{q}_1^{(2)}\ \mathbf{q}_2^{(1)}\ \mathbf{q}_2^{(2)}\right]. \tag{32}$$

The number of combinations of user grouping can be calculated and it is given by $\frac{U!}{\frac{U}{2}!2^{\frac{U}{2}}}$.

C. Proposed Solution for Multiple RF Chains

Before solving the problem, we give the general form of $\mathbf{F}$. Any $\mathbf{F}$ satisfying the first constraint of Problem 3 and 4 in (26) and (30) can be written as

$$\mathbf{F} = \mathbf{H}^H(\mathbf{H}\mathbf{H}^H)^{-1}\mathbf{G} + \mathbf{F}^\perp, \tag{33}$$

where each column of $\mathbf{F}^\perp$ is in the null space of $\mathbf{H}$. Then $\mathbf{F}$ can be described as

$$\mathbf{F} = \Big[\underbrace{g_1^{(1)} e^{j\phi_1^{(1)}} \mathbf{q}_1^{(1)} + g_2^{(1)} e^{j\phi_2^{(1)}} \mathbf{q}_2^{(1)}}_{\text{Column 1}} \cdots \underbrace{g_1^{(\frac{U}{2})} e^{j\phi_1^{(\frac{U}{2})}} \mathbf{q}_1^{(\frac{U}{2})} + g_2^{(\frac{U}{2})} e^{j\phi_2^{(\frac{U}{2})}} \mathbf{q}_2^{(\frac{U}{2})}}_{\text{Column } \frac{U}{2}}\Big] + \mathbf{F}^\perp. \tag{34}$$

The transmit power is related to the Frobenius norm of $\mathbf{F}$ explained as follows: From (24), the transmit power is

$$\mathbb{E}\left\{||\mathbf{Fs}||^2\right\} = \mathrm{tr}(\mathbf{F}\mathbb{E}\left\{\mathbf{ss}^H\right\}\mathbf{F}^H). \tag{35}$$

Assume that $\mathbb{E}\left\{\mathbf{ss}^H\right\} = \frac{1}{N_\mathrm{s}}\mathbf{I}_{N_\mathrm{s}}$. The transmit power becomes $\frac{1}{N_\mathrm{s}}\mathrm{tr}(\mathbf{FF}^H)$. Because of $N_\mathrm{s} = \frac{U}{2}$, the transmit power can be written as $\frac{2}{U}\|\mathbf{F}\|_F^2$. The Frobenius norm of $\mathbf{F}$ is given by

$$\begin{aligned}\|\mathbf{F}\|_F^2 = \sum_{k=1}^{\frac{U}{2}} &|g_1^{(k)}|^2 \|\mathbf{q}_1^{(k)}\|^2 + |g_2^{(k)}|^2 \|\mathbf{q}_2^{(k)}\|^2 + \\ &2 g_1^{(k)} g_2^{(k)} \Re\{\mathbf{q}_2^{(k)H} \mathbf{q}_1^{(k)} e^{j(\phi_1^{(k)} - \phi_2^{(k)})}\} + \|\mathbf{F}^\perp\|_F^2.\end{aligned} \tag{36}$$

Proposed solution for Problem 3

From (36), Problem 3 can be reformulated as

$$\begin{aligned}&\left(\tilde{\mathbf{G}}, \tilde{\mathbf{F}}, \tilde{\phi}_u^{(k)} \quad u = 1 \text{ and } 2, \quad k = 1,2,\cdots,\frac{U}{2}\right)\\ &= \arg\min \sum_{k=1}^{\frac{U}{2}} |g_1^{(k)}|^2 \|\mathbf{q}_1^{(k)}\|^2 + |g_2^{(k)}|^2 \|\mathbf{q}_2^{(k)}\|^2\\ &\qquad + 2 g_1^{(k)} g_2^{(k)} \Re\{\mathbf{q}_2^{(k)H} \mathbf{q}_1^{(k)} e^{j(\phi_1^{(k)} - \phi_2^{(k)})}\} + \|\mathbf{F}^\perp\|_F^2\\ &\quad \text{s.t. } \mathbf{HF} = \mathbf{G}.\end{aligned} \tag{37}$$

Proposition 5: Given the target SERs for users and thus the corresponding beamforming gains $g_u^{(k)}$ are fixed, the best beamformer that minimizes the transmit power for each grouping case is similar to Proposition 1 as follows:

$$\mathbf{F} = \Big[g_1^{(1)} e^{j\phi_1^{(1)}} \mathbf{q}_1^{(1)} + g_2^{(1)} e^{j\phi_2^{(1)}} \mathbf{q}_2^{(1)} \cdots \; g_1^{(\frac{U}{2})} e^{j\phi_1^{(\frac{U}{2})}} \mathbf{q}_1^{(\frac{U}{2})} + g_2^{(\frac{U}{2})} e^{j\phi_2^{(\frac{U}{2})}} \mathbf{q}_2^{(\frac{U}{2})}\Big]. \quad (38)$$

where the phases for each group are set according to

$$\phi_1^{(k)} - \phi_2^{(k)} = \pi - \mathrm{phase}({\mathbf{q}_2^{(k)}}^H \mathbf{q}_1^{(k)}), \quad k = 1, 2, \cdots, \frac{U}{2}. \quad (39)$$

Finally, the best grouping can be obtained by searching exhaustively for all $\frac{U!}{\frac{U}{2}!2^{\frac{U}{2}}}$ grouping cases.

Proof: To minimize total transmit power is to minimize the term $2g_1^{(k)} g_2^{(k)} \Re\{{\mathbf{q}_2^{(k)}}^H \mathbf{q}_1^{(k)} e^{j(\phi_1^{(k)} - \phi_2^{(k)})}\}$ for each group. Thus the phase solution in (39) can be obtained by the similar way as Appendix A. ■

Proposed solution for Problem 4

From (36), Problem 4 can be reformulated as

$$\begin{aligned}
&\left(\tilde{\mathbf{G}}, \tilde{\mathbf{F}}^{\perp}, \tilde{g}_u^{(k)}, \tilde{\phi}_u^{(k)} \quad u = 1 \text{ and } 2, \quad k = 1, 2, \cdots, \frac{U}{2}\right) \\
&= \arg\min \sum_{k=1}^{\frac{U}{2}} \sum_{u=1}^{2} c_u^{(k)} \, \mathrm{Q}\left(\frac{\mathrm{d}_{\min} g_u^{(k)}}{2\sigma}\right), \\
&\quad \text{s.t. } \mathbf{HF} = \mathbf{G}, \\
&\quad \sum_{k=1}^{\frac{U}{2}} |g_1^{(k)}|^2 \|\mathbf{q}_1^{(k)}\|^2 + |g_2^{(k)}|^2 \|\mathbf{q}_2^{(k)}\|^2 \\
&\quad + 2g_1^{(k)} g_2^{(k)} \Re\{{\mathbf{q}_2^{(k)}}^H \mathbf{q}_1^{(k)} e^{j(\phi_1^{(k)} - \phi_2^{(k)})}\} \\
&\quad + \|\mathbf{F}^{\perp}\|_F^2 = \frac{U}{2} P_{\mathrm{T}}.
\end{aligned} \quad (40)$$

The phase solution for each group is like the 2-user case described in the following lemma.

Lemma 2: The optimal phase solution for Problem 4 in each group satisfies the following equation:

$$\phi_1^{(k)} - \phi_2^{(k)} = \pi - \mathrm{phase}({\mathbf{q}_2^{(k)}}^H \mathbf{q}_1^{(k)}), \textit{ for } k = 1, 2, \cdots, \frac{U}{2}, \quad \textit{for } \forall g_u^{(k)} > 0, \quad u = 1 \textit{ and } 2. \quad (41)$$

Proof: The proof is similar to that for Lemma 1. Each group conducts similar derivation and the result can be obtained. ■

Substituting the result of Lemma 2 into (40), the second constraint of Problem 4 in (40) can be rewritten as

$$\sum_{k=1}^{\frac{U}{2}} |g_1^{(k)}|^2 \|\mathbf{q}_1^{(k)}\|^2 + |g_2^{(k)}|^2 \|\mathbf{q}_2^{(k)}\|^2 - 2g_1^{(k)} g_2^{(k)} |{\mathbf{q}_2^{(k)}}^H \mathbf{q}_1^{(k)}| + \|\mathbf{F}^{\perp}\|_F^2 = \frac{U}{2} P_{\mathrm{T}}. \quad (42)$$

The gain solution for users in the same group meets a specific relationship introduced in the following lemma:

Lemma 3: The optimal gain solutions of Problem 4 in (30) for the users in the "same" group $g_1^{(k)}$ and $g_2^{(k)}$ satisfy

$$\begin{aligned}
&|g_1^{(k)}|^2 - |g_2^{(k)}|^2 \\
&= -\frac{2}{\gamma^2} \ln\left(\left(\frac{c_2^{(k)}}{c_1^{(k)}}\right) \frac{g_1^{(k)} \|\mathbf{q}_1^{(k)}\|^2 - g_2^{(k)} |{\mathbf{q}_2^{(k)}}^H \mathbf{q}_1^{(k)}|}{g_2^{(k)} \|\mathbf{q}_2^{(k)}\|^2 - g_1^{(k)} |{\mathbf{q}_2^{(k)}}^H \mathbf{q}_1^{(k)}|}\right), \\
&k = 1, \cdots, \frac{U}{2},
\end{aligned} \quad (43)$$

where $\gamma = \frac{d_{\min}}{2\sigma}$.

