Transactions Letters

Bit Allocation Schemes for MIMO Equal Gain Precoding

Chi-Liang Chao, Shang-Ho Tsai, and Terng-Yin Hsu

***Abstract*—We design equal gain precoders with scalar quantization in MIMO systems with limited feedback, which do not require the predefined codebook. In such precoders, bit allocation can be used to quantize the precoding vector/matrix to further improve the system performance. There are two conventional bit allocation schemes. One is the uniform bit allocation and the other is the optimal bit allocation obtained by using exhaustive search. The uniform bit allocation is simple but turns out to have obvious performance degradation. On the other hand, the exhaustive search method leads to the optimal performance. However, its computational complexity is extremely high and may be somewhat impractical to be realized. In this paper, we propose two bit allocation schemes for the MIMO equal gain precoder with scalar quantization. Both of the proposed methods are with low complexity and their performance is close to the optimal scheme. Consequently, the proposed bit allocations provide good trade-off between performance and complexity.**

***Index Terms*—Bit allocation, MIMO, scalar quantized (SQ), equal gain precoder, limited feedback.**

I. Introduction

MULTIPLE-INPUT multiple-output (MIMO) techniques enable higher spectral efficiency and improve robustness against channel fading. Among MIMO techniques, the closed-loop MIMO can provide extra gain compared to the open-loop MIMO thanks to the provision of the channel state information at the transmitter (CSIT). However, the tremendous feedback amount of full CSIT is somewhat impractical in most wireless systems. This drawback motivates the research of limited feedback precoding techniques [1]-[4], where only quantized precoding vectors from the receiver is conveyed back to the transmitter. The quantization techniques can be categorized to vector quantization (VQ) and scalar quantization (SQ). VQ in general performs better than SQ. However, SQ is done on a per-antenna basis. Hence there is no need to search exhaustively to determine the closest codeword. Moreover, the storage effort for the codebook in both transmitter and receiver can also be waived if SQ is used. The performance analysis of quantization techniques can be found in [5]-[9]. In [10] the authors contributed to the analysis of the received SNR and mutual information loss for the random vector quantization (RVQ) scheme in correlated MIMO channels. Although SQ enjoys lower computational complexity than VQ, for small values of feedback bits, however, the performance of SQ may degrade a lot. By assigning suitable quantization bits to the elements of precoding vectors, bit allocation for precoding vectors can be used to further improve the system performance. In [11] the authors proposed a bit allocation scheme for multiuser systems with VQ which employs multi-resolution codebook. In [8], the authors proposed to use exhaustive search and uniform bit allocation for the scalar quantized equal gain precoder in single user systems; the former method searches all possible bit allocations and decides the one with the best gain effect. This method can achieve the optimal performance but encounter huge computational complexity. The latter method uniformly allocates bits to the equal gain precoder. This method has low complexity but suffers from considerable performance degradation.

In this paper, two bit allocation algorithms are proposed to overcome the drawbacks of conventional bit allocations in equal gain precoder. Let B be the total feedback bits, the proposed bit allocation algorithms show the following advantages: *1)* Compared to the exhaustive search algorithm, the proposed schemes remarkably reduce the computational complexity from exponential order to polynomial order in B. Also, the corresponding performance is close to the exhaustive search algorithm with moderate feedback bits. *2)* The proposed schemes greatly outperform the uniform algorithm with acceptable extra computational complexity. *3)* From simulation results, if bit allocation table is available in the transmitter, we found SQ equal gain precoding with proposed bit allocations outperform the VQ precoding without bit allocation, *e.g.* [2], in the same B. *4)* It is worthy to emphasize one of the proposed algorithms does not need multiplication operations when generating the feedback information and performing precoding in the transmitter, which remarkably reduces the computational and implementational complexity.

The rest of the paper is organized as follows. Sec. II presents the system model. Sec. III shows the bit allocation of the precoding vector. Two proposed bit allocation algorithms are

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developed in Sec. IV. In Sec. V, simulation results are given, and we make the conclusion in Sec. VI.

