Equal Gain Transmission with Antenna Selection in MIMO Communications

Shang-Ho Tsai, *Member, IEEE*

Abstract—**The beamforming vectors of an equal gain transmission (EGT) contains phase information only and thereby enjoys several implementational advantages when compared to the optimal scheme, *i.e.* maximum ratio transmission (MRT). The implementational advantages make EGT a promising solution for simple transceiver design while offering a performance comparable to that of MRT. This solution motivates us to explore how close the performance can be between EGT and MRT. The maximum SNR loss between EGT and MRT is known to be 1.05 dB in MISO channels. However, little is known about the SNR loss in MIMO channels, since no closed-form solution is available for the best EGT in MIMO channels. In this work, a suboptimal closed-form EGT design for MIMO channels is proposed and its performance is analyzed. Interestingly, the maximum SNR loss between the proposed EGT and the MRT (both employing MRC in receiver) in MIMO channels is shown to be approximately 1.05 dB as well. Moreover, instead of applying conventional all transmit antennas, this study proposes to adopt antenna selection, to further improve the performance of EGT. Two antenna selection algorithms are proposed and the corresponding performance is analyzed. When the proposed antenna selection algorithms are applied to EGT, the SNR loss between EGT and MRT can be reduced to as low as 0.45-0.65 dB, with the numbers of transmit antennas ranging from 4 to 8. One of the proposals with fixed number of transmit antennas not only outperforms conventional EGT but also requires fewer number of RF (radio frequency) components; also, it employs constant power in each transmit antenna like EGT does. As a result, hardware complexity can be reduced by this proposal. Furthermore, design strategies to apply the proposed EGT and antenna selection algorithms in systems with limited feedback are also suggested.**

Index Terms—**Equal gain transmission, EGT, hybrid selection, (transmit) antenna selection, TAS, beamforming, precoding, MRT, Lloyd codebook, MIMO.**

I. INTRODUCTION

MIMO beamforming/precoding techniques have recently received extensive attention in wireless communication systems. If all the elements of a beamforming vector have equal magnitude, it is usually called the equal gain transmission (EGT) [1],[2]. Unlike the optimal beamforming, *i.e.* maximum ratio transmission (MRT), which demands the feedback of both magnitude and phase information for its beamforming vector, the beamforming vector of an EGT contains phase information only and hence enjoys several advantages described as follows: First, in a TDD (Time-division Duplex) system, where complete channel information is known to the transmitter, using EGT can greatly relax the design effort for power amplifier (PA), since the corresponding peak to average power ratio (PAPR) in EGT is much lower than that in MRT [3]. Second, in a wireless system where sufficient feedback is possible due to the low mobility of users and base stations, the use of EGT not only can ease PA design, but also can save half of the feedback redundancy [2]. Although EGT performs worse than the optimal MRT, the potential implementational benefits of EGT mentioned above motivate us to explore the following questions: *1.)* What is the maximum performance loss between EGT and MRT? *2.)* Could the performance loss be reduced by applying a good but simple design on the EGT? *3.)* Suppose we have the solutions for Question 2. What are the design strategies to use the solutions in non-TDD systems, where only limited feedback is available?

The answer for Question 1 has been solved for MISO channels, since the solution for the best EGT in MISO channels can be easily obtained from the channel phase (see e.g. [2],[3]); the maximum SNR loss between the best EGT and MRT was shown to be 1.05 dB in MISO channels [4]. For MIMO channels, however, no simple closed-form solution is currently available for the best MIMO EGT [1],[3]; hence, little is known about the performance loss between EGT and MRT in MIMO channels. In this study, a suboptimal EGT design for MIMO channels is proposed and its performance is theoretically analyzed. The maximum SNR loss between the proposed EGT and the MRT is shown to be approximately 1.05 dB in MIMO channels with an approximation error that is less than 0.003 dB.

Could the maximum 1.05 dB performance loss be further reduced, as Question 2 mentioned? To answer the question, it is shown in this paper that using all transmit antennas in EGT does not guarantee the best performance; that is, for a fixed transmit power constraint, wasting power in the transmit antennas with bad channel conditions is less effective than allocating the power in those with good conditions. For this reason, we propose to select antennas properly, rather than always using all transmit antennas, to further improve the performance of EGT; the best performance can be achieved by selecting L out of the N_t branches, where N_t is the number of transmit antennas, the value L and the L selected branches depend on channel conditions, and are generally different for different channel realizations. This selection concept is similar to antenna selection [5],[6],[7],[8] or hybrid selection [9],[10],[11],[12]. However, some differences need to be pointed out; that is, the antenna selection problems generally

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The author is with the Department of Electrical Engineering, National Chiao Tung University, Hsinchu, Taiwan. R.O.C. (e-mail: shanghot@mail.nctu.edu.tw).

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attempt to select a predetermined number L of branches from N_t so as to maximize the channel capacity via the singular value decomposition (SVD). The value L in antenna selection is hence fixed and the transmit power is not equal in individual branches due to the use of the SVD.

The computational complexity to select the best L branches from N_t antennas is generally high, see *e.g.* [8]. To enhance the performance of EGT while maintaining a low computational complexity, two simple antenna selection algorithms are proposed for EGT in this study. The corresponding performance is also analyzed theoretically; the SNR gap between MRT and the proposed EGT with antenna selection can be reduced to as low as 0.45 dB for $N_t = 4$, and to 0.65 dB for $N_t = 8$ in MIMO channels. Additionally, as mentioned earlier, the number L of the selected antennas in EGT should be determined according to channel conditions, so as to maximize the SNR. However, if a slight performance loss is tolerable (less than 0.1 dB loss for $N_t \geq 16$ compared to flexible L), a fixed number L_o of the strongest branches can be used in the proposed MIMO EGT. L_o for different N_t is derived. The theoretical result shows that the suggested L_o is independent of the number of receive antennas. It is worth to point out that the proposed EGT with fixed number of transmit antennas not only outperforms conventional EGT but also requires fewer number of RF (radio frequency) components; moreover each of the selected transmit antennas employs constant power like conventional EGT does. As a result, this scheme enjoys better performance as well as lower hardware complexity than conventional EGT.

In non-TDD systems where only limited feedback is available, design strategies for the proposed EGT and antenna selection algorithms are also suggested for answering Question 3. Simulation results show that with fewer number of feedback bits, the proposed EGT and antenna selection designs can still outperform conventional EGT.

The rest of this paper is organized as follows: The system model and background of beamforming are presented in Sec. II. The proposed EGT in MIMO channels and its performance analysis are introduced in Sec. III. Antenna selection algorithms are proposed for EGT and the corresponding performance is analyzed theoretically in Sec. IV. Remarks on the proposed EGT and antenna selection algorithms are given in Sec. VI, and the corresponding design strategies to use these proposals in a limited-feedback environment are discussed in Sec. VII. Simulation results are provided in Sec. VIII. Finally, concluding remarks are given in Sec. IX.

II. System Model and Preliminaries

The block diagram of a beamforming system is shown in Fig. 1. When beamforming is used in TDD systems, the transmitter is assumed to know complete channel state information (CSI); while in non-TDD systems, the transmitter is assumed to know only partial CSI. Let the number of transmit antennas be N_t. At the first stage, one transmit symbol x is multiplied by a $N_t \times 1$ beamforming vector $\mathbf{f}$. The square of the L_2 norm for the beamforming vector is usually normalized to unity, *i.e.* $\|\mathbf{f}\|_2^2 = 1$, where $\|\mathbf{f}\|_2^2 = |f_1|^2 + |f_2|^2 + \cdots |f_{N_t}|^2$ [13]. For an EGT beamforming vector, all the elements in $\mathbf{f}$ have

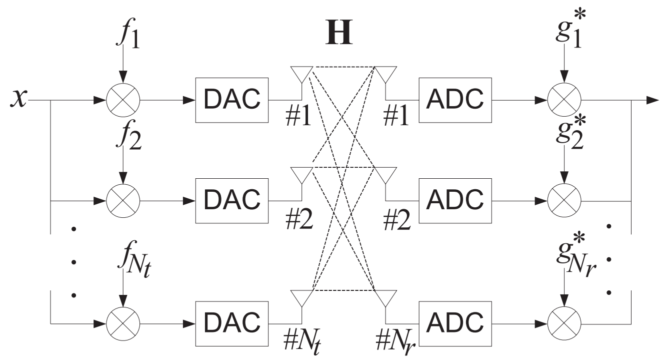

Fig. 1. A MIMO beamforming system.

