Low-Overhead Cooperative Beamforming Under Imperfect Quantized SNR of Source-to-Relay Links

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***Abstract*-We consider a cooperative beamforming system with the amplify-and-forward protocol. To reduce the signaling overhead, each relay quantizes the SNR of the source-to-relay (S-R) link into one bit, which is then transmitted through a binary symmetric channel (BSC) with a known crossover probability to the destination. Given the set of the error-corrupted one-bit messages received at the destination, the beamforming design criterion is the maximization of the expected receive SNR, averaged over the conditional bit-flipping distributions of BSCs. We derive an analytic expression for the considered SNR metric, which is a complicated function of the beamforming weights. To facilitate analysis, we further derive a tractable lower bound for the conditional average SNR. By conducting maximization with respect to this lower bound, a closed-form sub-optimal beamformer can be obtained as a solution to a generalized eigenvalue problem. Computer simulations are used to illustrate the performance of the proposed scheme.**

***Index Terms*—**Cooperative communications; amplify and forward; relay; beamforming; overhead reduction; quantization.

I. INTRODUCTION

Cooperative communications is a well-known technique for exploiting distributed spatial resource for performance enhancement in modern wireless networks [1-2]. To realize various performance gains benefiting from user collaboration, the channel state information (CSI) of the source-to-relay (S-R) and relay-to-destination (R-D) links is needed at the relays/destination [1-2] in order to facilitate the design of efficient signal relaying schemes. As a result, communication overheads dedicated to link CSI transmission or feedback become necessary. Considering that energy efficiency is a critical demand in next-generation wireless systems, the reduction, or minimization, of intra-network communication is rather crucial. The development of low-overhead information relaying techniques therefore has been an emerging research topic [3-9]. Besides, almost all existing studies of cooperative communications rely on the idealized assumption that the transmission and reception of inter-node link CSI are perfect. However, such a design paradigm may not be realistic in many large-scale system scenarios. For example, in wireless sensor networks wherein small-size sensor nodes with limited power resources are deployed far away from the fusion center (FC), the transmission of the local CSI via wireless links to the FC could be subject to severe path loss and deep fading [10], resulting in distorted or even erroneous CSI received at the FC. Hence, in addition to the low-energy-consumption demand, designs of relay systems that further take account of imperfect CSI transmission/feedback are also of much importance. Such an issue, however, remains much to be investigated in the literature.

This paper presents an original study of the aforementioned problem in the cooperative beamforming setup. We consider the cooperative beamforming scheme employing the amplify-and-forward (AF) relaying protocol. To reduce the signaling overhead, each relay quantizes the signal-to-noise ratio (SNR) of the S-R link into one bit, which is then sent to the destination for beamforming design. Rather than assuming that the quantized bit is received at the destination without errors, we consider the general case that the one-bit message could be flipped by a binary symmetric channel (BSC) with a known nonzero crossover probability. Given the one-bit messages received from all relays, the beamforming design criterion adopted at the destination is the maximization of the receive SNR averaged with respect to the conditional channel flipping distributions. A closed-form formula for the proposed SNR metric is first derived. The formula is seen to be a highly nonlinear function of the beamforming factors, and direct maximization of this objective function is quite difficult. For analytic tractability, we further derive a lower bound of the conditional average SNR that can be expressed as a generalized Rayleigh quotient [11]. By conducting maximization with respect to this lower bound, a closed-form suboptimal beamformer can be obtained as the solution to a generalized eigenvalue problem. Computer simulations are used to illustrate the performance of the proposed solution. The rest of this paper is organized as follows. Section II is the preliminary. Section III presents the main results of this paper. Section IV is the conclusion.