Proof: From (40) and (42) and using a similar procedure as that in the two-user case by forming the Lagrange, taking the partial derivatives and set them to zeros, the results in (43) can be obtained. ■

The gain solutions for the users in the "different" groups of the problem in (30) is introduced in the following proposition.

Proposition 6: The optimal gain solutions of (30) for User 1 in Groups i and j, i.e., *$g_1^{(i)}$ and $g_1^{(j)}$, satisfy*

$$\begin{aligned}
&|g_1^{(i)}|^2 - |g_1^{(j)}|^2 \\
&= -\frac{2}{\gamma^2} \ln\left(\left(\frac{c_1^{(j)}}{c_1^{(i)}}\right) \frac{g_1^{(i)} \|\mathbf{q}_1^{(i)}\|^2 - g_2^{(i)} |{\mathbf{q}_2^{(i)}}^H \mathbf{q}_1^{(i)}|}{g_1^{(j)} \|\mathbf{q}_1^{(j)}\|^2 - g_2^{(j)} |{\mathbf{q}_2^{(j)}}^H \mathbf{q}_1^{(j)}|}\right), \\
&i, j = 1, \cdots, \frac{U}{2},
\end{aligned} \quad (44)$$

and the gain solutions for User 2 in Groups i and j, i.e., *$g_2^{(i)}$ and $g_2^{(j)}$, satisfy*

$$\begin{aligned}
&|g_2^{(i)}|^2 - |g_2^{(j)}|^2 \\
&= -\frac{2}{\gamma^2} \ln\left(\left(\frac{c_2^{(j)}}{c_2^{(i)}}\right) \frac{g_2^{(i)} \|\mathbf{q}_2^{(i)}\|^2 - g_1^{(i)} |{\mathbf{q}_2^{(i)}}^H \mathbf{q}_1^{(i)}|}{g_2^{(j)} \|\mathbf{q}_2^{(j)}\|^2 - g_1^{(j)} |{\mathbf{q}_2^{(j)}}^H \mathbf{q}_1^{(j)}|}\right), \\
&i, j = 1, \cdots, \frac{U}{2}.
\end{aligned} \quad (45)$$

Proof: See Appendix F. ■

Again, one can find simple closed-form solutions for Problem 4 using Lemma 3 and Proposition 6 in the high and the low SNR regimes as follows:

1) In High SNR Regime:

Lemma 4: As $\gamma \to \infty$, equal gain allocation is the optimal solution for the users in the "same" group, i.e.,

$$g_1^{(k)} = g_2^{(k)}, \quad k = 1, 2, \cdots, \frac{U}{2}. \quad (46)$$

Proof: See Appendix G ■

Lemma 5: As $\gamma \to \infty$, equal gain allocation is the optimal solution for the users in "different" groups, i.e.,

$$g_1^{(i)} = g_1^{(j)}, \quad i, j = 1, 2, \cdots, \frac{U}{2}. \quad (47)$$

Proof: See Appendix H ■

Proposition 7: Let the total transmit power be P_{T}. As $\gamma \to \infty$, the optimal beamformer that minimizes the weighted SER has the following format:

$$\mathbf{F} = \Big[g_1^{(1)} e^{j\phi_1^{(1)}} \mathbf{q}_1^{(1)} + g_2^{(1)} e^{j\phi_2^{(1)}} \mathbf{q}_2^{(1)} \cdots g_1^{(\frac{U}{2})} e^{j\phi_1^{(\frac{U}{2})}} \mathbf{q}_1^{(\frac{U}{2})} + g_2^{(\frac{U}{2})} e^{j\phi_2^{(\frac{U}{2})}} \mathbf{q}_2^{(\frac{U}{2})}\Big], \quad (48)$$

where the beamforming gains are equal described as

$$g_1^{(1)} = g_2^{(1)} = \cdots = g_1^{(\frac{U}{2})} = g_2^{(\frac{U}{2})} = \sqrt{\frac{\frac{U}{2}P_{\mathrm{T}}}{\sum_{k=1}^{\frac{U}{2}} \|\mathbf{q}_1^{(k)}\|^2 + \|\mathbf{q}_2^{(k)}\|^2 - 2|\mathbf{q}_2^{(k)H}\mathbf{q}_1^{(k)}|}}, \tag{49}$$

the phases for each group are set according to

$$\phi_1^{(k)} - \phi_2^{(k)} = \pi - \mathrm{phase}(\mathbf{q}_2^{(k)H}\mathbf{q}_1^{(k)}), \quad k = 1, 2, \cdots, \frac{U}{2}. \tag{50}$$

Finally, the best grouping can be obtained by searching exhaustively for all $\frac{U!}{\frac{U}{2}!2^{\frac{U}{2}}}$ *grouping cases.*

Proof: This is a direct result using a similar argument for the 2-user case as well as Lemma 4 and Lemma 5. ■

The proposed beamforming algorithm is summarized in Algorithm 1 (The notations are defined in Definition 1).

Definition 1: Symbols for Algorithm

$\mathbf{G}_i$*: the gain and phase matrix for the* i*th user grouping.* $\mathbf{F}_i$*: the best beamforming matrix in the* i*th user grouping.* P_i*: the weighted SER for the* i*th user grouping.*

Algorithm 1 Weighted SER Minimizing Beamforming Algorithm for High SNR Regime

Input: The number of users U; Total transmit power P_{T}; Channels of all users $\mathbf{h}_1^T, \mathbf{h}_2^T, \cdots, \mathbf{h}_U^T$; The noise standard deviation σ; The minimum distance of constellation $d_{\min}$; The weights for users $c_1^{(1)}, c_2^{(1)}, \cdots, c_1^{(\frac{U}{2})}, c_2^{(\frac{U}{2})}$; All the grouping cases $\mathbf{G}_1, \mathbf{G}_2, \cdots, \mathbf{G}_{\frac{U!}{\frac{U}{2}!2^{\frac{U}{2}}}}$;

Output: The optimal beamformer $\mathbf{F}$ that minimizes the weighted SER

1: **for** $i \le \frac{U!}{\frac{U}{2}!2^{\frac{U}{2}}}$ **do**
2: Set $\mathbf{q}_1^{(1)}, \mathbf{q}_2^{(1)}, \cdots, \mathbf{q}_2^{(\frac{U}{2})}$ according to grouping case $\mathbf{G}_i$ and channels $\mathbf{h}_1^T, \mathbf{h}_1^T, \cdots, \mathbf{h}_U^T$.
3: Set the gains and phases for $\mathbf{G}_i$ using Eqs. (49) and (50).
4: Set $\mathbf{F}_i$ using Eq. (48).
5: Calculate P_i using the weighted SER in (30).
6: **end for**
7: $m = \mathrm{argmin}_{i=1,\cdots,\frac{U!}{\frac{U}{2}!2^{\frac{U}{2}}}} P_i$
8: $\mathbf{F} = \mathbf{F}_m$
9: **return** $\mathbf{F}$;

2) In Low SNR Regime:

Lemma 6: As $\gamma \to 0$*, the optimal gain solutions for the users in "the same" group satisfy*

$$g_1^{(k)} = g_2^{(k)} \left(\frac{c_1^{(k)}\|\mathbf{q}_2^{(k)}\|^2 + c_2^{(k)}|\mathbf{q}_2^{(k)H}\mathbf{q}_1^{(k)}|}{c_2^{(k)}\|\mathbf{q}_1^{(k)}\|^2 + c_1^{(k)}|\mathbf{q}_2^{(k)H}\mathbf{q}_1^{(k)}|} \right), \quad k = 1, 2, \cdots, \frac{U}{2}. \tag{51}$$

Proof:

See Appendix I ■

For description simplicity, the following notations are defined according to Lemma 6.