equal magnitude, *i.e.* $f_i = e^{j\theta_i}/\sqrt{N_t}$. After the beamforming processing, the signal is transmitted to a $N_r \times N_t$ MIMO channel, denoted by $\mathbf{H}$. The elements of $\mathbf{H}$ is assumed to have i.i.d. complex Gaussian distribution with zero mean. The received signal vector is combined by a $N_r \times 1$ combining vector $\mathbf{g}$ and form a scalar z, *i.e.*

$$z = \mathbf{g}^\dagger \mathbf{H}\mathbf{f}x + \mathbf{g}^\dagger \mathbf{n}, \tag{1}$$

where $\mathbf{A}^\dagger$ is the Hermitian of $\mathbf{A}$ and $\mathbf{n}$ is a $N_r \times 1$ noise vector. Let the power of x be E_x and the noise power be N_0. The instantaneous SNR of z is shown to be

$$\rho = \frac{E_x}{N_0}\frac{|\mathbf{g}^\dagger \mathbf{H}\mathbf{f}|^2}{\|\mathbf{g}\|_2^2} = \frac{E_x}{N_0}|\mathbf{g}^\dagger \mathbf{H}\mathbf{f}|^2, \tag{2}$$

where $\|\mathbf{g}\|_2^2$ is assumed to be unity without losing the generality [1]. The optimal beamforming system is given by [14]: $\mathbf{f}_o = \mathbf{v}_1$ and $\mathbf{g}_o = \mathbf{u}_1$, where $\mathbf{u}_1$ and $\mathbf{v}_1$ are the left and the right singular vectors corresponding to the maximum singular value, σ_1, respectively; the optimal instantaneous SNR: $\rho_o = \frac{E_x}{N_0}\sigma_1^2$, can then be obtained using (2). The beamforming vector $\mathbf{f}_o$ and combining vector $\mathbf{g}_o$ are usually called MRT and MRC (maximum ratio combining), respectively.

For EGT and the corresponding MRC design, the problem to design the beamforming and combining vectors so as to maximize the instantaneous SNR of z is as follows:

$$(\mathbf{f}_e,\ \mathbf{g}_e) = \arg \max_{\mathbf{g},\ \mathbf{f}} |\mathbf{g}^\dagger \mathbf{H}\mathbf{f}|^2, \quad \text{with} \quad f_i = e^{j\theta_i}/\sqrt{N_t}. \tag{3}$$

The solution for (3) exists for MISO channels and is not unique. Let $\angle h_i = \psi_i$. One solution is (see [1],[3]):

$$\mathbf{f}_{e-miso} = \frac{1}{\sqrt{N_t}}\left(1\ e^{-j(\psi_2-\psi_1)}\ e^{-j(\psi_3-\psi_1)} \cdots e^{-j(\psi_{N_t}-\psi_1)}\right)^t$$

and

$$g_{e-miso} = \frac{\mathbf{h}^t \mathbf{f}_{e-miso}}{|\mathbf{h}^t \mathbf{f}_{e-miso}|},$$

where $\mathbf{A}^t$ is the transpose of $\mathbf{A}$. The instantaneous SNR: $\rho_{e-miso} = \frac{E_x}{N_0}\frac{\|\mathbf{h}\|_1^2}{N_t}$, can be obtained using the above solution and (2), where $\|\mathbf{x}\|_1^2$ is the square of the L_1 norm for vector $\mathbf{x}$ defined by $\|\mathbf{x}\|_1^2 = (|x_1| + |x_2| + \cdots |x_n|)^2$ [13]. In MISO channels, $\sigma_1^2 = \|\mathbf{h}\|_2^2$; hence the average SNR loss between MRT and EGT in MISO channels is given by [4]

$$\frac{E\{\rho_o\}}{E\{\rho_{e-miso}\}} = \frac{N_t E\{\|\mathbf{h}\|_2^2\}}{E\{\|\mathbf{h}\|_1^2\}} = \frac{N_t}{1 + (N_t - 1)\frac{\pi}{4}}. \tag{4}$$

The maximum SNR loss in (4) is 1.05 dB, when N_t tends to infinity [4]. It is worth pointing out that Brennan showed the SNR loss between the receive MRC and the receive EGC is also 1.05 dB [15]. In MIMO channels, unfortunately, there is no simple closed-form solution for (3) (see Sec. V in [1]); hence, little knowledge is available about the performance loss of EGT in MIMO channels. Is the SNR loss still at most 1.05 dB for EGT in MIMO channels? Interestingly, the answer is positive and this is introduced in the following section.

III. Proposed MIMO EGT and its Performance Analysis

A MIMO EGT that applies cyclic optimization skill was proposed by Zheng *et. al.* in [3]; the cyclic MIMO EGT requires computing phase vector iteratively to obtain the solution. In this section, we propose a suboptimal MIMO EGT that does not need iterations. Later simulation result shows the performance gap between the proposed MIMO EGT and the cyclic MIMO EGT is only around 0.01-0.035 dB. The proposed MIMO EGT design is motivated by the optimal EGT design in MISO channels. That is, in MISO channels, one optimal EGT design is $e^{-j\angle \mathbf{h}}/\sqrt{N_t}$, which uses the phase of $\mathbf{h}^*$. $\mathbf{h}^*/\|\mathbf{h}\|_2$ is actually the right singular vector of the MISO channel $\mathbf{h}^t$. Similarly, in MIMO channels, the phase of the right singular vector $\mathbf{v}_1$ (corresponding to the maximum singular value) of $\mathbf{H}$ may be used for MIMO EGT design. The proposed EGT, $\mathbf{f}_{e-mimo}$, and the corresponding MRC, $\mathbf{g}_{e-mimo}$, for MIMO channels are then as follows:

$$\mathbf{f}_{e-mimo} = \frac{1}{\sqrt{N_t}}\left(1 \; e^{j(\theta_2-\theta_1)} \; e^{j(\theta_3-\theta_1)} \cdots e^{j(\theta_{N_t}-\theta_1)}\right)^t, \tag{5a}$$

$$\mathbf{g}_{e-mimo} = \frac{\mathbf{H}\mathbf{f}_{e-mimo}}{\|\mathbf{H}\mathbf{f}_{e-mimo}\|_2}, \tag{5b}$$

where $(\theta_1 \;\; \theta_2 \;\; \cdots \theta_{N_t})$ is the phase of $\mathbf{v}_1$. From (2), the instantaneous SNR using (5) is

$$\rho_{e-mimo} = \frac{E_x}{N_0}\frac{\|\mathbf{H}\mathbf{f}_{e-mimo}\|_2^2}{N_t}. \tag{6}$$

Lemma 1: The instantaneous SNR of the proposed EGT in (5) is lower bounded by

$$\rho_{e-mimo} \geq \frac{E_x}{N_0}\frac{\sigma_1^2\|\mathbf{v}_1\|_1^2}{N_t}. \tag{7}$$

Proof. Let $\mathbf{p} = (e^{j\theta_1} \;\; e^{j\theta_2} \cdots e^{j\theta_{N_t}})^t/\sqrt{N_t}$. Using the fact that $\|\mathbf{u}_1\|_2^2$=1, and the Cauchy-Schwarz inequality, *i.e.* $|\mathbf{x}^\dagger\mathbf{y}|^2 \leq \|\mathbf{x}\|_2^2\|\mathbf{y}\|_2^2$, we have: $\left|\mathbf{u}_1^\dagger\mathbf{H}\mathbf{p}\right|^2 \leq \|\mathbf{u}_1\|_2^2\|\mathbf{H}\mathbf{p}\|_2^2 = \|\mathbf{H}\mathbf{p}\|_2^2$. Since $\|\mathbf{H}\mathbf{p}\|_2^2 = \|\mathbf{H}\mathbf{p}e^{-j\theta_1}\|_2^2 = \|\mathbf{H}\mathbf{f}_{e-mimo}\|_2^2$, it leads to

$$\left|\mathbf{u}_1^\dagger\mathbf{H}\mathbf{p}\right|^2 \leq \|\mathbf{H}\mathbf{f}_{e-mimo}\|_2^2. \tag{8}$$

The inequality: $\rho_{e-mimo} \geq \frac{E_x}{N_0}\frac{\left|\mathbf{u}_1^\dagger\mathbf{H}\mathbf{p}\right|^2}{N_t}$ can then be obtained from (6) and (8). The SVD of $\mathbf{H}$ can also be expressed as: $\mathbf{H} = \sum_{i=1}^r \sigma_i\mathbf{u}_i\mathbf{v}_i^\dagger$, where r is the rank of $\mathbf{H}$. The result in (7) can then be obtained by applying this decomposition into the above inequality. ■

The following lemmas are introduced for analyzing the lower bound in (7).