II. PRELIMINARY

A. Cooperative Beamforming System

We consider the cooperative beamforming system, in which L relays employ the AF protocol to collaboratively transmit the common source signal $x[n] \in \{-1,1\}$ to the destination. The received signal at the ith relay is

$$y_{s_i}[n] = \sqrt{P_s} h_{s,i} x[n] + v_i[n], \quad (2.1)$$

where P_s is the source transmit power, $h_{s,i} \sim \mathcal{CN}(0,\sigma_s^2)$ is the channel gain of the ith S-R link[1], and $v_i[n] \sim \mathcal{CN}(0,\sigma_v^2)$ is the receive noise at the ith relay. Based on (2.1), the SNR of the ith S-R link is thus

$$\gamma_{s_i} \triangleq \frac{P_s \left|h_{s,i}\right|^2}{\sigma_v^2}. \quad (2.2)$$

The received signal at the destination reads

$$y_d[n] = \sum_{i=1}^{L} h_{r,i} G_i g_i y_{s_i}[n] + w[n], \quad (2.3)$$

where $h_{r,i} \sim \mathcal{CN}(0,\sigma_r^2)$ denotes the ith R-D channel gain, $G_i = \dfrac{1}{h_{s,i}\sqrt{P_s(1+\gamma_{s_i}^{-1})}}$ is the power normalization factor, g_i is the ith beamforming weight, and $w[n] \sim \mathcal{CN}(0,\sigma_w^2)$ represents the receive noise at the destination. With (2.1), $y_d[n]$ in (2.3) can be expressed as

$$y_d[n] = \sum_{i=1}^{L} \frac{h_{r,i} g_i}{\sqrt{1+\xi_{s_i}}} x[n] + \sum_{i=1}^{L} \frac{h_{r,i} g_i \sqrt{\xi_{s_i}}}{\sqrt{1+\xi_{s_i}}} \overline{v}_i[n] + w[n], \quad (2.4)$$

where $\xi_{s_i} \triangleq 1/\gamma_{s_i}$ is the reciprocal of the SNR of the ith S-R link, and $\overline{v}_i[n] \sim \mathcal{CN}(0,1)$. To design the beamforming weights g_i's, one commonly used approach is to conduct SNR maximization based on the knowledge of the CSI of the S-R and R-D communication links. This paper focuses on the low-overhead cooperative beamforming scheme, wherein the ith relay quantizes the SNR of the ith S-R link (see (2.2)) into one bit $q_i \in \{0,1\}$. Assuming that (i) $\{q_1,\cdots,q_L\}$ are received at the destination without errors, and (ii) the CSI of all the R-D links is perfectly known at the destination, the SNR conditioned on either $x[n]=1$ or $x[n]=-1$, is shown to be [12][2]

$$\gamma_{dq}(\mathrm{g},\mathrm{h}_{\mathrm{r}},\mathrm{q}) = \frac{\left|\sum_{i=1}^{L} g_i h_{r,i} \phi(q_i)\right|^2}{\sum_{i=1}^{L} \left|g_i\right|^2 \left|h_{r,i}\right|^2 (1-\phi^2(q_i)) + \sigma_w^2}, \quad (2.5)$$

where $\mathrm{q} \triangleq [q_1,\cdots,q_L]^T$, $\mathrm{g} \triangleq [g_1,\cdots,g_L]^T$, $\mathrm{h}_{\mathrm{r}} \triangleq [h_{r,1},\cdots,h_{r,L}]^T$,

$$\phi(q_i) \triangleq \frac{\int_{t_{q_i}}^{t_{q_i+1}} \frac{1}{\sqrt{1+(1/\mu)}} \frac{1}{\overline{\gamma}_s} \exp(-\frac{\mu}{\overline{\gamma}_s}) d\mu}{\int_{t_{q_i}}^{t_{q_i+1}} \frac{1}{\overline{\gamma}_s} \exp(-\frac{\mu}{\overline{\gamma}_s}) d\mu} \quad (2.6)$$

is the mean of $1/\sqrt{1+\xi_{s_i}}$ given that $\gamma_{s_i} = 1/\xi_{s_i}$ belongs to the quantization interval associated with q_i (with t_{q_i} and t_{q_i+1} denoting the associated quantization thresholds), and $\overline{\gamma}_s \triangleq P_s \sigma_s^2 \sigma_v^{-2}$ is the average S-R SNR. The optimal g_i's, which maximize $\gamma_{dq}(\mathrm{g},\mathrm{h}_{\mathrm{r}},\mathrm{q})$ in (2.5) subject to the total power constraint