Definition 2: Best gain ratios for users in the same group
$r_{21}^{(k)}$*: the ratio* $\frac{g_1^{(k)}}{g_2^{(k)}}$*, denoted by* $\left(\frac{c_1^{(k)}\|\mathbf{q}_2^{(k)}\|^2 + c_2^{(k)}|\mathbf{q}_2^{(k)H}\mathbf{q}_1^{(k)}|}{c_2^{(k)}\|\mathbf{q}_1^{(k)}\|^2 + c_1^{(k)}|\mathbf{q}_2^{(k)H}\mathbf{q}_1^{(k)}|}\right)$.
$r_{12}^{(k)}$*: the ratio* $\frac{g_2^{(k)}}{g_1^{(k)}}$*, which is* $1/r_{21}^{(k)}$.

Proposition 8: As $\gamma \to 0$*, the optimal gain solutions for the users in "different" groups satisfy*

$$g_1^{(i)} = g_1^{(j)} \left(\frac{c_1^{(i)}(\|\mathbf{q}_1^{(j)}\|^2 - r_{12}^{(j)}|\mathbf{q}_2^{(j)H}\mathbf{q}_1^{(j)}|)}{c_1^{(j)}(\|\mathbf{q}_1^{(i)}\|^2 - r_{12}^{(i)}|\mathbf{q}_2^{(i)H}\mathbf{q}_1^{(i)}|)} \right), \quad i, j = 1, 2, \cdots, \frac{U}{2}. \tag{52}$$

Proof: See Appendix J ■

For description simplicity, the following notations are defined according to Proposition 8.

Definition 3: Best gain ratios for users in different groups
$r_1^{(j,i)}$*: the ratio* $\frac{g_1^{(i)}}{g_1^{(j)}}$*, denoted by* $\left(\frac{c_1^{(i)}(\|\mathbf{q}_1^{(j)}\|^2 - r_{12}^{(j)}|\mathbf{q}_2^{(j)H}\mathbf{q}_1^{(j)}|)}{c_1^{(j)}(\|\mathbf{q}_1^{(i)}\|^2 - r_{12}^{(i)}|\mathbf{q}_2^{(i)H}\mathbf{q}_1^{(i)}|)}\right)$.
$r_2^{(j,i)}$*: the ratio* $\frac{g_2^{(i)}}{g_2^{(j)}}$*, denoted by* $\left(\frac{c_2^{(i)}(\|\mathbf{q}_2^{(j)}\|^2 - r_{21}^{(j)}|\mathbf{q}_2^{(j)H}\mathbf{q}_1^{(j)}|)}{c_2^{(j)}(\|\mathbf{q}_2^{(i)}\|^2 - r_{21}^{(i)}|\mathbf{q}_2^{(i)H}\mathbf{q}_1^{(i)}|)}\right)$.

Corollary 1: The ratios $r_1^{(j,i)}$ *and* $r_2^{(j,i)}$ *are positive. Hence the proposed solutions are legal.*

Proof: See Appendix K ■

Proposition 9: Let the total transmit power be P_{T}*. As* $\gamma \to 0$*, the optimal beamformer that minimizes the weighted SER has the following format in Eq. (48), where the gains are given by*

$$g_1^{(k)} = \sqrt{\frac{\frac{U}{2}P_{\mathrm{T}}}{E}}, \quad \text{and } g_2^{(k)} = \sqrt{\frac{\frac{U}{2}P_{\mathrm{T}}}{F}}, \tag{53}$$

$$E = \sum_{i=1}^{\frac{U}{2}} |r_1^{(k,i)}|^2\|\mathbf{q}_1^{(i)}\|^2 + |r_1^{(k,i)}|^2|r_{12}^{(i)}|^2\|\mathbf{q}_2^{(i)}\|^2 - 2|r_1^{(k,i)}|^2|r_{12}^{(i)}||\mathbf{q}_2^{(i)H}\mathbf{q}_1^{(i)}|, \tag{54}$$

$$F = \sum_{i=1}^{\frac{U}{2}} |r_2^{(k,i)}|^2|r_{21}^{(i)}|^2\|\mathbf{q}_1^{(i)}\|^2 + |r_2^{(k,i)}|^2\|\mathbf{q}_2^{(i)}\|^2 - 2|r_2^{(k,i)}|^2|r_{21}^{(i)}||\mathbf{q}_2^{(i)H}\mathbf{q}_1^{(i)}|, \tag{55}$$

the phases for each group are set as

$$\phi_1^{(k)} - \phi_2^{(k)} = \pi - \mathrm{phase}(\mathbf{q}_2^{(k)H}\mathbf{q}_1^{(k)}), \quad k = 1, 2, \cdots, \frac{U}{2}. \tag{56}$$

The best solution can be determined by searching exhaustively for all $\frac{U!}{\frac{U}{2}!2^{\frac{U}{2}}}$ *grouping cases.*

Proof: See Appendix L ■

The proposed beamforming solution for Problem 4 in the low SNR regime is summarized in Algorithm 2.

V. Simulation Results

In this section, simulation results are provided to show the performance of the proposed system and algorithms. The mmwave channel model in [15] is used. The channel of User u is given by $\mathbf{h}_u^T = \sqrt{\frac{N_{\mathrm{t}}}{L}}\sum_{l=1}^{L}\alpha_l\mathbf{a}_{\mathrm{t}}(\theta_l)^H$, where L is the number of propagation paths, α_l is the complex gain

Algorithm 2 Weighted SER Minimizing Beamforming Algorithm for Low SNR Regime

Input: Same input as those in Algorithm 1.
Output: The optimal beamformer that minimizes the weighted SER $\mathbf{F}$

for $i \leq \frac{U!}{\frac{U}{2}!2^{\frac{U}{2}}}$ **do**
 Set $\mathbf{q}_1^{(1)}, \mathbf{q}_2^{(1)}, \cdots, \mathbf{q}_2^{(\frac{U}{2})}$ according to grouping case $\mathbf{G}_i$ and channels $\mathbf{h}_1^T, \mathbf{h}_1^T, \cdots, \mathbf{h}_U^T$.
 Set the best gains using Definitions 2 and 3.
 Set the gain and phase matrix $\mathbf{G}_i$ using Eqs. (53) and (56)
 Set $\mathbf{F}_i$ using Proposition 9.
 Calculate P_i by the weighted SER in Eq. (30).
end for
$m = \text{argmin}_{i=1,\cdots,\frac{U!}{\frac{U}{2}!2^{\frac{U}{2}}}} P_i$
$\mathbf{F} = \mathbf{F}_m$
return $\mathbf{F}$;

of the lth path, and θ_l is its angle of departure and it is assumed to be uniformly distributed. The complex gains α_l are assumed to be $\alpha_l \sim \mathcal{CN}(0,1)$. The vector $\mathbf{a}_\mathrm{t}(\theta_l)$ is the normalized transmit array response vectors at the corresponding angles of departure. For an N_t-element uniform linear array (ULA) on the y-axis, the array response vector $\mathbf{a}_\mathrm{t}(\theta_l)$ is $\frac{1}{\sqrt{N_\mathrm{t}}}\left[1, e^{j\frac{2\pi}{\lambda}d\sin(\theta_l)}, \cdots, e^{j(N_\mathrm{t}-1)\frac{2\pi}{\lambda}d\sin(\theta_l)}\right]^T$, where λ is the wavelength of mmWave frequency and d is the inter-element spacing. Assume that the channel is known by the base station [9]–[11]. In the simulation, the mmWave channel has 6 clusters for each user.

The sector angle of the transmitter is assumed to be $60^\circ-$wide. The inter-element spacing d of the antenna array is half-wavelength. $4-$QAM symbol for each stream is used and thus each user needs to detect a BPSK symbol. The weight for the SER of each user is assumed to be reciprocal of the number of users. Hence the weighted SER is reduced to the average SER. In each experiment, the number of realizations is 10^6.

A. Experiment 1: Performance Comparison of Proposed and Zero-Forcing Schemes

In this experiment, the performance of the proposed and the conventional zero-forcing beamforming (ZFBF) is compared. For the ZFBF, the beamforming matrix is $\mathbf{H}^H(\mathbf{H}\mathbf{H}^H)^{-1}$, where U RF chains are required and each user transmits BPSK symbols. On the other hand, the proposed scheme uses $\frac{U}{2}$ RF chains and a 4-QAM constellation is adopted for every two users. It is worth emphasizing that the requirement of RF chains of the ZFBF is higher than the proposed scheme since more RF chains are needed.