Lemma 2: Let σ_i and $\mathbf{v}_i$ be the singular values and the right singular vectors of $\mathbf{H}$, where $\mathbf{H}^\dagger\mathbf{H}$ is with the Wishart distribution. Then, σ_i is distributed independently of $\mathbf{v}_i$. Moreover, $\mathbf{v}_i$ has the conditional Haar invariant distribution (see pp. 536-538 in [16]). ■

Lemma 3: Let $\mathbf{h}^t$ be a $1 \times N_t$ vector with i.i.d. complex Gaussian distributed elements. Then, the following approximation can be made:

$$E\left\{\frac{\|\mathbf{h}\|_1^2}{\|\mathbf{h}\|_2^2}\right\} \approx \frac{E\left\{\|\mathbf{h}\|_1^2\right\}}{E\left\{\|\mathbf{h}\|_2^2\right\}} = 1 + \frac{\pi}{4}(N_t - 1). \tag{9}$$

Proof. Please see Appendix. ■

Note that the approximation in (9) is very accurate. The approximation error is less than 0.003 dB, verified by the Monte Carlo simulation for $N_t \leq 64$.

Theorem 1: The maximum SNR loss between the MRT and the proposed EGT in MIMO channels is approximately 1.05 dB.

Proof. From (7), the average SNR loss between the MRT and the proposed EGT in MIMO channels is upper bounded by: $\frac{E\{\rho_o\}}{E\{\rho_{e-mimo}\}} \leq \frac{N_t E\{\sigma_1^2\}}{E\{\sigma_1^2\|\mathbf{v}_1\|_1^2\}}$. From Lemma 2, the distribution of σ_1 and $\mathbf{v}_1$ are independent; hence $E\left\{\sigma_1^2\|\mathbf{v}_1\|_1^2\right\} = E\left\{\sigma_1^2\right\}E\left\{\|\mathbf{v}_1\|_1^2\right\}$. The upper bound can therefore be rewritten as $\frac{E\{\rho_o\}}{E\{\rho_{e-mimo}\}} \leq \frac{N_t}{E\{\|\mathbf{v}_1\|_1^2\}}$. Moreover, $\mathbf{v}_1$ and $\mathbf{h}^*/\|\mathbf{h}\|_2$ both have the conditional Haar invariant distribution, because $\mathbf{v}_1$ and $\mathbf{h}^*/\|\mathbf{h}\|_2$ are the right singular vectors of $\mathbf{H}$ and $\mathbf{h}^t$, respectively. Hence, the upper bound can be expressed as $\frac{E\{\rho_o\}}{E\{\rho_{e-mimo}\}} \leq \frac{N_t}{E\{\|\mathbf{h}\|_1^2/\|\mathbf{h}\|_2^2\}}$. From Lemma 3, the upper bound may be approximated by

$$\frac{E\{\rho_o\}}{E\{\rho_{e-mimo}\}} \leq \frac{N_t}{1 + (N_t - 1)\frac{\pi}{4}}. \tag{10}$$

The term $N_t/(1+(N_t-1)\frac{\pi}{4})$ is the same with that in (4), and it is a monotonically increasing function of N_t. The maximum value is approximately 1.05 dB as N_t approaches ∞. ■

The SNR loss in MIMO channels may be even smaller than that in MISO channels, since the loss in (10) is an approximate upper bound with approximation error that is less than 0.003 dB, while that in (4) is an equality.

IV. Proposed Transmission Antenna Selection (TAS) for EGT

When the transmit power is fixed, selecting transmit antennas can move the transmission power of the unselected antennas to the selected antennas. Consequently, antenna selection can improve the performance of EGT. Similar concept was used to improve channel capacity in [5]. In this section, two TAS algorithms with low computational complexity for EGT are proposed.

A. Proposed TAS for MISO EGT

For the EGT with TAS in MISO channels, the problem is to find a subvector $\mathbf{h}_s$ from $\mathbf{h}$ so that the resulting instantaneous SNR is maximized, *i.e.* $\rho_{e-miso-as} = \frac{E_x}{N_0}\max_{\mathbf{h}_s \subset \mathbf{h}}\left\{\frac{\|\mathbf{h}_s\|_1^2}{l_o}\right\}$, where l_o is the number of elements in $\mathbf{h}_s$. At first glance, the required number of searches seems to be $2^{N_t} - 1$ (minus 1 skips the case that all antennas are off). However, the computational complexity can be greatly reduced by reordering the elements of the channel vector according to their power or magnitude. This is described in Algorithm 1.

Algorithm 1 Proposed TAS for EGT in MISO channels.

1: Reorder the MISO channel $\mathbf{h}$ according to the power or magnitude. Let the reordered channel vector (in increasing order) be $(h_{n_1}\ h_{n_2}\ \cdots h_{n_{N_t}})$.

2: $l_o = \arg\max_{1\le l\le N_t}\left\{\frac{1}{l}(\sum_{k=N_t-l+1}^{N_t}|h_{n_k}|)^2\right\}$.

3: Select the strongest l_o branches from the reordered channel as the $N_t \times 1$ beamforming vector, denoted by $\mathbf{f}_{e-miso-as}$. The n_k-th element of $\mathbf{f}_{e-miso-as}$ is thus given by

$$[\mathbf{f}_{e-miso-as}]_{n_k} = \begin{cases} \frac{e^{-j\angle h_{n_k}}}{\sqrt{l_o}}, & N_t - l_o + 1 \le k \le N_t; \\ 0, & \text{otherwise.} \end{cases}$$

If the Bubble sorting [17] is used in Step 1, the worst computational order is $\mathcal{O}(N_t^2)$. For Step 2, the computational order is $\mathcal{O}(N_t)$. Hence, the overall computational order for Algorithm 1 is $\mathcal{O}(N_t^2)$. The resulting instantaneous SNR is

$$\rho_{e-miso-as} = \frac{E_x}{N_0}\max_{1\le L\le N_t}\left\{\frac{1}{L}(\sum_{i=N_t-L+1}^{N_t}|h_{n_i}|)^2\right\}. \quad (11)$$

B. Proposed TAS for MIMO EGT

For the EGT in MIMO channels, the goal of TAS is to find the best submatrix $\mathbf{H}_s$ from $\mathbf{H}$ such that $\rho_{e-mimo-as} = \frac{E_x}{N_0}\max_{\mathbf{g}_e,\mathbf{f}_e}\left\{\frac{|\mathbf{g}_e^\dagger\mathbf{H}_s\mathbf{f}_e|^2}{l_o}\right\}$, where l_o is the number of columns in $\mathbf{H}_s$, and the elements of $\mathbf{f}_e$ are either 0 or $e^{j\phi_i}/\sqrt{l_o}$. Again, no simple closed-form solution is available. Additionally, even if there was a closed-form solution, selecting the best submatrix also demands a lot of computational effort. To overcome this problem, we notice that once the combining vector $\mathbf{g}_e$ is determined, the equivalent channel $\mathbf{g}_e^\dagger\mathbf{H}$ may be regarded as a $1\times N_t$ MISO channel vector; the proposed TAS in Algorithm 1 can then be applied to reduce the computational complexity for this equivalent MISO channel. The procedure is introduced as follows: First, let the combining vector be $\mathbf{g}_e = \mathbf{u}_1$. The equivalent MISO channel is hence $\mathbf{u}_1^\dagger\mathbf{H} = \sigma_1\mathbf{v}_1^\dagger$. Then, apply Algorithm 1 to this equivalent channel and determine the corresponding TAS for EGT, *i.e.* $\mathbf{f}_{e-mimo-as}$. Finally, replace the initial combining vector $\mathbf{u}_1^\dagger$ by $\mathbf{f}_{e-mimo-as}^\dagger\mathbf{H}^\dagger$, since this is the MRC and it results in higher instantaneous SNR using the following property: $\|\mathbf{H}\mathbf{f}_{e-mimo-as}\|_2^2 \ge |\mathbf{u}_1^\dagger\mathbf{H}\mathbf{f}_{e-mimo-as}|^2$, where the derivation of the above inequality is similar to that in Lemma 1. The proposed TAS algorithm for MIMO EGT is concluded in Algorithm 2:

Algorithm 2 Proposed TAS for EGT in MIMO channels.

1: Conduct the SVD to the $N_r \times N_t$ MIMO channel $\mathbf{H}$. Let $\mathbf{u}_1$ be the left singular vector corresponding to the largest singular value. Let the equivalent channel be $\tilde{\mathbf{h}} = \mathbf{u}_1^\dagger\mathbf{H}$.

2: Reorder the equivalent channel $\tilde{\mathbf{h}}$ according to the power or magnitude. Let the reordered channel vector (in increasing order) be $(\tilde{h}_{n_1}\ \tilde{h}_{n_2}\ \cdots \tilde{h}_{n_{N_t}})$.

3: $l_o = \arg\max_{1\le l\le N_t}\left\{\frac{1}{l}(\sum_{k=N_t-l+1}^{N_t}|\tilde{h}_{n_k}|)^2\right\}$.