$$\sum_{i=1}^{L} \left|g_i\right|^2 = P_d, \quad (2.7)$$

are shown to be [12]

$$g_i \propto \frac{h_{r,i}^{-1}\phi(q_i)}{1+\xi_{r_i} - \phi^2(q_i)}, \quad \text{where} \quad \xi_{r_i} \triangleq 1/\gamma_{r_i} = \frac{\sigma_w^2}{P_d \left|h_{r,i}\right|^2}.$$

(2.8)

B. Problem Statement

The main purpose of this paper is to study the problem of cooperative beamforming design, under the assumption that the transmitted one-bit message q_i from each relay is subject to communication channel impairments. More specifically, it is assumed that q_i is sent over a BSC with a crossover probability p_i, $1 \le i \le L$. From the perspective of SNR maximization, we propose a new beamforming design method which takes account of the imperfect reception of $\{q_1,\cdots,q_L\}$.

III. MAIN RESULTS

This section introduces the main results of this paper. Section III-A derives the conditional average SNR, which is the proposed design metric for the beamforming factors. Section III-B then derives a lower bound of the considered SNR metric. An analytic suboptimal beamforming scheme is also obtained via the maximization of the lower bound. Computer simulations are given in section III-C to illustrate the performance of the proposed solution.

A. Conditional Average SNR

Let $\hat{q}_i \in \{0,1\}$ be the received quantized message associated with q_i, $1 \le i \le L$. Conditioned on the $\hat{\mathrm{q}} = [\hat{q}_i,\cdots,\hat{q}_L]^T$, the main purpose is to derive the conditional SNR averaged with respect to all possible transmitted $\tilde{\mathrm{q}} = [\tilde{q}_1 \cdots \tilde{q}_L]^T$'s that are flipped to $\hat{\mathrm{q}}$ by the BSC. Recall that, the SNR conditioned on $\hat{\mathrm{q}} = \mathrm{q} = [q_1 \cdots q_L]^T$ is given by $\gamma_{dq}(\mathrm{g},\mathrm{h}_{\mathrm{r}},\mathrm{q})$ in (2.5). Hence, the expected $\gamma_{dq}(\mathrm{g},\mathrm{h}_{\mathrm{r}},\tilde{\mathrm{q}})$ given $\hat{\mathrm{q}}$ is thus

1. The notation $\mathcal{CN}(0,\sigma^2)$ denotes the complex Gaussian random variable with zero mean and variance σ^2.

2. For a finite L, the number of relays, an analytic SNR formula is difficult to find [12]. The expression (2.5) was obtained in [12] based on asymptotic analyses in the regime $L \to \infty$.

$$E_{\tilde{q}|\hat{q}}\left[\gamma_{dq}(\mathrm{g},\mathrm{h}_\mathrm{r},\tilde{\mathrm{q}}) \mid \hat{\mathrm{q}}\right] = \sum_{\tilde{\mathrm{q}}} \gamma_{dq}(\mathrm{g},\mathrm{h}_\mathrm{r},\tilde{\mathrm{q}}) \times \Pr(\tilde{\mathrm{q}} \mid \hat{\mathrm{q}}), \quad (3.1)$$

where $\Pr(\tilde{\mathrm{q}}|\hat{\mathrm{q}})$ denotes the probability that $\tilde{\mathrm{q}}$ is flipped into $\hat{\mathrm{q}}$. That is, the conditional average SNR is obtained by averaging over all possible transmitted $\tilde{\mathrm{q}}$'s given the received $\hat{\mathrm{q}}$.