Let us see how the performance is affected by the different number of transmit antennas first, *i.e.,* $N_\mathrm{t} = 8$ and 16. Let $U = 2$. Fig. 3 shows the performance comparison in the low SNR regime while Fig. 4 shows that in the high SNR regime. Observing from the figures, the proposed schemes achieve comparable performance with the ZFBF when $N_t = 8$. The advantage of the ZFBF becomes pronounced when the number of antennas increases. When $N_t = 16$, the ZFBF outperforms the proposed schemes by around 2 dB.

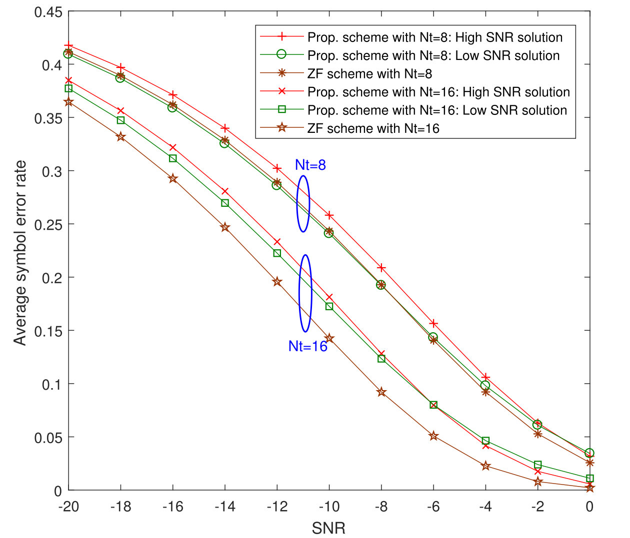

Fig. 3. Performance comparison between the proposed and ZFBF for various number of antennas in low SNR region.

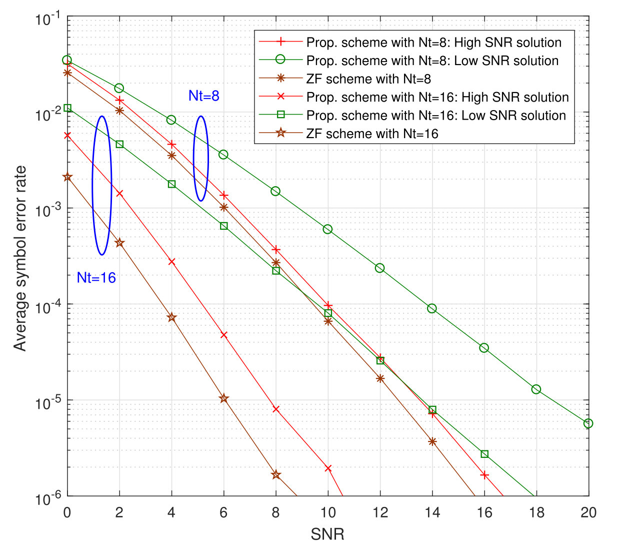

Fig. 4. Performance comparison between the proposed and ZFBF for various number of antennas in high SNR region.

Now let us evaluate how performance is affected when the number U of users increases, *i.e.,* increasing the value of U from 2 to 4. Let $N_\mathrm{t} = 8$. Fig. 5 and Fig. 6 show the performance comparison in the low and high SNR regimes, respectively. Observe that the proposed scheme has an advantage when the number of users increases. More specifically, when the value of U increases from 2 to 4, the performance of the proposed scheme outperforms the ZFBF up to 2.5 dB in both the low and high SNR regimes.

From the above discussion, the performance improvement of the proposed scheme is more pronounced when the number of users increases and the number of antennas decreases. To see this, let $N_\mathrm{t} = 6$ and $U = 4$, Fig. 7 shows the corresponding performance comparison. The proposed algorithm

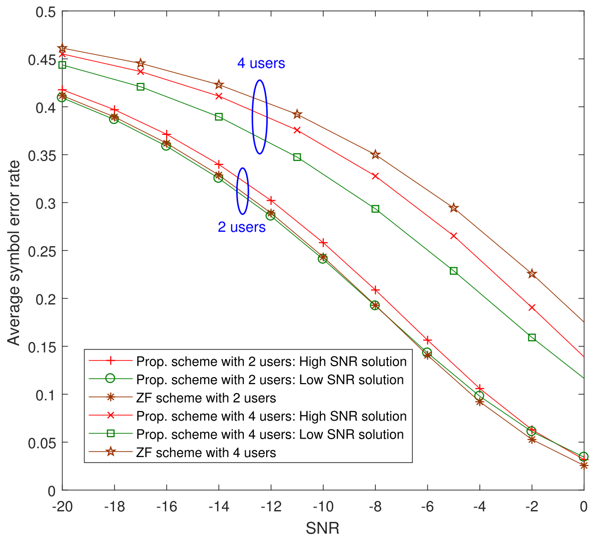

Fig. 5. Performance comparison between the proposed and ZFBF for various number of users in low SNR region.

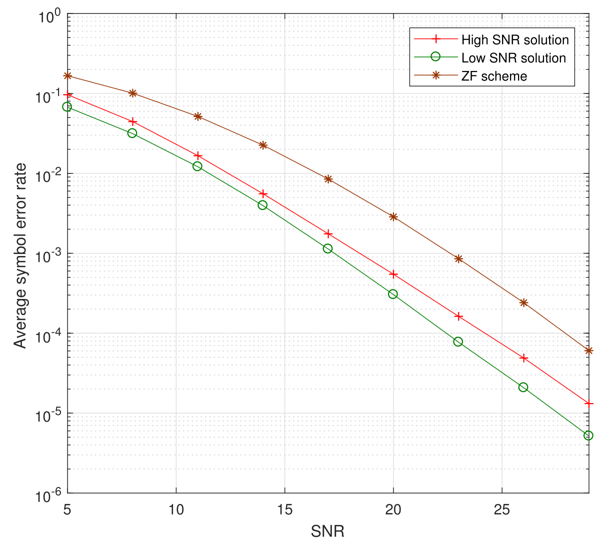

Fig. 7. Performance comparison between the proposed and ZFBF for $N_{\mathrm{t}} = 6$ and $U = 4$.

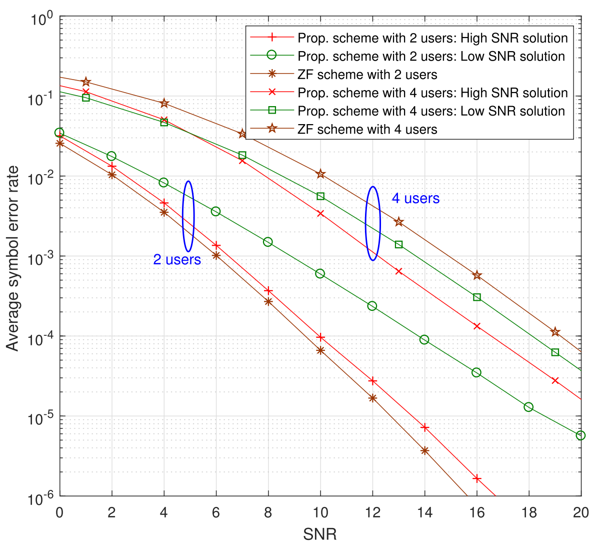

Fig. 6. Performance comparison between the proposed and ZFBF for various number of users in high SNR region.

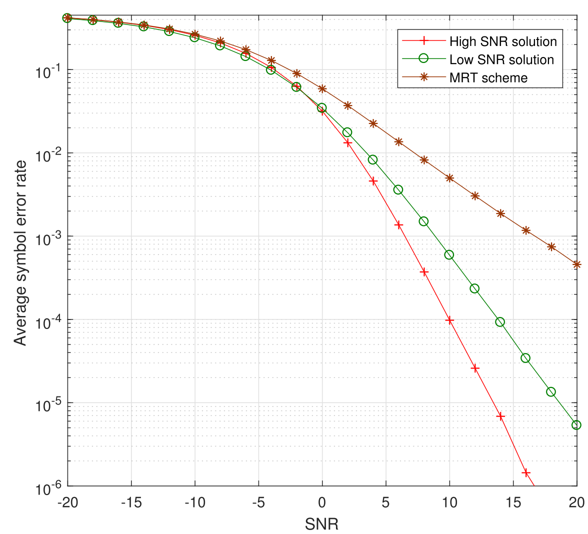

Fig. 8. Performance comparison between the proposed and MRT schemes for $N_{\mathrm{t}} = 8$ and $U = 2$.

for low SNR regime performs the best in this case and it outperforms the ZFBF more than 5 dB. This example demonstrates the advantage of the proposed scheme. More specifically, in several parameter settings, the proposed scheme can achieve better performance than the ZFBF; at the same time, the corresponding hardware cost is lower than the ZFBF because fewer RF chains are required.