4: Select the strongest l_o branches from the reordered channel as the $N_t \times 1$ beamforming vector, denoted by $\mathbf{f}_{e-mimo-as}$. The n_k-th element of $\mathbf{f}_{e-mimo-as}$ is thus given by

$$[\mathbf{f}_{e-mimo-as}]_{n_k} = \begin{cases} \frac{e^{-j\angle\tilde{h}_{n_k}}}{\sqrt{l_o}}, & N_t - l_o + 1 \le k \le N_t; \\ 0, & \text{otherwise.} \end{cases}$$

Algorithm 2 is a suboptimal solution. In fact, the performance can be improved if we iteratively use $\mathbf{g}_e = \mathbf{H}\mathbf{f}_{e-mimo-as}$ as a new combining vector and perform antenna selection on the new equivalent MISO channel $\mathbf{g}_e^\dagger\mathbf{H}$. However, later simulation result show the improvement is only around 0.002-0.02 dB. Hence, Algorithm 2 without using iteration may already provide a sufficiently good performance. The computational order of the proposed TAS for MIMO EGT in Algorithm 2 is still $\mathcal{O}(N_t^2)$. Moreover, let the reordered vector of $\mathbf{v}_1$ be $(v_{n_1}\ v_{n_2}\ \cdots\ v_{n_{N_t}})^t$. The instantaneous SNR of the EGT with TAS is lower bounded by

$$\begin{aligned}\rho_{e-mimo-as} &\ge \frac{E_x}{N_0}|\sigma_1\mathbf{v}_1^\dagger\mathbf{f}_{e-mimo-as}|^2 \\ &= \frac{E_x}{N_0}\sigma_1^2\max_{1\le L\le N_t}\left\{\frac{1}{L}(\sum_{i=N_t-L+1}^{N_t}|v_{n_i}|)^2\right\}.\end{aligned} \quad (12)$$

The lower bound of the resulting instantaneous SNR in Algorithms 1 and 2, *i.e.* (11) and (12), are analyzed theoretically in the following section.

V. Performance Analysis of Proposed TAS for EGT

A. Performance analysis of proposed TAS in MISO EGT.

Consider the performance of Algorithm 1 first. From (11), the average SNR is $E\{\rho_{e-miso-as}\}$. Obtaining a closed-form representation for $E\{\rho_{e-miso-as}\}$ may not be easy, since the values $\frac{1}{L}(\sum_{i=N_t-L+1}^{N_t}|h_{n_i}|)^2$ corresponding to different L are not i.i.d. The analysis for non-i.i.d. order statistics is complicated (see Chapter 5 in [18]). Here a lower bound for $E\{\rho_{e-miso-as}\}$ is derived instead. $E\{\rho_{e-miso-as}\}$ can be lower bounded by

$$E\{\rho_{e-miso-as}\} \ge \frac{E_x}{N_0}\max_{1\le L\le N_t}E\left\{\frac{1}{L}(\sum_{i=N_t-L+1}^{N_t}|h_{n_i}|)^2\right\}. \quad (13)$$

That is, the lower bound is achieved by selecting a fixed number L_o of branches, where L_o can be obtained by using the Monte Carlo simulation of the following equation.

$$L_o = \arg\max_{1\le L\le N_t}\left\{E\{\frac{1}{L}(\sum_{i=N_t-L+1}^{N_t}|h_{n_i}|)^2\}\right\}. \quad (14)$$

L_o for different N_t is tabulated in Tab. I, which is obtained by running 600,000 channel realizations using (14). The EGT with fixed number L_o of transmit branches using Tab. I is called the "EGT with fixed TAS" for short.

TABLE I
OPTIMAL L FOR FIXED NUMBER OF TRANSMIT BRANCHES.

N_t	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17
L_o	3	3	4	5	6	6	7	8	8	9	10	11	11	12	13

N_t	18	19	20	21	22	23	24	25	26	27	28	29	30	31	32
L_o	14	14	15	16	17	17	18	19	20	20	21	22	23	23	24

Using Jensen's Inequality, *i.e.* $E\{g(x)\} \geq g(E\{x\})$, (13) can be rewritten as

$$\begin{aligned} E\{\rho_{e-miso-as}\} &\geq \frac{E_x}{N_0}\frac{1}{L_o}(E\{\sum_{i=N_t-L_o+1}^{N_t}|h_{n_i}|\})^2 \\ &= \frac{E_x}{N_0 L_o}(\sum_{i=N_t-L_o+1}^{N_t}E\{|h_{n_i}|\})^2 \quad (15) \end{aligned}$$

Obtaining the lower bound for $E\{\rho_{e-miso-as}\}$ in (15) requires $\sum_{i=N_t-L_o+1}^{N_t}E\{|h_{n_i}|\}$, which is actually the same problem of deriving the mean SNR of the receive EGC with the L_o strongest branches. Deriving the PDF of $\sum_{i=N_t-L_o+1}^{N_t}|h_{n_i}|$ is difficult; in fact, even a closed-form PDF result for $\sum_{i=1}^{L}|h_i|$ has not been solved for $L > 2$, see [9], [10] and [15]. However, deriving an accurate approximation for the mean of $\sum_{i=N_t-L_o+1}^{N_t}|h_{n_i}|$ may be possible. The derivation needs the following lemma.

Lemma 4: Let $\mu_{N_t:r}$ be the mean of the r-th smallest value from N_t samples, and $\mu_{i:i}$ be the mean of the maximum value from i samples. $\mu_{N_t:r}$ can be expressed by (see p. 45 in [18])

$$\mu_{N_t:r} = \sum_{i=r}^{N_t}(-1)^{i-r}\binom{i-1}{r-1}\binom{N_t}{i}\mu_{i:i}. \quad (16)$$

■

If the distribution is Rayleigh, $\mu_{N_t:N_t} = E\{|h_{n_{N_t}}|\}$.

Theorem 2: Let $\mathbf{h}^t$ be a $1 \times N_t$ channel vector whose elements have i.i.d. complex Gaussian distribution with zero mean and variance σ_h^2. The average SNR of the EGT with TAS can be approximately lower bounded by

$$\begin{aligned} &E\{\rho_{e-miso-as}\} \geq \frac{E_x}{N_0}\frac{1}{L_o}(\sum_{i=N_t-L_o+1}^{N_t}E\{|h_{n_i}|\})^2 \\ &\approx \frac{E_x}{N_0}\frac{\sigma_h^2}{L_o}\left(\sum_{r=N_t-L_o+1}^{N_t}\sum_{i=r}^{N_t}(-1)^{i-r}\binom{i-1}{r-1}\binom{N_t}{i}\right. \\ &\left.\cdot\left(\sqrt{\alpha(i)} - \frac{1}{8}\beta(i)\sqrt{\frac{1}{\alpha^3(i)}}\right)\right)^2, \quad (17) \end{aligned}$$

where

$$\alpha(i) = (1 + \sum_{n=2}^{i}\frac{1}{n}) \text{ and } \beta(i) = (1 + \sum_{n=2}^{i}\frac{1}{n^2}). \quad (18)$$

Proof: The mean value of the function $g(x)$ can be approximated by (see p. 150 in [19])

$$E\{g(x)\} \approx g(E\{x\}) + g''(E\{x\})\frac{Var\{x\}}{2}. \quad (19)$$

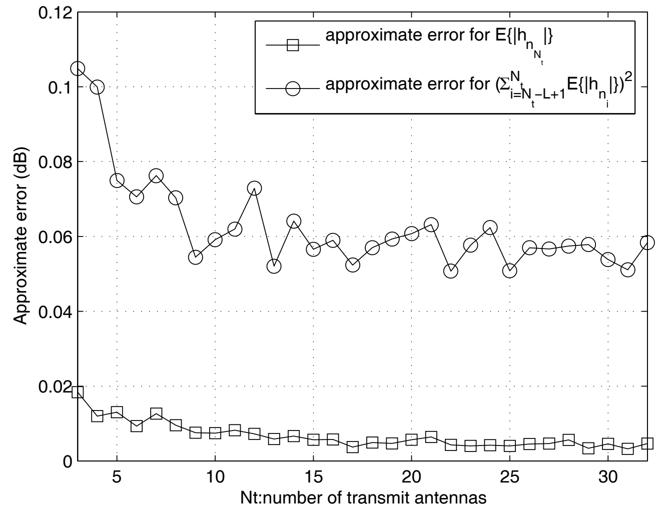

Fig. 2. Approximation error for the mean value of the order statistics from Rayleigh distribution.