II. System Model

The MIMO precoding systems considered in this work have N_t transmit and N_r receive antennas. Assume incomplete channel information is available at the transmitter via limited feedback. At first, the modulated symbol $x \in \mathbb{C}$ is multiplied by the precoding vector $\mathbf{w} \in \mathbb{C}^{N_t \times 1}$. For equal gain precoding, the precoding vector can be expressed as [2]

$$\mathbf{w} = \frac{\mathbf{1}}{\sqrt{\mathbf{N_t}}}\left[\mathbf{1}\ \mathrm{e}^{\mathbf{j}\theta_\mathbf{2}}\ ...\ \mathrm{e}^{\mathbf{j}\theta_{\mathbf{N_t}}}\right]^\mathbf{T}. \tag{1}$$

After the precoding, the transmitting symbol vector, $\mathbf{s} = [s_1\ s_2\ ...\ s_{N_t}]^T$, can be expressed as $\mathbf{s} = \mathbf{w}x$. Then, $\mathbf{s}$ is transmitted to the MIMO channel. The MIMO channel is $\mathbf{H} \in \mathbb{C}^{\mathbf{N_r} \times \mathbf{N_t}}$, and the (i,j)th element is represented by h_{ij}. At the receiver, the receiving vector $\mathbf{r} = [r_1\ r_2\ ...\ r_{N_r}]^T$ from the MIMO channel is given by $\mathbf{r} = \mathbf{H}\mathbf{w}x + \mathbf{n}$, where $\mathbf{n} \in \mathbb{C}^{\mathbf{N_r} \times \mathbf{1}}$ is a noise vector and its elements is assumed to be i.i.d. complex Gaussian with variance σ_n^2. Then $\mathbf{r}$ is multiplied by a receiving vector $\mathbf{z} \in \mathbb{C}^{1 \times N_r}$ to form $\hat{x}$, i.e. $\hat{x} = \mathbf{zHw}x + \mathbf{zn}$. To achieve the best performance, the MRC technique [1] is applied and the receiving vector is $\mathbf{z} = (\mathbf{Hw})^H$. Thus

$$\hat{x} = \gamma x + \mathbf{zn} \tag{2}$$

where $\gamma = ||\mathbf{Hw}||_2^2$ is the gain effect due to the space diversity and the precoding.

III. Bit Allocation of Precoding Vector

We use scalar quantization with equal step size. From (1) we can quantize $(\theta_2, ..., \theta_{N_t})$. Let the quantized precoding vector be $\hat{\mathbf{w}} = \frac{1}{\sqrt{N_t}}[1\ \mathrm{e}^{j\hat{\theta}_2} ...\ \mathrm{e}^{j\hat{\theta}_{N_t}}]^T$, where $\hat{\theta}_i = (2\pi n_i)/2^{b_i},\ 0 \leqslant n_i \leqslant 2^{b_i} - 1$; n_i is the feedback index for θ_i, and b_i is the number of quantization bits for θ_i. For MISO channels, the optimal equal gain precoder has closed-form solutions [1],[8],[9]. For MIMO channels, however, there is no closed-form optimal solution available for equal gain precoder. Nevertheless, the work in [8] proposed to iteratively calculate the solution for MIMO channel via cyclic method, and we may regard this solution as the optimal solution in MIMO channels. When we obtain the optimal solution, we can quantize θ_i of the optimal equal gain precoder to the closest available values $\hat{\theta}_i$. Then, we need to send the index set $(n_2, n_3, ..., n_{N_t})$ from the receiver to the transmitter, which totally requires $B = \sum_{i=2}^{N_t} b_i$ feedback bits.

Bit allocation for precoding vector is a problem of deciding the quantization bit set for θ_i to further improve the system performance. Conventional bit allocation schemes were proposed in [8] including *1) exhaustive search scheme*: This scheme searches all the possible bit allocation sets to find the optimal set $\left\{b_i^{opt}\right\}_{i=2}^{N_t}$ that maximizes $\hat{\gamma} = ||\mathbf{H}\hat{\mathbf{w}}||_2^2$. This method can achieve the best performance. For a given bit budget B, it needs to search totally $\begin{pmatrix} B + N_t - 2 \\ B \end{pmatrix} = ((B + N_t - 2)!)\,/\,((B)!\,(N_t - 2)!)$ possible sets[1]. The computational complexity is with the exponential order in B (see p. 15 in [12]). *2) uniform scheme:* This scheme attempts to make b_i equal. For example, let $\bar{b} = \lfloor (B)/(N_t - 1) \rfloor$, $\tilde{b} = \bar{b} + 1$ and $\bar{N} = B - \bar{b}(N_t - 1)$. Starting from the second antenna, it assigns $\{b_i\}_{i=2}^{\bar{N}+1} = \tilde{b}$ for the first $\bar{N}$ phases $\{\theta_i\}_{i=2}^{\bar{N}+1}$, and $\{b_i\}_{i=\bar{N}+2}^{N_t} = \bar{b}$ bits for the other $(N_t - 1) - \bar{N}$ phases $\{\theta_i\}_{i=\bar{N}+2}^{N_t}$. The complexity is constant and independent of B. However, its performance degrades considerably, compared to the optimal scheme.