B. Experiment 2: Performance Comparison Between the Proposed Solution and MRT Schemes

To show the advantage of the proposed weighted SER minimizing algorithm in Algorithms 1 and 2, we conduct a simulation that applies the MRT (maximum ratio transmission) in the proposed system. Here, the MRT uses the same number of RF chains and constellation design as for the proposed scheme. The MRT beamformer is designed as $\frac{(\mathbf{h}_1^T+\mathbf{h}_2^T)^H}{\|\mathbf{h}_1^T+\mathbf{h}_2^T\|}$. Let $N_{\mathrm{t}} = 8$ and $U = 2$, Fig. 8 shows the corresponding performance comparison. Observing from the figure, the proposed weighted SER algorithm greatly outperforms the MRT beamformer. This shows the fact that the performance advantage of the proposed scheme mainly comes from the well designed SER minimizing algorithm.

C. Experiment 3: The Effect of User Grouping

In this experiment, we demonstrate how the user grouping affects the performance when there are multiple RF chains. Let $U = 4$ and $N_{\mathrm{t}} = 8$. Figs. 9 and 10 show the performance for low and high SNR regimes, respectively, where the curve without user grouping is to let Users 1 and 2 in the same group and thus Users 3 and 4 are always in the same group. Observe that the performance improvement with user grouping is more pronounced in the high SNR region.

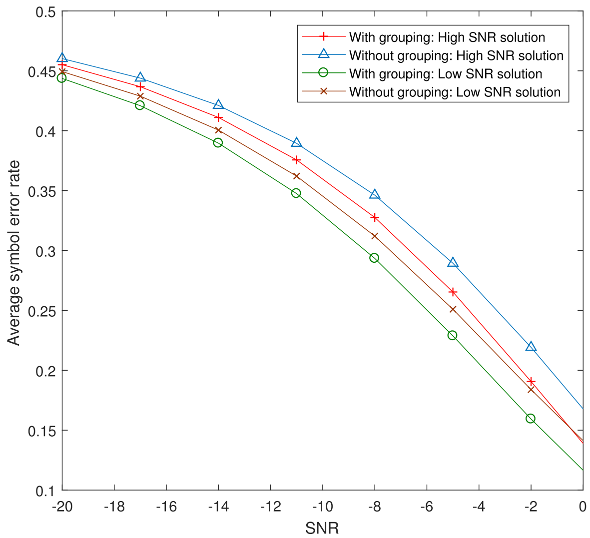

Fig. 9. The effect of grouping in low SNR region with $N_\mathrm{t} = 8$ and $U = 4$.

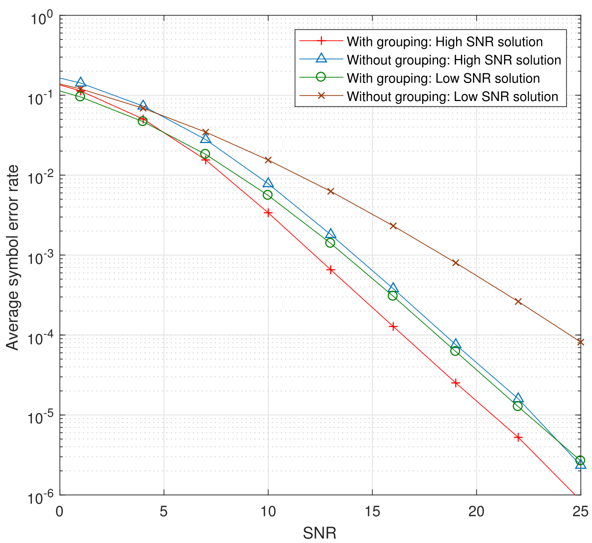

Fig. 10. The effect of grouping in high SNR region with $N_\mathrm{t} = 8$ and $U = 4$.

D. Experiment 4: Performance Comparison With the Quantization Effect

In practical systems, the gain and phase of each antenna element in the antenna array have finite resolution. That is, quantization effect should be considered. In this experiment, we compare the performance of the proposed and the ZFBF schemes with the quantization effect. The numbers of quantization bits used for gain and phase for each antenna are $B = 3$ and 4. Thus each antenna has $2B = 6$ and 8 quantization bits. Let $U = 2$ and $N_\mathrm{t} = 8$. Figs. 11 and 12 show the performance comparison with the quantization effect for $B = 3$ and $B = 4$, respectively. Observe that the proposed scheme is more robust than the ZFBF with the quantization effect. More specifically, when $B = 3$, the proposed scheme greatly outperforms the ZFBF (see the triangular and cross curves in 11). As the value of B increases to 4, the performance gap between the two schemes becomes smaller than that for $B = 3$ from Fig. 12.

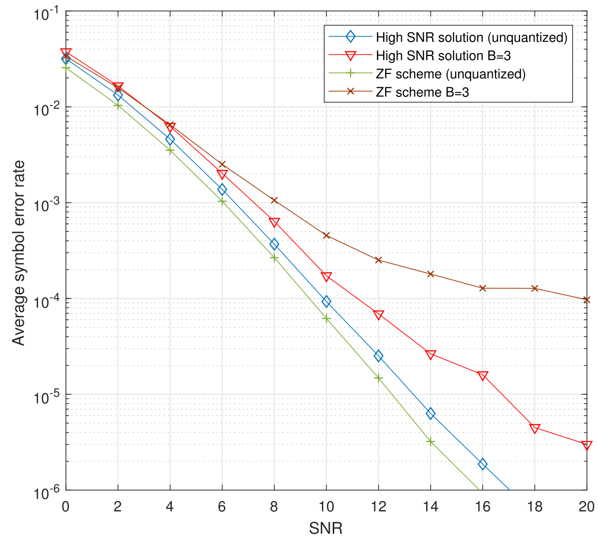

Fig. 11. Performance comparison with quantized effect for $B = 3$ with $N_\mathrm{t} = 8$ and $U = 2$.

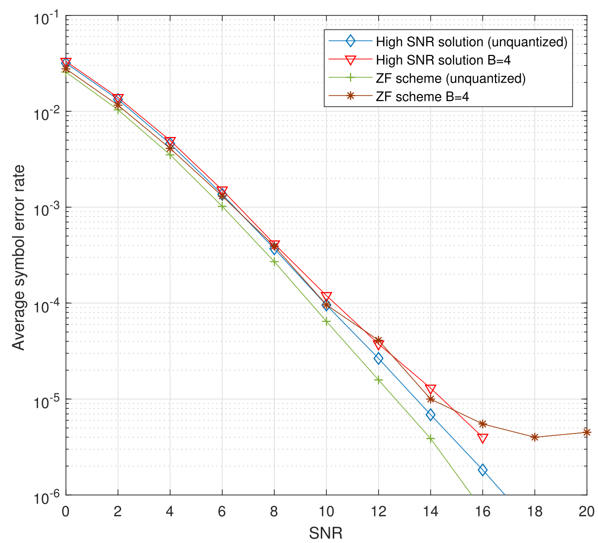

Fig. 12. Performance comparison with quantized effect for $B = 4$ with $N_\mathrm{t} = 8$ and $U = 2$.

VI. Conclusion

We have proposed a new multiuser beamforming system that can reduce the number of RF chains by half compared to the conventional zero-forcing beamforming scheme. Based on the recently developed new antenna array, in which gain and phase of each antenna element is adjustable, we have designed beamforming patterns for the proposed system that can minimize the transmit power and the weighted SERs, separately. Simulation results have shown that the proposed scheme outperforms the conventional zero-forcing beamforming scheme; while the required number of RF chains is lower than the zero-forcing scheme thanks to the reduction of the RF chains. Considering practical implementations with the quantization effect, the proposed system has shown to be more robust to this effect than the zero-forcing scheme. The superior advantages in the performance and hardware cost make the proposed scheme a potential candidate for the beamforming solutions of future communications systems.