Letting $g(x) = \sqrt{x}$ leads to the following approximation:

$$\begin{aligned} E\{|h_{n_{N_t}}|\} &\approx \sqrt{E\{|h_{n_{N_t}}|^2\}} - \frac{1}{8}Var\{|h_{n_{N_t}}|^2\} \\ &\cdot\sqrt{\frac{1}{(E\{|h_{n_{N_t}}|^2\})^3}}. \quad (20) \end{aligned}$$

The value of $E\{\sum_{i=N_t-L+1}^{N_t}|h_{n_i}|^2\}$ was first derived in [11], and later a simpler derivation using virtual branch analysis was given in [12] to find the follows:

$$E\{\sum_{i=N_t-L+1}^{N_t}|h_{n_i}|^2\} = L(1 + \sum_{n=L+1}^{N_t}\frac{1}{n})\sigma_h^2, \quad (21a)$$

$$Var\{\sum_{i=N_t-L+1}^{N_t}|h_{n_i}|^2\} = L(1 + \sum_{n=L+1}^{N_t}\frac{1}{n^2})\sigma_h^4. \quad (21b)$$

Letting $L = 1$ in (21) and using (18) results in the following equalities:

$$E\{|h_{n_{N_t}}|^2\} = \alpha(N_t)\sigma_h^2 \text{ and } Var\{|h_{n_{N_t}}|^2\} = \beta(N_t)\sigma_h^4. \quad (22)$$

Applying (22) to (20) leads to $E\{|h_{n_{N_t}}|\} \approx \sigma_h\left(\sqrt{\alpha(N_t)} - \frac{1}{8}\beta(N_t)\sqrt{\frac{1}{\alpha(N_t)^3}}\right)$. The result in (17) can therefore be reached, by using (15), (16) and the above approximation. ■

The approximation of $E\{|h_{n_{N_t}}|\}$ is accurate. The approximation error for $E\{|h_{n_{N_t}}|\}$ is shown in Fig. 2, by the Monte Carlo simulation for 600,000 channel realizations. For $N_t \geq 8$, the approximation error is less than 0.01 dB. Additionally, the approximation error of $(\sum_{i=N_t-L_o+1}^{N_t}E\{|h_{n_i}|\})^2$ in (17) is less than 0.1 dB for $N_t > 3$. From Theorem 2, the SNR loss between MRT and the proposed EGT with TAS in MISO channels is approximately upper bounded by

$$\frac{E\{\rho_o\}}{E\{\rho_{e-miso-as}\}} \leq \frac{L_o E\{\|\mathbf{h}\|_2^2\}}{(\sum_{i=N_t-L_o+1}^{N_t}E\{|h_{n_i}|\})^2}. \quad (23)$$

B. Performance analysis of proposed TAS in MIMO EGT.

Using Lemma 2 and similar procedure for MISO case, (12) can be expressed as

$$E\{\rho_{e-mimo-as}\} \geq \frac{E_x}{N_0}E\{\sigma_1^2\}\max_{1\leq L\leq N_t}\left\{E\{\frac{1}{L}(\sum_{i=N_t-L+1}^{N_t}|v_{n_i}|)^2\}\right\} \quad (24)$$

The lower bound of $E\{\rho_{e-mimo-as}\}$ is achieved by using fixed TAS with fixed number L_o of antennas. L_o can be determined by running the Monte Carlo simulation of the following equation:

$$L_o = \arg\max_{1\leq L\leq N_t}\left\{E\{\frac{1}{L}(\sum_{i=N_t-L+1}^{N_t}|v_{n_i}|)^2\}\right\}. \quad (25)$$

When L_o is applied to (24), the resulting SNR is exactly the average SNR of MIMO EGT with fixed TAS, which can be regarded as the lower bound SNR for MIMO EGT with TAS. A much easier alternative to determine L_o for MIMO channels is introduced in the following theorem.

Theorem 3: Assume the equivalent channel $\tilde{\mathbf{h}} = \mathbf{u}_1\mathbf{H}^\dagger$ is used to perform antenna selection. Let L_o be the best fixed number of transmit antennas for EGT with fixed TAS. The L_o values are independent of number of receive antennas; that is, the L_o values in Tab. I originally designed for MISO channels are also the best values for MIMO channels.

Proof. From (25), $\mathbf{v}_1$ has the same distribution with $\mathbf{h}^*/\|\mathbf{h}\|_2$. Hence, L_o is determined by

$$L_o \equiv \arg\max_{1\leq L\leq N_t}\left\{E\{\frac{1}{L}(\sum_{i=N_t-L+1}^{N_t}\frac{|h_{n_i}|}{\|\mathbf{h}\|_2})^2\}\right\}$$
$$\equiv \arg\max_{1\leq L\leq N_t}\left\{E\{\frac{1}{L}(\sum_{i=N_t-L+1}^{N_t}|h_{n_i}|)^2\}\right\}. \quad (26)$$

The theorem is proved since the final expression in (26) is the same with that in (14). ■

With the result in Theorem 3, if the equivalent channel $\tilde{\mathbf{h}} = \mathbf{u}_1\mathbf{H}^\dagger$ is used to perform antenna selection, there is no need to determine L_o for different N_r in MIMO channels. This not only simplifies the off-line computational effort but also reduces the memory requirement to store L_o for different N_r, if EGT with fixed TAS is used. From (24) and using the fact that $\mathbf{h}^*/\|\mathbf{h}\|_2$ and $\mathbf{v}_i$ have the same distribution, the SNR loss is upper bounded by

$$\frac{E\{\rho_o\}}{E\{\rho_{e-mimo-as}\}} \leq \frac{L_o}{(\sum_{i=N_t-L_o+1}^{N_t}E\{|h_{n_i}|/\|\mathbf{h}\|_2\})^2}. \quad (27)$$

VI. Remarks on EGT with TAS and EGT with Fixed TAS

For description convenience, let us call the proposed EGT with TAS as “EGT+TAS”, and EGT with fixed TAS as“EGT+fixed TAS”. The proposed EGT+fixed TAS has several advantages. First, its SNR loss is smaller than EGT. Second, each antenna employs constant power like EGT does; hence, design effort for PAs can be greatly reduced, compared to MRT. Third, except for $N_t \leq 3$, the required number of RF components is fewer than that of EGT because $L_0 < N_t$.

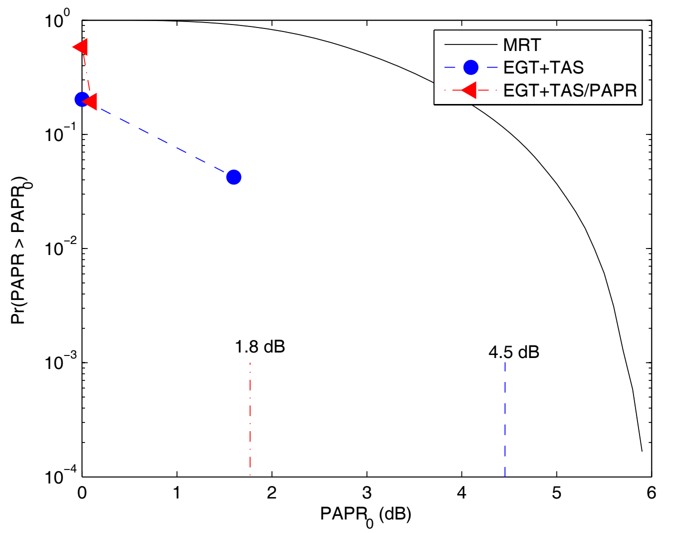

Fig. 3. PAPR comparison of MRT and EGT+TAS, and the corresponding PAPR reduction scheme.

Although the proposed EGT+TAS outperforms EGT+fixed TAS, it does not have the second and the third advantages of EGT+fixed TAS. EGT+TAS does not have constant power since each antenna needs to support a dynamic range of transmit power E_x/L, where $L = 1,\ 2, \cdots N_t$. Nevertheless, even if EGT+TAS does not have constant power, its PAPR is still smaller than MRT. The PAPR comparison between EGT+TAS and MRT for $N_t = 4$ and QPSK modulation is shown in Fig. 3. Let $E_x = 1$; the power levels for EGT+TAS are $1/4$, $1/3$, $1/2$ and 1. Since the four power levels do not occur in equal probability, we use Monte Carlo simulation to obtain the average power, which is 0.3535. Hence, the PAPR can be obtained by using the formula $\frac{E_x/L}{0.3535}$, and they are -1.49 dB, -0.24 dB, 1.52 dB and 4.53 dB, which are shown in Fig. 3. From the figure, EGT+TAS (circle-curve) outperforms MRT (solid-curve) in terms of PAPR.