IV. Proposed Algorithms

The proposed bit allocation algorithms are designed to maximize the average of received SNR with acceptable complexity. We briefly summarize the derivation and re-express the average of received SNR in [9] below. Considering a $1 \times N_t$ MISO channel $\mathbf{h}$, its ith coefficient is h_i and ϕ_i is the phase of h_i. The instantaneous received SNR due to scalar quantization is $\hat{\rho}_e = \frac{\hat{\gamma}\sigma_x^2}{\sigma_n^2}$, where $\hat{\gamma} = \hat{\mathbf{w}}^H\mathbf{h}^H\mathbf{h}\hat{\mathbf{w}}$ and σ_x^2 and σ_n^2 are the variances of x and n. $\hat{\gamma}$ can be derived as $\hat{\gamma} = \frac{1}{N_t}\left(\sum_{i=1}^{N_t}|h_i|^2 + \sum_{i=1,j\neq i}^{N_t} h_i^* h_j e^{j(\hat{\theta}_j - \hat{\theta}_i)}\right)$ and the average of received SNR is given by

$$\mathbb{E}_\mathbf{h}\left\{\hat{\rho}_e\right\} = \frac{\sigma_x^2}{\sigma_n^2 N_t}\left(\begin{array}{l}\sum_{i=1}^{N_t}\mathbb{E}_\mathbf{h}\left\{|h_i|^2\right\} + \\ \sum_{i=1,j\neq i}^{N_t}\mathbb{E}_\mathbf{h}\left\{\Re\left\{h_i^* h_j e^{j(\hat{\theta}_j - \hat{\theta}_i)}\right\}\right\}\end{array}\right). \tag{3}$$

Let $\varepsilon_i = \hat{\theta}_i - \theta_i$ and $\varepsilon_j = \hat{\theta}_j - \theta_j$ be the phase quantization errors for the ith and jth precoding elements of $\hat{\mathbf{w}}$ respectively. Since $\theta_i = -(\phi_i - \phi_1)$ and $\theta_j = -(\phi_j - \phi_1)$, we can rewrite (3) as

$$\mathbb{E}_\mathbf{h}\left\{\hat{\rho}_e\right\} = \frac{\sigma_x^2}{\sigma_n^2 N_t}\left(\begin{array}{c}\sum_{i=1}^{N_t}\mathbb{E}_\mathbf{h}\left\{|h_i|^2\right\} + \\ 2\sum_{j=2}^{N_t}\mathbb{E}_\mathbf{h}\left\{|h_1^* h_j|\right\}\mathbb{E}_\mathbf{h}\left\{\cos\left(\varepsilon_j\right)\right\} + \\ \sum_{i=2,j\neq 1,i}^{N_t}\mathbb{E}_\mathbf{h}\left\{|h_i^* h_j|\right\}\mathbb{E}_\mathbf{h}\left\{\cos\left(\varepsilon_j - \varepsilon_i\right)\right\}\end{array}\right). \tag{4}$$

From (4), there are two methods to maximize the SNR in MISO channels; that is, *1)* minimizing the quantization errors, and *2)* minimizing the differences of the quantization errors of all precoding element pairs. Method 1 can be done by increasing B. Our proposed bit allocations are motivated by Method 2 that can provide more efficient solutions for limited B.

For MIMO cases, Zheng et. al. in [8] proposed a cyclic algorithm to obtain a near optimal equal gain precoder. The

[1]This number arises as a result of tabulating all the unordered arrangements of $(N_t + B - 2)$ 'objects' that include $N_t - 2$ antenna positions and B bit positions (see p. 15 in [12]).

cyclic algorithm uses the closed-form MISO optimal solution iteratively; that is, it iteratively calculates the precoding vector $\mathbf{w}$ and the receiving vector $\mathbf{z}$ using the following equations until it converges:

$$\mathbf{w} = 1/(\sqrt{N_t})e^{j\angle(\mathbf{zH})^H} \quad \text{and} \quad \mathbf{z}^H = \frac{\mathbf{Hw}}{||\mathbf{Hw}||_2}. \tag{5}$$