Appendix A
Proof of Proposition 1

Proof: Any solution which satisfies $\mathbf{H}\mathbf{f} = \begin{bmatrix} g_1 e^{j\phi_1} \\ g_2 e^{j\phi_2} \end{bmatrix}$ can be expressed as

$$\begin{aligned} \mathbf{f} &= \mathbf{H}^H(\mathbf{H}\mathbf{H}^H)^{-1} \begin{bmatrix} g_1 e^{j\phi_1} \\ g_2 e^{j\phi_2} \end{bmatrix} + \mathbf{f}^\perp \\ &= \begin{bmatrix} \mathbf{q}_1 & \mathbf{q}_2 \end{bmatrix} \begin{bmatrix} g_1 e^{j\phi_1} \\ g_2 e^{j\phi_2} \end{bmatrix} + \mathbf{f}^\perp \\ &= g_1 e^{j\phi_1}\mathbf{q}_1 + g_2 e^{j\phi_2}\mathbf{q}_2 + \mathbf{f}^\perp. \end{aligned} \tag{57}$$

where $\mathbf{f}^\perp$ denotes any vector in the null space of $\mathbf{H}$. Since $\mathrm{col}(\mathbf{H}^H)$ and $\mathrm{null}(\mathbf{H})$ are orthogonal complements, the 2-norm of $\mathbf{f}$ can be written as

$$\begin{aligned} \|\mathbf{f}\|^2 = g_1^2\|\mathbf{q}_1\|^2 + g_2^2\|\mathbf{q}_2\|^2 \\ +2g_1g_2\Re\{\mathbf{q}_2^H\mathbf{q}_1 e^{j(\phi_1-\phi_2)}\} + \|\mathbf{f}^\perp\|^2. \end{aligned} \tag{58}$$

Because the SERs for users are fixed, the gains g_1 and g_2 are also fixed. The adjustable parameters are ϕ_1, ϕ_2 and $\mathbf{f}^\perp$. From Eqs. (9) and (58), Problem 1 can be solved by setting $\mathbf{f}^\perp = \mathbf{0}$ and the phase as that in (11), which completes the proof. ■

Appendix B
Proof of Lemma 1

Proof: Assume the optimal gains are g_1 and g_2, and g_1 and g_2 forms a ratio defined by $g_r = \frac{g_2}{g_1}$, where $g_r > 0$. We prove that $\phi_1 - \phi_2 = \pi - \mathrm{phase}(\mathbf{q}_2^H\mathbf{q}_1)$ is the optimal phase setting for arbitrary combinations of g_1 and g_2 as follows. Assume any gain ratio $g_r > 0$ is given. From Eq. (58), let the transmit power be P_T, *i.e.,*

$$g_1^2\|\mathbf{q}_1\|^2+g_2^2\|\mathbf{q}_2\|^2+2g_1g_2\Re\{\mathbf{q}_2^H\mathbf{q}_1 e^{j(\phi_1-\phi_2)}\}+\|\mathbf{f}^\perp\|^2 = P_\mathrm{T}. \tag{59}$$

Substitute $g_2 = g_1 g_r$ into Eq. (59) and rearrange the equation,

$$\begin{aligned} &\Longrightarrow g_1^2(\|\mathbf{q}_1\|^2 + g_r^2\|\mathbf{q}_2\|^2 + 2g_r\Re\{\mathbf{q}_2^H\mathbf{q}_1 e^{j(\phi_1-\phi_2)}\}) \\ &\ = \ P_\mathrm{T} - \|\mathbf{f}^\perp\|^2, \\ &\Longrightarrow g_1 = \sqrt{\frac{P_\mathrm{T} - \|\mathbf{f}^\perp\|^2}{\|\mathbf{q}_1\|^2 + g_r^2\|\mathbf{q}_2\|^2 + 2g_r\Re\{\mathbf{q_2}^H\mathbf{q}_1 e^{j(\phi_1-\phi_2)}\}}}. \end{aligned} \tag{60}$$

Note that the largest value of g_1 leads to the largest value of g_2 due to the fixed gain ratio. From (60), the largest values of g_1 and g_2 occur when $\mathbf{f}^\perp = \mathbf{0}$ and $\phi_1 - \phi_2 = \pi - \mathrm{phase}(\mathbf{q}_2^H\mathbf{q}_1)$. Because the Q function is strictly decreasing, the largest gains guarantee the lowest value of the weighted SER. ■

Appendix C
Proof of Proposition 2

Proof: To decide the gains g_1 and g_2 in this constrained optimization problem, defining the Lagrange multiplier:

$$\begin{aligned} L(g_1, g_2, \lambda) &= c_1\,\mathrm{Q}\left(\frac{\mathrm{d_{min}}g_1}{2\sigma}\right) + c_2\,\mathrm{Q}\left(\frac{\mathrm{d_{min}}g_2}{2\sigma}\right) \\ &\quad +\lambda(g_1^2\|\mathbf{q}_1\|^2 + g_2^2\|\mathbf{q}_2\|^2 - 2g_1g_2|\mathbf{q}_2^H\mathbf{q}_1| \\ &\quad +\|\mathbf{f}^\perp\|^2 - P_\mathrm{T}). \end{aligned} \tag{61}$$

Let the partial derivatives of $L(g_1, g_2, \lambda)$ for all variables be 0 [35]. We have

$$\frac{\partial L(g_1,g_2,\lambda)}{\partial g_1} = 0, \quad \frac{\partial L(g_1,g_2,\lambda)}{\partial g_2} = 0, \quad \frac{\partial L(g_1,g_2,\lambda)}{\partial \lambda} = 0. \tag{62}$$

From (62), we obtain

$$-c_1\frac{\gamma}{\sqrt{2\pi}}e^{-\frac{\gamma^2 g_1^2}{2}} + \lambda(2g_1\|\mathbf{q}_1\|^2 - 2g_2|\mathbf{q}_2^H\mathbf{q}_1|) = 0, \tag{63}$$

$$-c_2\frac{\gamma}{\sqrt{2\pi}}e^{-\frac{\gamma^2 g_2^2}{2}} + \lambda(2g_2\|\mathbf{q}_2\|^2 - 2g_1|\mathbf{q}_2^H\mathbf{q}_1|) = 0, \tag{64}$$

$$g_1^2\|\mathbf{q}_1\|^2 + g_2^2\|\mathbf{q}_2\|^2 - 2g_1g_2|\mathbf{q}_2^H\mathbf{q}_1| + \|\mathbf{f}^\perp\|^2 - P_\mathrm{T} = 0. \tag{65}$$

From (63) and (64), λ can be described by

$$\lambda = \frac{c_1\frac{\gamma}{\sqrt{2\pi}}e^{-\frac{\gamma^2 g_1^2}{2}}}{2g_1\|\mathbf{q}_1\|^2 - 2g_2|\mathbf{q}_2^H\mathbf{q}_1|} = \frac{c_2\frac{\gamma}{\sqrt{2\pi}}e^{-\frac{\gamma^2 g_2^2}{2}}}{2g_2\|\mathbf{q}_2\|^2 - 2g_1|\mathbf{q}_2^H\mathbf{q}_1|}. \tag{66}$$

(66) can be rearranged as

$$\begin{aligned} \frac{c_1 e^{-\frac{\gamma^2 g_1^2}{2}}}{c_2 e^{-\frac{\gamma^2 g_2^2}{2}}} &= \frac{g_1\|\mathbf{q}_1\|^2 - g_2|\mathbf{q}_2^H\mathbf{q}_1|}{g_2\|\mathbf{q}_2\|^2 - g_1|\mathbf{q}_2^H\mathbf{q}_1|}, \\ \Longrightarrow e^{-\frac{\gamma^2(g_1^2-g_2^2)}{2}} &= \left(\frac{c_2}{c_1}\right)\frac{g_1\|\mathbf{q}_1\|^2 - g_2|\mathbf{q}_2^H\mathbf{q}_1|}{g_2\|\mathbf{q}_2\|^2 - g_1|\mathbf{q}_2^H\mathbf{q}_1|}, \end{aligned} \tag{67}$$

Eq. (67) leads to the result in (15) ■

Appendix D
Proof of Proposition 3

Proof: As $\gamma \to \infty$, the RHS of Eq. (15) approximates to 0 as

$$g_1^2 - g_2^2 \approx 0. \tag{68}$$

This implies equal gain allocation minimizes the weighted SER. Substituting $g_1 = g_2$ into Eq. (65), the constraint can be rewritten as

$$g_2^2(\|\mathbf{q}_1\|^2 + \|\mathbf{q}_2\|^2 - 2|\mathbf{q}_2^H\mathbf{q}_1|) = P_\mathrm{T} - \|\mathbf{f}^\perp\|^2. \tag{69}$$