To fix the idea, let us define[3]

$$S_l(\hat{\mathrm{q}}) \triangleq \left\{\tilde{\mathrm{q}} \Big| \sum_{i=1}^{L} \tilde{q}_i \oplus \hat{q}_i = l\right\},\ 0 \le l \le L, \quad (3.2)$$

which denotes the set consisting of all possible $\tilde{\mathrm{q}}$'s that differ from $\hat{\mathrm{q}}$ in exactly l bits; there are thus $C_l^L = \dfrac{L!}{l!(L-l)!}$ possible $\tilde{\mathrm{q}}$'s in $S_l(\hat{\mathrm{q}})$. Associated with each $\tilde{\mathrm{q}} \in S_l(\hat{\mathrm{q}})$, we further collect all indices at which $\tilde{q}_i$ differs from $\hat{q}_i$ to obtain

$$I_l(\tilde{\mathrm{q}},\hat{\mathrm{q}}) \triangleq \left\{i \middle| \tilde{q}_i \ne \hat{q}_i\right\}. \quad (3.3)$$

With (3.2) and (3.3), the conditional average SNR is given as

$$\overline{\gamma}_{dq}(\mathrm{g},\mathrm{h}_\mathrm{r},\hat{\mathrm{q}}) = \sum_{l=0}^{L} \sum_{\tilde{\mathrm{q}} \in S_l(\hat{\mathrm{q}})} \Pr(\tilde{\mathrm{q}}|\hat{\mathrm{q}}) \gamma_{dq}(\mathrm{g},\mathrm{h}_\mathrm{r},\tilde{\mathrm{q}}), \quad (3.4)$$

where

$$\Pr(\tilde{\mathrm{q}}|\hat{\mathrm{q}}) = \left(\prod_{k \in I_l(\tilde{\mathrm{q}},\hat{\mathrm{q}})} p_k\right)\left(\prod_{m \in I_l^c(\tilde{\mathrm{q}},\hat{\mathrm{q}})} (1-p_m)\right), \quad (3.5)$$

and $I_l^c(\tilde{\mathrm{q}},\hat{\mathrm{q}})$ denotes the complement of $I_l(\tilde{\mathrm{q}},\hat{\mathrm{q}})$. Through further manipulations an explicit formula of $\overline{\gamma}_{dq}(\mathrm{g},\mathrm{h}_\mathrm{r},\hat{\mathrm{q}})$ is shown in the following theorem (details omitted due to space limitation).

Theorem 3.1: The conditional average SNR (3.4) admits the following form

$$\overline{\gamma}_{dq}(\mathrm{g},\mathrm{h}_\mathrm{r},\hat{\mathrm{q}}) = \sum_{l=0}^{L}\left\{\sum_{k_1=1}^{L}\sum_{k_2=k_1+1}^{L}\cdots\sum_{k_{l-1}=k_{l-2}+1}^{L}\sum_{k_l=k_{l-1}+1}^{L}\frac{\left|\sum_{i=1}^{L} c_i(l,k_1,\cdots,k_l) g_i\right|^2}{\sum_{i=1}^{L}\left|g_i\right|^2 d_i(l,k_1,\cdots,k_l)}\right\} \quad (3.6)$$

where

$$c_i(l,k_1,\cdots,k_l) \triangleq \sqrt{\eta \prod_{j=1}^{l} p_{k_j}\left(\prod_{j=1}^{l}(1-p_{k_j})\right)^{-1}}\, h_{r,i}\phi(\rho_i), \quad (3.7)$$

in which $\eta \triangleq \prod_{l=1}^{L}(1-p_l)$ and $\phi(\cdot)$ is defined in (2.6),

$$d_i(l,k_1,\cdots,k_l) \triangleq \left|h_{r,i}\right|^2\left[1-\phi^2(\rho_i)\right] + \frac{\sigma_w^2}{P_d}, \quad (3.8)$$

and

$$\rho_i \triangleq \begin{cases} \hat{q}_i^t = \hat{q}_i \oplus 1 & ,i = k_1,k_2,\cdots,k_l \\ \hat{q}_i & \text{, otherwise.} \end{cases} \quad (3.9)$$

□

To maximize the conditional average SNR $\overline{\gamma}_{dq}(\mathrm{g},\mathrm{h}_\mathrm{r},\hat{\mathrm{q}})$ given in (3.6) with respect to the beamforming weights g_i's, we shall first rewrite $\overline{\gamma}_{dq}(\mathrm{g},\mathrm{h}_\mathrm{r},\hat{\mathrm{q}})$ in a more tractable form. Through further rearranging the indices in the multiple summations in (3.6), $\overline{\gamma}_{dq}(\mathrm{g},\mathrm{h}_\mathrm{r},\hat{\mathrm{q}})$ can be expressed as a single sum of Rayleigh quotients. This is established in the next theorem (details omitted due to space limitation).