Let $\mathbf{w}_o$ and $\mathbf{z}_o$ be the converged precoding and receiving vectors, respectively. By letting $\mathbf{z}_o\mathbf{H}$ be the effective MISO channel $\mathbf{h}_{eff}$, the gain effect $\gamma_{eff} = \mathbf{w}_o^H \mathbf{h}_{eff}^H \mathbf{h}_{eff} \mathbf{w}_o$ and the average SNR for MIMO channels can be written as $\mathbb{E}_{\mathbf{h}}\{\rho_e\} = \frac{\gamma_{eff}\sigma_x^2}{\sigma_n^2}$. Since the cyclic algorithm actually obtains the MIMO solution by iteratively updating the optimal MISO solution, following the same ways in the MISO environments, we can maximize the average SNR in the MIMO environment as well. Therefore, our proposed bit allocations applied to both MISO and MIMO cases. Let us introduce the proposed bit allocations as follows:

1. Proposed bit allocation scheme 1 (BA1): This scheme is a simple but efficient method. From (4), when a bit is allocated to the antenna with the largest quantization error, the quantization error of this antenna has high probability to be greatly reduced, and so does ε_j, $j = 2, ..., N_t$. Therefore, BA1 based on *maximum-quantization-error-first criterion* is described below.

Algorithm: Bit Allocation Scheme 1 (BA1)
Step 0. Initialization: $\hat{\Theta} \leftarrow [0\ 0 \cdots 0]^T$; $j \leftarrow 0$;
$b_i \leftarrow 0$, $\varepsilon_i \leftarrow \hat{\theta}_i - \theta_i$, for $2 \leqslant i \leqslant N_t$;
Step 1. $c \leftarrow \arg\max_i(|\varepsilon_i|)$; $b_c \leftarrow b_c + 1$; $\Delta \leftarrow 2\pi/2^{b_c}$;
$\tilde{n}_c \leftarrow \theta_c/\Delta$;
$\varepsilon_c \leftarrow (\lceil \tilde{n}_c \rfloor - \tilde{n}_c) \cdot \Delta$; // $\lceil \cdot \rfloor$ *denotes the round operation.*
$j \leftarrow j + 1$;
Step 2. **if** $j < B$ **then** goto step 1; **else** end the program.

BA1 is simple. However, it does not necessarily minimize the differences of the quantization errors for all transmit antenna pairs, especially when B is not sufficiently large. This motivates us to develop more sophisticated bit allocation schemes as follows.

2. Proposed bit allocation scheme 2 (BA2): BA2 attempts to investigate more potential bit allocation sets with high gain effects, compare the gain effects, and then choose the set with the largest gain effect. First, BA2 employs a table $\mathbf{\Pi}$ with dimension $(N_t - 1) \times (B + 1)$ to record all possible quantization errors for transmit antennas $i = 2,\ 3, \cdots N_t$ for all bit assignments $b_i = 0,\ 1,\ \cdots B$. For example, $\mathbf{\Pi}(i,j)$ is the quantization error of the ith antenna after allocating j bits; it can be written as

$$\mathbf{\Pi}(i,j) = \frac{2\pi}{2^j}\left(\lceil \tilde{n}_i \rfloor - \tilde{n}_i\right) \tag{6}$$

where $\tilde{n}_i = \theta_i/(2\pi/2^j)$. Second, BA2 investigates the table with at most $(N_t - 1)(B + 1)$ iterations, then select a bit allocation set in each iteration, and calculate the gain effect. Finally, BA2 compares the resulting gain effects and selects the bit allocation set with the largest gain. The algorithm BA2 is summarized below and an example is given later in Example 1.