If a slight performance degradation is allowed, the PAPR for EGT+TAS can be considerably reduced. That is, if the numbers of selected antennas that are fewer than P are not considered in EGT+TAS, where P is a non-negative integer, the performance does not degrade much while the PAPR can be reduced. Taking $N_t = 4$ for instance, if $P = 1$, the possible numbers of selected antennas are 2, 3 and 4; selecting only one antenna is not considered in this case. Tab. II shows the suggested P values for different N_t that degrade system performance by less than 0.01 dB and 0.025 dB, respectively. Please note that ‘less than’ is used because actual performance loss is smaller than these two values. For instance, for $N_t = 8$, the actual loss by letting $P = 3$ is 0.006 dB (< 0.01 dB), and letting $P = 4$ is 0.02 dB (< 0.025 dB). Using this method and letting $P = 1$, the PAPR performance of EGT+TAS with $N_t = 4$ is greatly improved as shown in the triangular-curve in Fig. 3; the power levels in this case are $1/4$, $1/3$ and $1/2$, and the average power is 0.33. Hence the corresponding PAPR are -1.21 dB, 0.04 dB and 1.8 dB. For description convenience,

TABLE II
SUGGESTED P FOR EGT+TAS WITH PAPR REDUCTION.

N_t	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17
P within 0.01 dB loss	0	0	1	1	2	3	4	4	5	6	7	8	8	9	10
P within 0.025 dB loss	0	1	1	2	3	4	5	5	6	7	8	9	9	10	11
N_t	18	19	20	21	22	23	24	25	26	27	28	29	30	31	32
P within 0.01 dB loss	11	11	12	13	14	15	15	16	17	18	18	19	20	21	22
P within 0.025 dB loss	12	13	14	14	15	16	17	18	18	19	20	21	22	23	23

we called this method "EGT+TAS/PAPR" in the following sections.

VII. DESIGN STRATEGIES FOR NON-TDD SYSTEMS

For all the four proposed EGT schemes, *i.e.* the MIMO EGT in Sec. III, EGT+fixed TAS, EGT+TAS, and EGT+TAS/PAPR, multiplying $e^{j\phi}$ does not affect the resulting instantaneous SNR, *i.e.* $\|e^{j\phi}\mathbf{H}\mathbf{f}_e\|_2^2 = \|\mathbf{H}\mathbf{f}_e\|_2^2$, where $\mathbf{f}_e$ denotes the proposed beamforming vectors. Take EGT+TAS in MIMO channels for instance; some elements of the equivalent MISO channel $\tilde{\mathbf{h}}$ may not be selected if EGT+TAS is applied. Let $\hat{\mathbf{h}}$ be the $l_o \times 1$ vector that eliminates the unselected elements of $\tilde{\mathbf{h}}$ without reordering. Let $\hat{h}_1$ be the first element of $\hat{\mathbf{h}}$. The n_k-th element of $\mathbf{f}_{e-mimo-as}$ becomes (refer to Algorithm 2)

$$[\mathbf{f}_{e-mimo-as}]_{n_k} = \begin{cases} \frac{e^{-j(\angle \tilde{h}_{n_k} - \angle \hat{h}_1)}}{\sqrt{l_o}}, & N_t - l_o + 1 \leq k \leq N_t; \\ 0, & \text{otherwise.} \end{cases} \tag{28}$$

From (28), the phase quantization for antenna 1 of the selected antennas can be waived.

To conduct beamforming, the transmitter needs to know *1.)* antenna indices of the selected antennas, and *2.)* quantized beamforming vectors. Let us discuss these two items as follows:

Item 1: Index representations of selected antennas. For EGT+TAS, it demands N_t bits to indicate the selection status of each antenna. When N_t is large, many bits are required. Fortunately, both EGT+TAS/PAPR and EGT+fixed TAS can somewhat overcome this issue. For EGT+TAS/PAPR, the number of selected antennas should be greater than P. Hence, there are totally $2^{N_t} - \sum_{i=0}^{P} \binom{N_t}{i}$ possible selection combinations. Taking $N_t = 4$ and $P = 1$ for instance, if each bit indicates the selection status of one antenna and 1 represents the status "selected", then, 0000, 1000, 0100, 0010 and 0001 denote the selection combinations that are not considered. Hence there are totally 11 selection combinations in this case. For EGT+fixed TAS, the number of selected antenna L_o is fixed. Thus there are $\binom{N_t}{L_o}$ possible selection combinations. Take $N_t = 4$ for instance; $L_o = 3$ by Tab. I; hence there are $\binom{4}{3} = 4$ possible selection combinations.

Now consider Item 2. Phase information can be quantized by vector quantization (VQ) or scalar quantization (SQ) as follows.

Item 2.1: Quantizing beamforming vectors using VQ. For VQ, we use the optimal Lloyd codebook for EGT proposed by Murthy and Rao in [2]. Let the number of Lloyd codewords be M, the code rates B including Items 1 and 2 are $\log_2 M 2^{N_t}$ for EGT+TAS, $\log_2 M(2^{N_t} - \sum_{i=0}^{P} \binom{N_t}{i})$ for EGT+TAS/PAPR, and $\log_2 M \binom{N_t}{L_o}$ for EGT+fixed TAS.

It is worth to point out that the numbers of Lloyd codewords for EGT and for the proposed EGT with antenna selection schemes are different. For instance, if $N_t = 4$ and $B = 10$, the number of Lloyd codewords for EGT is 1024, but that for EGT+fixed TAS is 256, since 2 of the 10 bits are used for antenna selection. Therefore, the proposed EGT+fixed TAS not only requires smaller memory to store the codebook (thus fewer computations in exhaustive search), but also requires fewer number of RF components than EGT. Moreover, later simulation results show that with smaller B, EGT+fixed TAS can still outperform EGT.

Item 2.2: Quantizing beamforming vectors using SQ. Next, let us discuss SQ. Previous works allocate integer bits in each antenna to perform scalar quantization, *e.g.* [3],[4]; hence the number of quantization levels in each antenna is 2^{b_i}, where b_i is the assigned bit number in antenna i, $1 < i \leq N_t$. Although such scalar quantization always leads to integer bits to represent the codewords, it may not be efficient due to the following reasons: First, if $b_i = b$ for $1 < i \leq N_t$, each antenna has the same bits. The allowable code rate in this case is $(N_t - 1)b$. Second, if each antenna has different bits, the transmitter needs to know bit allocation table in advance [3]. To overcome these issues, we may let the number of quantization levels in each antenna be a positive integer C. Since there are $N_t - 1$ elements to be quantized, there are totally C^{N_t-1} codewords. For instance, if $C = 5$ and $N_t = 4$, there are $5^3 = 125$ codewords. Such SQ can be conducted as follows: Let $\Delta = 2\pi/C$. The quantized phase $\hat{\phi}_{n_k}$ is obtained by the following equation:

$$\hat{\phi}_{n_k} = (\text{round}\{\phi_{n_k}/\Delta\}) \cdot \Delta, \text{ for } N_t - l_o + 1 \leq k \leq N_t,$$

where round{.} is the rounding function. Therefore, the code rates B including Item 1 and Item 2 are $\log_2 C^{N_t-1} 2^{N_t}$ for EGT+TAS, $\log_2 C^{N_t-1}(2^{N_t} - \sum_{i=0}^{P} \binom{N_t}{i})$ for EGT+TAS/PAPR, and $\log_2 C^{N_t-1} \binom{N_t}{L_o}$ for EGT+fixed TAS. Taking EGT+fixed TAS with $N_t = 8$ for instance, $L_o = 6$; letting $C = 3$ leads to the code rate $B = 15.9$.

VIII. SIMULATION RESULT

Two kinds of performance curves were shown, *i.e.* the SNR loss and the BER (bit error rate). The simulation was conducted using the following settings: The channel coefficients were assumed to be i.i.d. complex Gaussian distributed with zero mean. 60,000 different channel realizations were used to evaluate the performance. The 16-QAM modulation was used for the BER evaluation. The notation mTnR denotes $N_t = m$ and $N_r = n$.