Rearranging Eq. (69), the gains can be expressed by

$$\Longrightarrow g_1 = g_2 = \sqrt{\frac{P_\mathrm{T} - \|\mathbf{f}^\perp\|^2}{\|\mathbf{q}_1\|^2 + \|\mathbf{q}_2\|^2 - 2|\mathbf{q}_2^H\mathbf{q}_1|}}. \tag{70}$$

Maximizing the numerator of the RHS of Eq. (70) in the radical sign is equivalent to maximizing the values of g_1 and g_2. Letting $\mathbf{f}^\perp = \mathbf{0}$ minimize the weighted SER and this completes the proof. ■

APPENDIX E
PROOF OF PROPOSITION 4

Proof: As $\gamma \approx 0$, the LHS of Eq. (67) approaches 1. We obtain

$$g_1 c_2 \|\mathbf{q}_1\|^2 - g_2 c_2 |\mathbf{q}_2^H \mathbf{q}_1| = g_2 c_1 \|\mathbf{q}_2\|^2 - g_1 c_1 |\mathbf{q}_2^H \mathbf{q}_1|. \quad (71)$$

Rearranging Eq. (71), the relationship between g_1 and g_2 which minimizes the weighted SER is given by

$$g_1 = g_2 \left(\frac{c_1 \|\mathbf{q}_2\|^2 + c_2 |\mathbf{q}_2^H \mathbf{q}_1|}{c_2 \|\mathbf{q}_1\|^2 + c_1 |\mathbf{q}_2^H \mathbf{q}_1|} \right). \quad (72)$$

Substituting Eq. (72) into the constraint in Eq. (65) leads to

$$g_2^2 B = P_\mathrm{T} - \|\mathbf{f}^\perp\|^2, \; g_1^2 C = P_\mathrm{T} - \|\mathbf{f}^\perp\|^2. \quad (73)$$

Then g_1 and g_2 can be expressed as

$$g_2 = \sqrt{\frac{P_\mathrm{T} - \|\mathbf{f}^\perp\|^2}{B}} \quad and \; g_1 = \sqrt{\frac{P_\mathrm{T} - \|\mathbf{f}^\perp\|^2}{C}}. \quad (74)$$

Maximizing the numerator of the RHSs of Eqs. (74) in the radical signs is equivalent to maximizing the values of g_1 and g_2. Letting $\mathbf{f}^\perp = \mathbf{0}$ minimizes the weighted SER and this completes the proof. ■

APPENDIX F
PROOF OF PROPOSITION 6

Proof: Similar to the two-user case in (66), the following left and right-hand side terms both equal to the Lagrange multiplier:

$$\begin{aligned} &\frac{c_1^{(i)} \frac{\gamma}{\sqrt{2\pi}} e^{-\frac{\gamma^2 |g_1^{(i)}|^2}{2}}}{2g_1^{(i)} \|\mathbf{q}_1^{(i)}\|^2 - 2g_2^{(i)} |\mathbf{q}_2^{(i)H} \mathbf{q}_1^{(i)}|} \\ &= \frac{c_1^{(j)} \frac{\gamma}{\sqrt{2\pi}} e^{-\frac{\gamma^2 |g_1^{(j)}|^2}{2}}}{2g_1^{(j)} \|\mathbf{q}_1^{(j)}\|^2 - 2g_2^{(j)} |\mathbf{q}_2^{(j)H} \mathbf{q}_1^{(j)}|}, \end{aligned} \quad (75)$$

$$\Longrightarrow \frac{c_1^{(i)} e^{-\frac{\gamma^2 |g_1^{(i)}|^2}{2}}}{c_1^{(j)} e^{-\frac{\gamma^2 |g_1^{(j)}|^2}{2}}} = \frac{g_1^{(i)} \|\mathbf{q}_1^{(i)}\|^2 - g_2^{(i)} |\mathbf{q}_2^{(i)H} \mathbf{q}_1^{(i)}|}{g_1^{(j)} \|\mathbf{q}_1^{(j)}\|^2 - g_2^{(j)} |\mathbf{q}_2^{(j)H} \mathbf{q}_1^{(j)}|}, \quad (76)$$

$$\begin{aligned} &\Longrightarrow e^{-\frac{\gamma^2 (|g_1^{(i)}|^2 - g_1^{(j)}|^2)}{2}} \\ &= \left(\frac{c_1^{(j)}}{c_1^{(i)}} \right) \frac{g_1^{(i)} \|\mathbf{q}_1^{(i)}\|^2 - g_2^{(i)} |\mathbf{q}_2^{(i)H} \mathbf{q}_1^{(i)}|}{g_1^{(j)} \|\mathbf{q}_1^{(j)}\|^2 - g_2^{(j)} |\mathbf{q}_2^{(j)H} \mathbf{q}_1^{(j)}|}, \end{aligned} \quad (77)$$

Manipulating (77) leads to the result in (44). Similar derivations can be used to obtain (45). ■

APPENDIX G
PROOF OF LEMMA 4

Proof: Similar to Eq. (68) in Proposition 3 of the two-user system, as $\gamma \to \infty$, Eq. (43) in Lemma 3 is approximated by

$$|g_1^{(k)}|^2 - |g_2^{(k)}|^2 \approx 0, \quad k = 1, 2, \cdots, \frac{U}{2}. \quad (78)$$

This leads to Eq. (46) because $g_1^{(k)}$ and $g_2^{(k)}$ are positive for every k. ■

APPENDIX H
PROOF OF LEMMA 5

Proof: As $\gamma \to \infty$, Eq. (44) in Proposition 6 becomes

$$|g_1^{(i)}|^2 - |g_1^{(j)}|^2 \approx 0, \quad i, j = 1, 2, \cdots, \frac{U}{2}. \quad (79)$$

Again, this results in Eq. (47) because $g_1^{(i)}$ and $g_2^{(j)}$ are positive for every i and j. ■

APPENDIX I
PROOF OF LEMMA 6

Proof:

Similar to Eq. (72) for proving Proposition 4 of the two-user case, as $\gamma \approx 0$, the gain relationship in Lemma 3 becomes

$$\begin{aligned} &\left(\frac{c_2^{(k)}}{c_1^{(k)}} \right) \frac{g_1^{(k)} \|\mathbf{q}_1^{(k)}\|^2 - g_2^{(k)} |\mathbf{q}_2^{(k)H} \mathbf{q}_1^{(k)}|}{g_2^{(k)} \|\mathbf{q}_2^{(k)}\|^2 - g_1^{(k)} |\mathbf{q}_2^{(k)H} \mathbf{q}_1^{(k)}|} = 1, \\ &\Longrightarrow g_1^{(k)} c_2^{(k)} \|\mathbf{q}_1^{(k)}\|^2 - g_2^{(k)} c_2^{(k)} |\mathbf{q}_2^{(k)H} \mathbf{q}_1^{(k)}| \\ &= g_2^{(k)} c_1^{(k)} \|\mathbf{q}_2^{(k)}\|^2 - g_1^{(k)} c_1^{(k)} |\mathbf{q}_2^{(k)H} \mathbf{q}_1^{(k)}|, \\ &k = 1, 2, \cdots, \frac{U}{2}, \end{aligned} \quad (80)$$

and this leads to the result in (51) after manipulations. ■

APPENDIX J
PROOF OF PROPOSITION 8

Proof: As $\gamma \approx 0$, Eq. (77) for proving Proposition 6 can be reduced to

$$\left(\frac{c_1^{(j)}}{c_1^{(i)}} \right) \frac{g_1^{(i)} \|\mathbf{q}_1^{(i)}\|^2 - g_2^{(i)} |\mathbf{q}_2^{(i)H} \mathbf{q}_1^{(i)}|}{g_1^{(j)} \|\mathbf{q}_1^{(j)}\|^2 - g_2^{(j)} |\mathbf{q}_2^{(j)H} \mathbf{q}_1^{(j)}|} = 1, \quad (81)$$

$$\begin{aligned} &\Longrightarrow g_1^{(i)} c_1^{(j)} \|\mathbf{q}_1^{(i)}\|^2 - g_2^{(i)} c_1^{(j)} |\mathbf{q}_2^{(i)H} \mathbf{q}_1^{(i)}| \\ &= g_1^{(j)} c_1^{(i)} \|\mathbf{q}_1^{(j)}\|^2 - g_2^{(j)} c_1^{(i)} |\mathbf{q}_2^{(j)H} \mathbf{q}_1^{(j)}|. \end{aligned} \quad (82)$$