Theorem 3.2: Let $\overline{\gamma}_{dq}(\mathrm{g},\mathrm{h}_\mathrm{r},\hat{\mathrm{q}})$ be defined in (3.6). Then we have

$$\overline{\gamma}_{dq}(\mathrm{g},\mathrm{h}_\mathrm{r},\hat{\mathrm{q}}) = \sum_{m=1}^{M}\frac{\left|\mathrm{c}_m^H\mathrm{g}\right|^2}{\mathrm{g}^H\mathrm{D}_m\mathrm{g}}, \quad (3.10)$$

in which $M = \sum_{l=0}^{L} C_l^L$, and, for each[4] $1 \le m \le M$,

$$\mathrm{c}_m^H \triangleq [c_1(l,k_1,\cdots,k_l)\cdots c_L(l,k_1,\cdots,k_l)], \quad (3.11)$$

$$\mathrm{D}_m \triangleq diag\left\{d_1(l,k_1,\cdots,k_l),\cdots,d_L(l,k_1,\cdots,k_l)\right\}, \quad (3.12)$$

for certain $l,k_1,\cdots,k_l$. Given a particular set of indices $l,k_1,\cdots,k_l$ in the multiple summations in (3.6), the corresponding index m in (3.10) is determined according to

$$m = \delta(l) + \sum_{s_0=0}^{l-1} C_{s_0}^L + \sum_{\lambda=1}^{l-1}\sum_{s_\lambda=k_{\lambda-1}+1}^{k_\lambda-1} C_{l-\lambda}^{L-s_\lambda} + (k_l - k_{l-1}), \quad (3.13)$$

where $k_0 = 0$ and $\delta(\cdot)$ denotes the Kronecker delta function. □

Based on (3.10), the beamforming weights can be obtained by solving the following optimization problem

$$\text{Maximize} \quad \sum_{m=1}^{M}\frac{\left|\mathrm{c}_m^H\mathrm{g}\right|^2}{\mathrm{g}^H\mathrm{D}_m\mathrm{g}} \quad \text{s.t.} \quad \left\|\mathrm{g}\right\|_2^2 = P_d. \quad (3.14)$$

where P_d denotes the total transmit power. However, since the cost function in (3.14) is a highly nonlinear function of g, a closed-form solution to (3.14) is hard to find. This thus motivates us to devise alternate approaches to finding (possibly suboptimal) beamforming weights.

B. Closed-Form Suboptimal Solution

To facilitate analysis, we go on to derive in the following theorem a tractable lower bound for $\overline{\gamma}_{dq}(\mathrm{g},\mathrm{h}_\mathrm{r},\hat{\mathrm{q}})$. By conducting maximization with respect to this lower bound, we can then obtain a closed-form suboptimal solution (details omitted due to space limitation).

Theorem 3.3: Let c_m and D_m be defined in (3.11) and (3.12). The following inequality holds:

$$\sum_{m=1}^{M}\frac{\left|\mathrm{c}_m^H\mathrm{g}\right|^2}{\mathrm{g}^H\mathrm{D}_m\mathrm{g}} \ge \frac{\mathrm{g}^H\mathrm{c}\mathrm{c}^H\mathrm{g}}{\mathrm{g}^H\mathrm{D}\mathrm{g}}, \quad (3.15)$$

where $\mathrm{c} \triangleq \sum_{m=1}^{M}\mathrm{c}_m$ and $\mathrm{D} \triangleq \sum_{m=1}^{M}\mathrm{D}_m$. □