Example 1: **SNR Loss as a function of N_t:** The SNR loss between MRT and various EGT schemes are shown in Figs. 4-5, for $N_r = 1$ (MISO channel) and $N_r = 2$ (MIMO channel), respectively. Observations are summarized as follows: *1.)* As N_t increases, the SNR loss of EGT increases (toward the upper bound of 1.05 dB). When $N_t = 32$, the loss is around 1.01 dB for MISO channel and that is around 0.9 dB for MIMO channels. This is reasonable because the SNR loss of the proposed EGT in MIMO channels is smaller than that

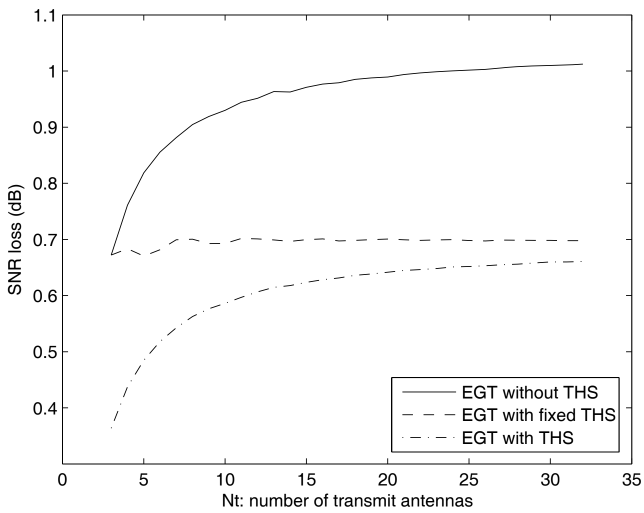

Fig. 4. SNR loss as a function of the number of transmit antennas for $N_r = 1$.

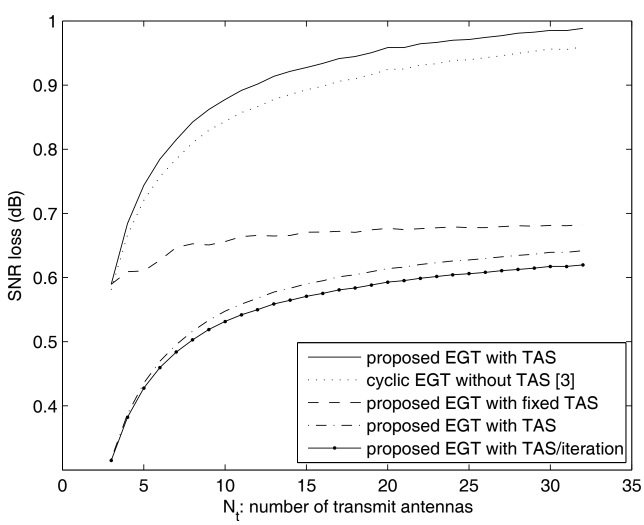

Fig. 5. SNR loss as a function of the number of transmit antennas for $N_r = 2$.

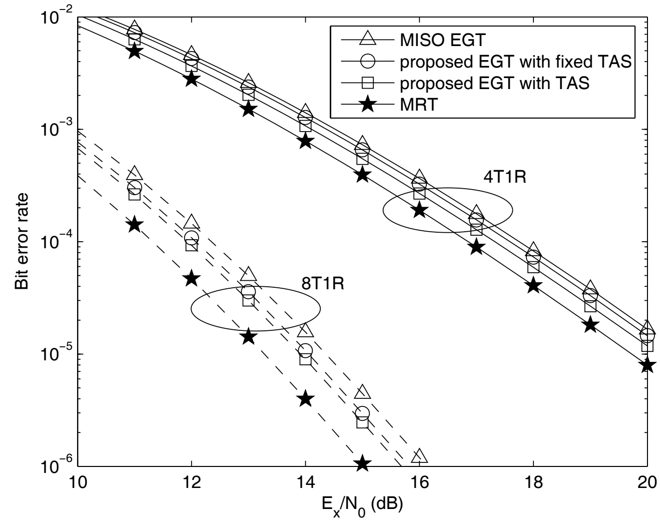

Fig. 6. BER comparison of MRT and the EGT proposals for $N_r = 1$.

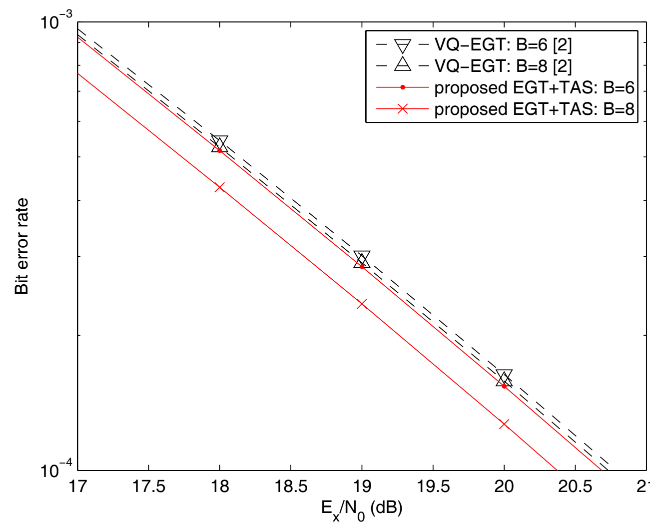

Fig. 7. BER comparison of VQ-EGT and VQ-EGT+TAS in a 3T1R channel.

in MISO channels (see Sec. III). *2.)* The SNR loss between MRT and the proposed EGT+TAS is around 0.32-0.64 dB in MIMO channels and around 0.35-0.67 dB in MISO channel; the improvement of the EGT+TAS is hence more pronounced in MIMO channels than in MISO channels. *3.)* The SNR loss between the proposed EGT+TAS and EGT+fixed TAS is less than 0.1 dB for $N_t \geq 16$. More specifically, TAS can improve the performance of EGT by around 0.3-0.35 dB, and fixed TAS can improve the performance of EGT by up to 0.3 dB. *4.)* From Fig. 5, for large N_t, the performance gap between the proposed EGT and the cyclic EGT in [3], which requires iterations and could be served as a benchmark for MIMO EGT, is only around 0.035 dB. *5.)* If iterations are applied in the proposed EGT+TAS, the performance can be slightly improved by around 0.02 dB, where the number of iterations is 20 in this example.

Example 2: **BER comparison of MRT, EGT, EGT+fixed TAS and EGT+TAS:** The BER of MRT and proposed EGT schemes, as a function of SNR for $N_r = 1$ is shown in Fig. 6. For $N_t = 8$, the performance gap between MRT and EGT+TAS is less than 0.65 dB and that between MRT and EGT+fixed TAS is less than 0.8 dB; the above performance gaps decreases from 0.65 to 0.45 dB, and from 0.8 to 0.7 dB, as N_t decreases from 8 to 4, respectively.

Example 3: **BER comparison of EGT designs with VQ:** In this example, BER performance of various EGT designs are compared. The Lloyd codebooks for EGT proposed in [2] is used. Let $N_t = 3$ and $N_r = 1$. Fig. 7 shows the performance of EGT and the proposed EGT+TAS. The code rate (number of feedback bits) is B. For EGT+TAS, 3 bits of B are used to indicate the selection status. From Fig. 7, EGT+TAS with $B = 6$ outperforms EGT with $B = 8$. Moreover, EGT+TAS with $B = 8$ outperforms EGT with $B = 8$ by around 0.4 dB.

Now let $N_t = 4$ and $N_r = 1$. Fig. 8 shows the performance of EGT, EGT+fixed TAS and EGT+TAS/PAPR. For

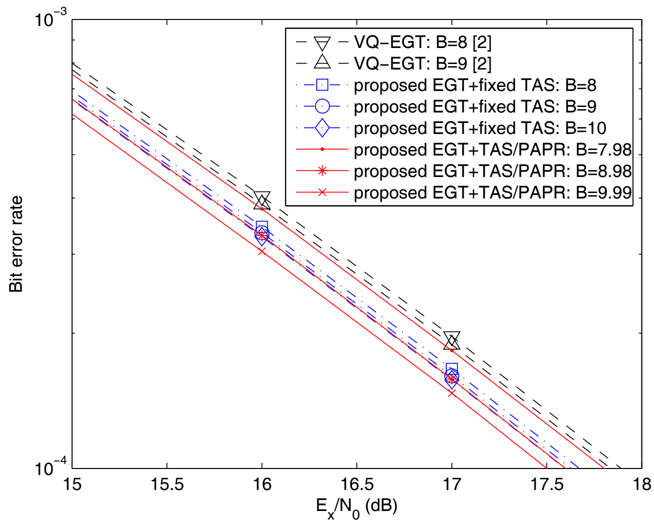

Fig. 8. BER comparison of VQ-EGT, VQ-EGT+TAS and VQ-EGT+TAS/PAPR in a 4T1R channel.