Algorithm: Bit Allocation Scheme 2 (BA2)
Step 0. Initialization: $\hat{\Theta} \leftarrow [0\ 0 \cdots 0]^T$; $b_i \leftarrow 0$, for $2 \leqslant i \leqslant N_t$;
Let $\mathbf{\Pi}(i,j)$ be the quantization error obtained from (6), for $2 \leqslant i \leqslant N_t$ and $0 \leqslant j \leqslant B$;
Let $\mathcal{S}$ be the set of antenna(s) with some bit(s) allocated; Set $\mathcal{S} = \emptyset$;
Step 1. Table investigation:
for $k = 1$ **to** $(N_t - 1)(B + 1)$ **do**
ignore the $k - 1$ smallest $|\mathbf{\Pi}(i,j)|$ investigated table elements
// *some elements may be equal in* $\mathbf{\Pi}$, *hence the number of ignored elements* $\geq k - 1$
if the number of ignored elements $\geq (N_t - 1)(B + 1)$ // *finish all investigations*
goto Step 2;
end if
// *record the min. quantization errors of antenna* $2 \sim N_t$.
$e_i \leftarrow \min_j(|\mathbf{\Pi}(i,j)|)$, for $2 \leqslant i \leqslant N_t$;
// *record the min. bit(s) required for the corresponding min. quantization errors.*
$col_i \leftarrow \arg\min_j(|\mathbf{\Pi}(i,j)|)$, for $2 \leqslant i \leqslant N_t$;
loop forever
if $\exists i \notin \mathcal{S}$, $2 \leqslant i \leqslant N_t$ // *exist some antenna(s) without bit(s) allocated*
$idx \leftarrow \arg\min_{i, i \notin \mathcal{S}}(e_i)$;
else // *all antennas are allocated*
perform uniform bit allocation for the remaining bit budget; leave loop;
end if
// *allocate bit(s)* b_{idx} *to antenna* idx.
if $\sum_{i=2}^{N_t} b_i + col_{idx} < B$ // *remaining bit budget* $>$ *bit(s) to be allocated.*
$\mathcal{S} \leftarrow \mathcal{S} \cup \{idx\}$; $b_{idx} \leftarrow col_{idx}$;
else // *remaining bit budget* $\leq$ *bit(s) to be allocated.*
$b_{idx} \leftarrow B - \sum_{i=2}^{N_t} b_i$; leave loop;
end if
end loop
calculate the gain effect $\hat{\gamma_k} = ||\mathbf{H}\hat{\mathbf{w}}_k||_2^2$ by the candidate bit allocation set $\{b_i\}_{i=1}^{N_t}$, where $\hat{\mathbf{w}}_k$ is the precoding vector decided in kth iteration.
end for
Step 2. compare the gain effects computed from all investigations and select the final bit allocation set $\{b_i\}_{i=1}^{N_t}$ with the maximum one;
Step 3. according to the final bit allocation set, compute the final quantized phase vector $\hat{\Theta}$.

If we constrain the elements of the precoding vectors in BA2 to be $\{\pm 1, \pm j\}$, the complexity of BA2 can be further reduced as follows:

3. Special case of BA2 (BA2f): In Step 1 of BA2, if the elements of $\hat{\mathbf{w}}_k$ are limited to be $\{\pm 1, \pm j\}$, the complex

TABLE I
COMPLEXITY COMPARISON AMONG VARIOUS BIT ALLOCATION SCHEMES

	Complexity	
	# of iterations	per iteration
Exhaustive Search	$\mathcal{O}\left(\left(\frac{e(B+N_t-2)}{B}\right)^B\right)$	$\mathcal{O}((N_r)(N_t))$ complex multiplications.
Uniform	$\mathcal{O}(1)$	$\mathcal{O}(1)$
BA1	$\mathcal{O}(B)$	$\mathcal{O}(1)$ $\arg\max_i(\lvert\varepsilon_i\rvert)$ operation
BA2	$\mathcal{O}((N_t-1)(B+1))$	$\mathcal{O}((N_r)(N_t))$ complex multiplications.
BA2f	$\mathcal{O}((N_t-1)3)$	$\mathcal{O}((N_r)(N_t))$ sign and conjugate operations.

multiplications in $\mathbf{H}\hat{\mathbf{w}}_k$ to calculate the gain effect $\hat{\gamma}_k$ can be replaced by sign and conjugate operations, which greatly reduces the computational complexity. Moreover, there is no need to use multiplications for the precoding operation $\hat{\mathbf{w}}x$ in the transmitter; note that x can be M-QAM symbols. Similar concept to constrain the elements of $\hat{\mathbf{w}}$ can also be found in the standardization literature, see *e.g.* [13]-[15], where the finite alphabet concept was indicated as an important property in the precoder design. We consider the resulting scheme as a special case of BA2, and call it BA2f for short. BA2f first constructs the quantization error table $\mathbf{\Pi}$ with dimension $(N_t-1)\times 3$, which means the number of table columns is shrunk into 3 ($b_i = 0, 1$ and 2). Then, BA2f inherits BA2 and investigates $\mathbf{\Pi}$ with fewer columns, which guarantee $b_i \leq 2$ and each element in $\hat{\mathbf{w}}_k$ is in alphabet $\{\pm 1, \pm j\}$. Compared to BA2, BA2f not only reduces the computational complexity but also owns the advantages of lower memory requirement due to the smaller quantization error table $\mathbf{\Pi}$ and fewer investigations for $\mathbf{\Pi}$. The penalty is a small but tolerable performance degradation, as will be demonstrated later in Sec. V.