From Definition 2, substituting $g_2^{(i)} = g_1^{(i)} r_{12}^{(i)}$ and $g_2^{(j)} = g_1^{(j)} r_{12}^{(j)}$ into Eq. (82) leads to

$$\begin{aligned} &g_1^{(i)} c_1^{(j)} \|\mathbf{q}_1^{(i)}\|^2 - g_1^{(i)} r_{12}^{(i)} c_1^{(j)} |\mathbf{q}_2^{(i)H} \mathbf{q}_1^{(i)}| \\ &= g_1^{(j)} c_1^{(i)} \|\mathbf{q}_1^{(j)}\|^2 - g_1^{(j)} r_{12}^{(j)} c_1^{(i)} |\mathbf{q}_2^{(j)H} \mathbf{q}_1^{(j)}|, \\ &i, j = 1, 2, \cdots, \frac{U}{2}. \end{aligned}$$

The result in (52) can be obtained by manipulating (83). ■

APPENDIX K
PROOF OF COROLLARY 1

Proof: We prove that the numerators and denominators of $r_1^{(j,i)}$ and $r_2^{(j,i)}$ are positive. For $r_1^{(j,i)}$, substituting the gain ratios for users in the same group $r_{12}^{(j)}$ defined in Definition 2 into the numerator of $r_1^{(j,i)}$ defined in Definition 3 yields

$$\begin{aligned} &c_1^{(i)} (\|\mathbf{q}_1^{(j)}\|^2 - r_{12}^{(j)} |\mathbf{q}_2^{(j)H} \mathbf{q}_1^{(j)}|) \\ &= c_1^{(i)} (\|\mathbf{q}_1^{(j)}\|^2 - \left(\frac{c_2^{(j)} \|\mathbf{q}_1^{(j)}\|^2 + c_1^{(j)} |\mathbf{q}_2^{(j)H} \mathbf{q}_1^{(j)}|}{c_1^{(j)} \|\mathbf{q}_2^{(j)}\|^2 + c_2^{(j)} |\mathbf{q}_2^{(j)H} \mathbf{q}_1^{(j)}|} \right) \\ &|\mathbf{q}_2^{(j)H} \mathbf{q}_1^{(j)}|), \end{aligned} \quad (83)$$

and the last term can be rewritten as

$$\begin{aligned} c_1^{(i)} \Bigg(& \frac{c_1^{(j)}\|\mathbf{q}_1^{(j)}\|^2\|\mathbf{q}_2^{(j)}\|^2 + c_2^{(j)}\|\mathbf{q}_1^{(j)}\|^2|\mathbf{q}_2^{(j)H}\mathbf{q}_1^{(j)}|}{c_1^{(j)}\|\mathbf{q}_2^{(j)}\|^2 + c_2^{(j)}|\mathbf{q}_2^{(j)H}\mathbf{q}_1^{(j)}|} \\ & - \frac{c_2^{(j)}\|\mathbf{q}_1^{(j)}\|^2|\mathbf{q}_2^{(j)H}\mathbf{q}_1^{(j)}| + c_1^{(j)}|\mathbf{q}_2^{(j)H}\mathbf{q}_1^{(j)}|^2}{c_1^{(j)}\|\mathbf{q}_2^{(j)}\|^2 + c_2^{(j)}|\mathbf{q}_2^{(j)H}\mathbf{q}_1^{(j)}|}\Bigg) \\ = c_1^{(i)} & \frac{c_1^{(j)}(\|\mathbf{q}_1^{(j)}\|^2\|\mathbf{q}_2^{(j)}\|^2 - |\mathbf{q}_2^{(j)H}\mathbf{q}_1^{(j)}|^2)}{c_1^{(j)}\|\mathbf{q}_2^{(j)}\|^2 + c_2^{(j)}|\mathbf{q}_2^{(j)H}\mathbf{q}_1^{(j)}|}. \end{aligned} \tag{84}$$

Using Cauchy-Schwarz inequality, the numerator term $\|\mathbf{q}_1^{(j)}\|^2\|\mathbf{q}_2^{(j)}\|^2 - |\mathbf{q}_2^{(j)H}\mathbf{q}_1^{(j)}|^2 > 0$ if $\mathbf{q}_1^{(j)}$ and $\mathbf{q}_2^{(j)}$ are linear independent. Thus the numerator of $r_1^{(j,i)}$ is positive. Using the similar argument, one can conclude that the denominator is also positive. Therefore $r_1^{(j,i)}$ is positive. Similar procedure can be used to prove that $r_2^{(j,i)}$ is also positive. ■

Appendix L
Proof of Proposition 9

Proof: Using the ratios defined in Definitions 2 and 3, for User 1 in group k, we have

$$\begin{aligned} |g_1^{(k)}|^2 (\sum_{i=1}^{\frac{U}{2}} & |r_1^{(k,i)}|^2\|\mathbf{q}_1^{(i)}\|^2 + |r_1^{(k,i)}|^2|r_{12}^{(i)}|^2\|\mathbf{q}_2^{(i)}\|^2 \\ & -2|r_1^{(k,i)}|^2|r_{12}^{(i)}||\mathbf{q}_2^{(i)H}\mathbf{q}_1^{(i)}|) = \frac{U}{2}P_{\mathrm{T}} - \|\mathbf{F}^{\perp}\|_F^2. \end{aligned} \tag{85}$$

The gain can be written as

$$g_1^{(k)} = \sqrt{\frac{\frac{U}{2}P_{\mathrm{T}} - \|\mathbf{F}^{\perp}\|_F^2}{E}}. \tag{86}$$

Similarly for User 2 in group k, we have

$$\begin{aligned} |g_2^{(k)}|^2 (\sum_{i=1}^{\frac{U}{2}} & |r_2^{(k,i)}|^2|r_{21}^{(i)}|^2\|\mathbf{q}_1^{(i)}\|^2 + |r_2^{(k,i)}|^2\|\mathbf{q}_2^{(i)}\|^2 \\ & -2|r_2^{(k,i)}|^2|r_{21}^{(i)}||\mathbf{q}_2^{(i)H}\mathbf{q}_1^{(i)}|) = \frac{U}{2}P_{\mathrm{T}} - \|\mathbf{F}^{\perp}\|_F^2. \end{aligned} \tag{87}$$

The gain can be written as

$$g_2^{(k)} = \sqrt{\frac{\frac{U}{2}P_{\mathrm{T}} - \|\mathbf{F}^{\perp}\|_F^2}{F}}. \tag{88}$$

From Eqs. (86) and (88), by setting $\mathbf{F}^{\perp} = \mathbf{0}$, we obtain

$$g_1^{(k)} = \sqrt{\frac{\frac{U}{2}P_{\mathrm{T}}}{E}}, \quad and\ g_2^{(k)} = \sqrt{\frac{\frac{U}{2}P_{\mathrm{T}}}{F}}, \tag{89}$$

which maximizes gains and guarantees the lowest weighted SER. Considering all the grouping cases, the solution for Problem 4 in the low SNR regime can be concluded in this proposition. ■

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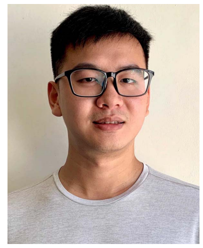

Kuo-Chen Ho was born in Hsinchu, Taiwan, in 1994. He received the B.S. degree in electrical and computer engineering and the M.S. degree in electrical and control engineering from National Chiao Tung University (NCTU), Hsinchu, in 2016 and 2018, respectively. His research interest includes signal processing for wireless communications.

Shang-Ho (Lawrence) Tsai (SM'12) was born in Kaohsiung, Taiwan. He received the Ph.D. degree in electrical engineering from the University of Southern California, CA, USA, in 2005. From 1999 to 2002, he was with Silicon Integrated Systems Corporation, where he was involved in the VLSI design for DMT-ADSL systems. He was a Visiting Fellow with the Department of Electrical Engineering, Princeton University, in 2013. From 2005 to 2007, he was with MediaTek Inc., where he was involved in the VLSI design for MIMO-OFDM systems and standard specifications for the IEEE 802.11n. In 2007, he joined the Department of Electrical Engineering, National Chiao Tung University, where he is currently a Professor. His research interests are in the areas of signal processing for communications, statistical signal processing, and signal processing for VLSI designs. He was a recipient of the Government Scholarship for Overseas Study from the Ministry of Education, Taiwan, from 2002 to 2005. He has been on the Editorial Board of the IEEE Signal Processing Repository since 2018.