With the aid of (3.15), a suboptimal beamformer can be obtained based on maximization of the lower bound derived in (3.15):

3. The notation $\oplus$ denotes the binary addition operation.

4. The dependence of the index m in (3.10) on $l,k_1,\cdots,k_l$ is omitted to simplify notation.

$$\text{Maximize} \quad \frac{\mathrm{g}^H \mathrm{c}\mathrm{c}^H \mathrm{g}}{\mathrm{g}^H \mathrm{D}\mathrm{g}} \quad \text{s.t.} \ \|\mathrm{g}\|_2^2 = P_d. \tag{3.16}$$

The solution to (3.16) is precisely the dominant eigenvector of $\mathrm{D}^{-1}\mathrm{c}\mathrm{c}^H$. Since the matrix $\mathrm{D}^{-1}\mathrm{c}\mathrm{c}^H$ is of rank-one, the solution to (3.16) is given by

$$\tilde{\mathrm{g}} = c_1 \mathrm{D}^{-1}\mathrm{c}, \tag{3.17}$$

where c_1 is chosen so that $\|\tilde{\mathrm{g}}\|_2^2 = P_d$.

C. Simulation Results

In this section computer simulations are used to illustrate the performance of the proposed method. We consider a cooperative beamforming system with four relays ($L = 4$). The channel gains of both the S-R and R-D links are i.i.d. random variables drawn from $\mathcal{CN}(0,1)$; the cross-over probability of the BSC is chosen as $p_i \sim U(0.05, 0.1)$, the uniform distribution over the interval $[0.05, 0.1]$. The quantization threshold is designed according to the rule in [12, p-4779]. The total power of transmit beamforming is set to be $P_d = 1$. In each Monte-Carlo run, a sequence of $T = 5000$ BPSK source symbols is generated. For fixed average S-R SNR $\overline{\gamma}_s = 20$ dB, Figure 1 compares the BER curves of the proposed beamformer (3.17) with the solution in [12] at various average R-D SNR, defined to be $\overline{\gamma}_d \triangleq P_d \sigma_r^2 / \sigma_w^2 = \sigma_w^{-2}$ [2]. As can be seen from the figure, the proposed scheme outperforms the method in [12]; the result is not unexpected since the solution in [12] is designed by assuming that the one-bit message is received at the destination without errors.

IV. CONCLUSIONS

We study the problem of low-overhead cooperative beamforming design, under the assumption that the SNR of each S-R link is quantized into one bit, which is transmitted through a BSC to the destination. Given the received one-bit message, the perfromance measure is the conditional receive SNR averaged with respect to the conditional bit-flipping distributions. A closed-form expression for the considered objective function is derived, and is seen to be a complicated function of the beamforming coefficients. A tractable lower bound of the conditional average SNR is derived. By conducting maximization of this lower bound, a suboptimal beamformer can be obtained by solving a generalized eigenvalue problem. Simulation results confirmed the performance advantage of the proposed design as compared with an existing solution designed in accordance with the idealized assumption. Our report has presented an original study of the cooperative beanforming systems in the presence of CSI transmission/feedback errors. We will extend the current results to the scenario in which high-resolution quantizers are adopted at relays. Moreover, the effect of the estimation errors of the R-D link CSI will also be further taken into consideration in the future study.

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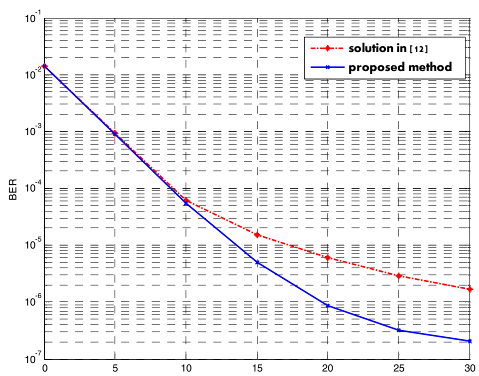

Figure 1. Simulated BER of the proposed beamformer (3.17) and the solution in [12].

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