BA1, BA2 and BA2f present trade-off between computational complexity and performance. Among prior art (exhaustive search, uniform), BA1, BA2 and BA2f, the overall complexity comparison is summarized in Table I and the performance evaluations are illustrated in Sec. V.

Example 1: Proposed bit allocation schemes. Let $N_t = 4$, $N_r = 2$, $B = 4$, and the MIMO channel matrix $\mathbf{H}$ be

$$\mathbf{H} = \begin{bmatrix} 0.6926 + 0.6930j & -0.1878 - 0.8427j \\ -0.3878 - 0.3097j & -0.0681 + 0.9662j \\ -0.9763 - 0.6171j & -0.5150 + 0.0632j \\ 1.3336 + 0.1751j & -2.0799 + 0.2878j \end{bmatrix}^T.$$

The cyclic algorithm in [8] can be applied in the MIMO systems to obtain the unquantized equal gain precoding vector, $\mathbf{w} = \frac{1}{\sqrt{4}}$ [1.0000 -0.9871+0.1600j -0.9622+0.2723j 0.3766+0.9264j]^T. As discussed in Sec. IV, the proposed bit allocation schemes can be applied in this MIMO precoding vector $\hat{\mathbf{w}} = e^{j\hat{\Theta}}/\sqrt{N_t}$. Let us use BA1 and let $\hat{\Theta}^{(i)} = [0\ \hat{\theta}_2\ \hat{\theta}_3\ \hat{\theta}_4]^T$ be the quantized phase vector at the ith iteration; we can obtain $\hat{\Theta}^{(1)} = 2\pi[0\ 0\ 0\ 0]^T, \hat{\Theta}^{(2)} = 2\pi[0\ 0\ 1/2\ 0]^T, \hat{\Theta}^{(3)} = 2\pi[0\ 1/2\ 1/2\ 0]^T$ and $\hat{\Theta}^{(4)} = 2\pi[0\ 1/2\ 1/2\ 1/4]^T$. Since $B = 4$, $\hat{\Theta}^{(4)}$ is the solution.

Now consider BA2. BA2 first constructs a quantization error table $\mathbf{\Pi}_{BA2}$ with dimension $(N_t-1)\times(B+1)$ as shown in Table II. BA2 iterates at most $(N_t-1)(B+1)$ table investigations because some duplicated table elements are skipped; and then selects a candidate of bit allocation set in each investigation. In Step 1, the first investigation includes the following two actions: *1)* It calculates the quantization error vector $\mathbf{e} = [e_2\ e_3\ e_4]^T = [0.1607\ 0.1170\ 0.0066]^T$ and the corresponding bit vector $\mathbf{col} = [col_2\ col_3\ col_4]^T = [1\ 4\ 4]^T$; *2)* then it selects e_4 and its bit value $col_4 = 4$ in the first loop. Because $col_4 = 4 = B$, it leaves the loop in Step 1 and selects the bit allocation set $\{b_i\}_{i=1}^{N_t} = [0\ 0\ 0\ 4]^T$. Similarly, the second investigation ignores the table element $\lvert\mathbf{\Pi}_{BA2}(4,4)\rvert = 0.0066$ because it is the smallest one. Then it computes $\mathbf{e} = [0.1607\ 0.1170\ 0.3861]^T$ and $\mathbf{col} = [1\ 4\ 2]^T$, and selects the bit allocation set $[0\ 0\ 4\ 0]^T$. After all investigations are done, it determines the most suitable bit allocation set by comparing the gain effects obtained from all investigations. In this example, the final bit allocation set is $[0\ 1\ 1\ 2]^T$ and the final quantized phase vector is $2\pi[0\ 1/2\ 1/2\ 1/4]^T$. Note that with parallel processing, the investigations can be handled simultaneously to reduce the processing time considerably. Also, since there are 7 duplicated table elements in $\mathbf{\Pi}_{BA2}$, only $15 - 7 = 8$ investigations are required. Now consider BA2f. BA2f constructs a quantization error table $\mathbf{\Pi}_{BA2f}$ with dimension $(N_t-1)\times 3$ as shown in Table III. The first investigation selects the bit allocation set $[0\ 1\ 1\ 2]^T$. The second investigation initially selects $[0\ 0\ 1\ 2]^T$ and then performs uniform bit allocation for the remaining bit budget to obtain $[0\ 1\ 1\ 2]^T$. After all investigations are done, the final bit allocation set is $[0\ 1\ 1\ 2]^T$ and the corresponding quantized phase vector is $2\pi[0\ 1/2\ 1/2\ 1/4]^T$. Compared to BA2, BA2f has smaller size of quantization error table and requires fewer table investigations. Also, all bit assignments are fewer than or equal to 2.