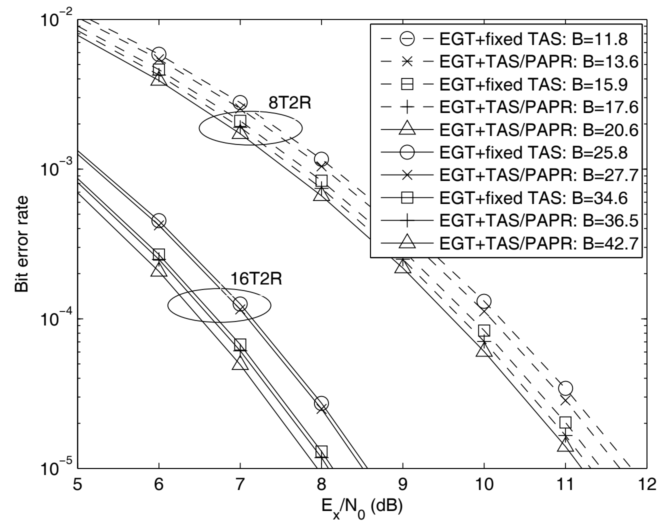

Fig. 9. BER comparison of the proposed SQ-EGT+TAS and SQ-EGT+TAS/PAPR in 8T2R and 16T2R channels.

EGT+fixed TAS, $L_o = 3$ for $N_t = 4$ according to Tab. I; hence, 2 bits, *i.e.* $\log_2 \binom{4}{3}$, of B are used to indicate the selection status. For EGT+TAS/PAPR, we let $P = 1$ and thus the performance degradation is less than 0.025 dB according to Tab. II; that is, there are $2^4 - \binom{4}{0} - \binom{4}{1} = 11$ possible selection combinations and this is equivalent to 3.46 bits; hence, if the numbers of Lloyd codewords are $M = 23$, 46 and 93, the corresponding code rates are $B = \log_2 11 \cdot 23 = 7.98$, $\log_2 11 \cdot 46 = 8.98$ and $\log_2 11 \cdot 93 = 9.99$, respectively. From Fig. 8, both the proposed EGT+fixed TAS with $B = 8$ and EGT+TAS/PAPR with $B = 7.98$ outperform EGT with $B = 9$; also, the number of required RF components for the proposed EGT+fixed TAS is smaller than that for EGT. Moreover, EGT+fixed TAS with $B = 8$ outperforms EGT+TAS/PAPR with $B = 7.98$. This is reasonable since EGT+TAS/PAPR uses $B = 3.46$ to indicate selection status, which is 1.46 bits more than that for EGT+fixed TAS. When B increases, more bits are used for Lloyd codebooks. In this case, EGT+TAS/PAPER with $B = 9.99$ outperforms EGT+fixed TAS with $B = 10$.

***Example 4:* Proposed EGT designs with SQ:** When the number of transmit antennas is large, constructing the optimal Lloyd codebook may become difficult. Scalar quantization is a suitable solution for this case. The BER performance of the proposed EGT designs with scalar quantization in 16T2R and 8T2R channels is shown in Fig. 9. For EGT+TAS/PAPR, we let the performance loss be less than 0.025 dB; the corresponding code rate can then be obtained using Tab. II and the discussion in Sec. VII. Take $N_t = 16$ for instance; $P = 10$ according to Tab. II, and there are $2^{16} - \sum_{i=1}^{10} \binom{16}{i} = 6886$ different selection combinations. If $C = 3$, the corresponding code rate is $B = \log_2 3^{(16-1)} \cdot 6886 = 36.5$, which is shown in the solid-plus curve in Fig. 9. Constructing optimal codebook in these cases might be difficult; moreover, the required memory and computational time are also prohibitive. In contrast, using scalar quantization can waive the memory; the corresponding computational complexity is also low.

IX. Conclusion

An EGT design for MIMO channels was proposed; the corresponding theoretical results showed that the maximum SNR loss between the proposed EGT and the optimal MRT (both employing MRC at the receiver) is approximately 1.05 dB in MIMO channels; the approximated result was compared to Morte Carlo simulation result, and the difference between the two results is less than 0.003 dB. Moreover, this study proposed to adopt antenna selection for further improving the performance of EGT. Using the proposed antenna selection algorithms in EGT, the SNR loss between EGT and MRT could be reduced to as low as to 0.45-0.65 dB for N_t ranging from 4 to 8 in MIMO channels. As a result, a transceiver with low complexity could be realized using the proposed EGT+TAS, EGT+TAS/PAPR and EGT+fixed TAS, in a penalty of slight performance degradation. Furthermore, we also discussed the design strategies to use the proposed EGT and antenna selection algorithms in non-TDD systems.

X. Appendix

A. *Proof of Lemma 3.*

Using the following equality:

$$\frac{\|\mathbf{h}\|_1^2}{\|\mathbf{h}\|_2^2} = 1 + \sum_{i=1}^{N_t} \sum_{j=1, j\neq i}^{N_t} \frac{|h_i||h_j|}{\|\mathbf{h}\|_2^2}, h_i, h_j \in \mathbf{h},$$

results in

$$E\left\{\frac{\|\mathbf{h}\|_1^2}{\|\mathbf{h}\|_2^2}\right\} = 1 + N_t(N_t-1)E\left\{\frac{|h_i||h_j|}{\|\mathbf{h}\|_2^2}\middle| h_i, h_j \in \mathbf{h}\right\}. \quad (29)$$

Let the mean values of x and y be μ_x and μ_y, respectively. The mean value of a function $g(x, y)$ of two random variables can be approximated by (see p. 215 in [19]):

$$E\{g(x,y)\} \approx g(\mu_x, \mu_y) + \frac{1}{2}\left(\frac{\partial^2 g(\mu_x,\mu_y)}{\partial x^2}\sigma_x^2 + 2\frac{\partial^2 g(\mu_x,\mu_y)}{\partial x \partial y}C_{xy} + \frac{\partial^2 g(\mu_x,\mu_y)}{\partial y^2}\sigma_y^2\right), \quad (30)$$

where C_{xy} is the covariance defined by $C_{xy} = E\{xy\} - \mu_x\mu_y$. Let $x = |h_i||h_j|$, $y = \|\mathbf{h}\|_2^2$ and $g(x,y) = x/y$; (30) can be rewritten as

$$E\left\{\frac{x}{y}\right\} \approx \frac{\mu_x}{\mu_y} + \frac{1}{2}\left(\frac{-2}{\mu_y^2}C_{xy} + \frac{2\mu_x}{\mu_y^3}\sigma_y^2\right), \qquad (31)$$

μ_x and μ_y were shown in [15] and [4] to be $\mu_x = \frac{\pi}{4}\sigma_h^2$ and $\mu_y = N_t\sigma_h^2$, respectively. Since y has a chi-square distribution, σ_y^2 is $\sigma_y^2 = N_t\sigma_h^4$. To determine C_{xy}, we first need $E\{xy\}$. From the definition of x and y, we have

$$\begin{aligned} E\{xy\} &= E\left\{\|\mathbf{h}\|_2^2|h_i||h_j| \;\middle|\; h_i, h_j \in \mathbf{h}\right\} \\ &= E\left\{|h_i|^3|h_j|\right\} + E\left\{|h_i||h_j|^3\right\} \\ &\quad + \sum_{k=1,k\neq i,j}^{N_t} E\left\{|h_k|^2|h_i||h_j|\right\} \\ &= 2E\left\{|h_i|^3\right\}E\left\{|h_j|\right\} \\ &\quad + (N_t-2)E\left\{|h_k|^2\right\}\left(E\left\{|h_i|\right\}\right)^2. \end{aligned} \qquad (32)$$

From [19], $E\left\{|h_i|^3\right\} = \frac{3}{4}\sqrt{\pi}\sigma_h^3$. Hence, we have

$$E\{xy\} = \frac{1}{4}N_t\pi\sigma_h^4 + \frac{1}{4}\pi\sigma_h^4 \text{ and } C_{xy} = \frac{\pi\sigma_h^4}{4}.$$

Consequently, $\left(\frac{-2}{\mu_y^2}C_{xy} + \frac{2\mu_x}{\mu_y^3}\sigma_y^2\right)$ in (31) is zero, and thus $E\left\{\frac{|h_i||h_j|}{\|\mathbf{h}\|_2^2}\middle| h_i, h_j \in \mathbf{h}\right\} = \frac{\pi}{4N_t}$. The result in (9) can hence be obtained using (29) and the above equality.

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Shang-Ho (Lawrence) Tsai received the Ph.D. degree in Electrical Engineering from the University of Southern California (USC), USA, in Aug. 2005. From June 1999 to July 2002, he was with the Silicon Integrated Systems Corp. (SiS), where he participated the VLSI design for DMT-ADSL systems. From Sep. 2005 to Jan. 2007, he was with the MediaTek Inc. (MTK) and participated the VLSI design for MIMO-OFDM systems. Since Feb. 2007, he joined the Department of Electrical and Control Engineering (now Department of Electrical Engineering) at the National Chiao Tung University where he is now an assistant professor. His research interests include signal processing for communications, particularly the areas of OFDM as well as MIMO systems. He is also interested in Ultra Wideband (UWB) and the VLSI design related to above topics. He was awarded a government scholarship for overseas study from the Ministry of Education, Taiwan, in 2002-2005.