TABLE II
THE QUANTIZATION ERROR TABLE, $\mathbf{\Pi}_{BA2}$.

	$b_i=0$	$b_i=1$	$b_i=2$	$b_i=3$	$b_i=4$
Ant. 2	2.9809	−0.1607	−0.1607	−0.1607	−0.1607
Ant. 3	2.8658	−0.2757	−0.2757	−0.2757	0.1170
Ant. 4	1.1847	1.1847	−0.3861	−0.3861	0.0066

TABLE III
THE QUANTIZATION ERROR TABLE, $\mathbf{\Pi}_{BA2f}$.

	$b_i=0$	$b_i=1$	$b_i=2$
Ant. 2	2.9809	−0.1607	−0.1607
Ant. 3	2.8658	−0.2757	−0.2757
Ant. 4	1.1847	1.1847	−0.3861

V. SIMULATION RESULTS

Let the elements of channel matrix $\mathbf{H}$ be i.i.d. complex Gaussian with zero mean and unit variance. The modulation is BPSK for Examples 2-3, and 16-QAM for Example 4.

Example 2: Comparison of optimal, uniform and proposed bit allocation algorithms. We compare the perfor-

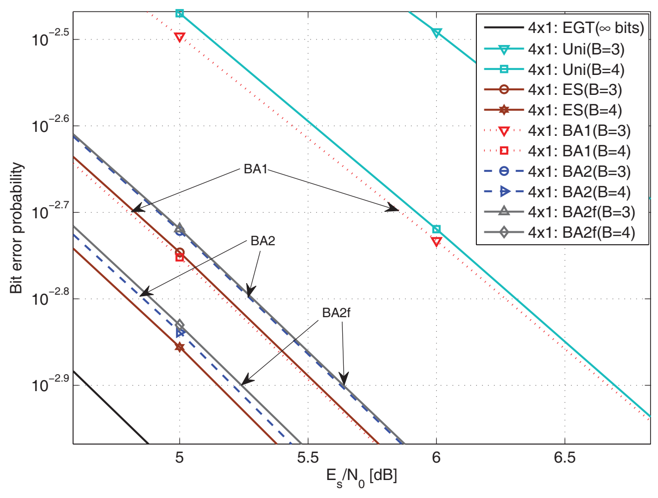

Fig. 1. BEP performance for the equal gain precoder with the optimal, the uniform and the proposed algorithms in a 4T1R channel.

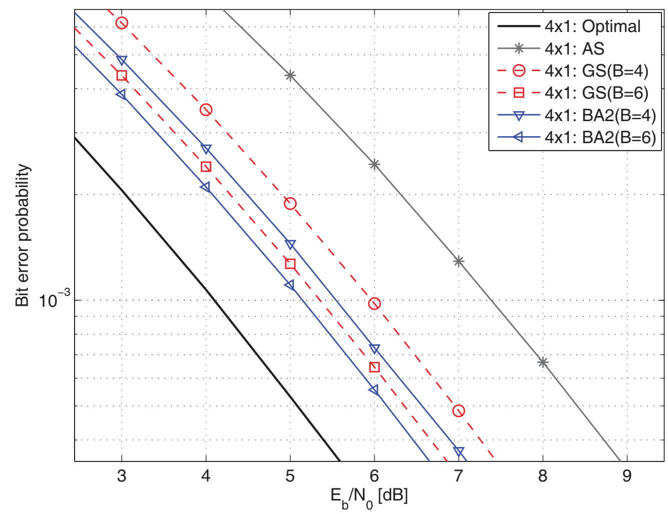

Fig. 2. BEP performance for different precoders in a 4T1R channel.

mance of the exhaustive search (ES), the uniform (Uni) and the proposed algorithms (BA1/BA2/BA2f) used in the equal gain precoder. Fig. 1 shows the bit error probability (BEP) comparison in a 4T1R (4-transmit and 1-receive) channel environment. We observe that BA2 and BA2f improve BA1's drawback that performs poorly at low bit budget such as $B = 3$. BA2 has the best performance, and BA2f owns low-complexity feature at the cost of slight performance degradation compared to BA2. Moreover, BA1, BA2, and BA2f all considerably outperform the uniform bit allocation. BA1 can approach the performance of the optimal bit allocation when a moderate B is given, for example $B = 5$. Furthermore, BA2 and BA2f can maintain small performance gap (≤ 0.2 dB) with the optimal bit allocation for all B's.

Example 3: Comparison of various precoders. We compare the performance of the following precoders: antenna selection (AS) scheme, Grassmannian (GS) precoder [2], equal gain (EG) precoder with the proposed bit allocation BA2, and the optimal precoder. The reason we choose BA2 is that GS precoder has been recognized as a precoder having excellent performance. We would like to show the comparison between GS precoder and the proposed precoder BA2. Fig. 2 shows the performance comparison in a 4T1R channel. We observe that the EG precoder with BA2 ($B = 4$ and 6) outperforms the Grassmannian precoder by around 0.4 dB and 0.2 dB respectively. Moreover, when $B = 6$, it outperforms the antenna selection scheme by around 2.2 dB. However, the penalty for the performance improvement is that the transmitter needs to know the bit allocation table for each antenna, since now each antenna may has different number of bits to quantize the phase, and this demands extra protocol effort. Compared to the optimal precoder without quantization effect, it degrades about 1 dB. The implementation advantages of the proposed bit allocation algorithm in MISO channels can be explained as follows: In MISO channels, the optimal closed-form unquantized precoding vector can be obtained directly from the channel. Using the unquantized precoding vector, the proposed bit allocation algorithm needs at most $(N_t-1)(B+1)$ iterations to generate the feedback information while the Grassmannian beamforming needs 2^B iterations to determine the feedback information. Therefore, when B increases, the iteration number grows linearly for the proposed EG precoder while that grows exponentially for the Grassmannian precoder. For example, letting $B = 6$ and $N_t = 4$, the proposed EG precoder needs at most 21 iterations while the Grassmannian precoder needs 64 iterations (64-codeword codebook is used).

Example 4: Proposed algorithms with large N_t. Let N_t increase from 4 to 8 and the modulation order be 16-QAM. Fig. 3 shows that when N_t and B increase, the performance gap between EG precoders with the exhaustive search algorithm and the uniform algorithm becomes larger, but that between EG precoders with the exhaustive search algorithm and the proposed algorithm, BA2f, still maintains quite close. Take $B = 3$ in a 4T2R channel and $B = 7$ in a 8T2R channel for instance, the gaps between the exhaustive search algorithm and BA2f are around 0.1-0.6 dB while the performance gap between the exhaustive search algorithm and uniform algorithm increases from 1.6 dB to 2.5 dB.

VI. Conclusion

We proposed two bit allocation schemes (BA1 and BA2) and BA2's special case, BA2f, for the scalar quantized equal gain precoder. Compared to the prior work, i.e. uniform and exhaustive search bit allocation methods, the proposed schemes not only significantly outperform the uniform method but also provide nearly optimal performance obtained by the exhaustive search method with moderate bits for BA1, and with arbitrary bits for BA2 and BA2f. They are all with affordable computational complexity. Let us conclude when to use the proposed schemes. 1) If the complexity is the main consideration, BA1 should be chosen. 2) If the performance is the critical issue, BA2 is a good choice. 3) If the performance and the complexity are both important, BA2f should be the right decision.

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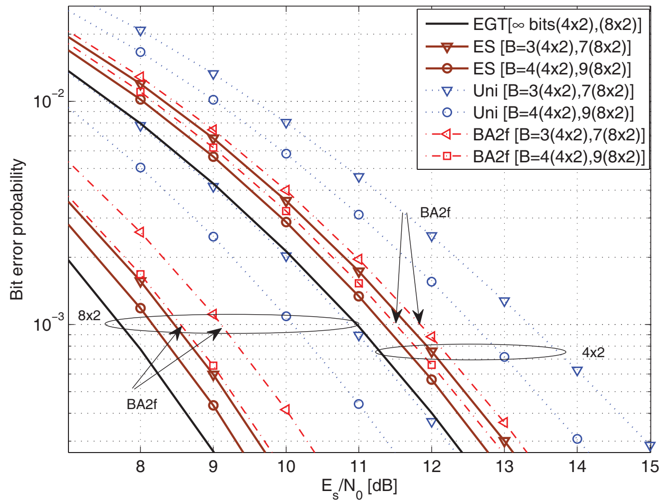

Fig. 3. BEP performance for the equal gain precoders with the optimal, the uniform and the proposed algorithms in 4T2R and 8T2R channels.

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