Performance Analysis and Algorithm Designs for Transmit Antenna Selection in Linearly Precoded Multiuser MIMO Systems

Pu-Hsuan Lin and Shang-Ho Tsai, *Senior Member, IEEE*

***Abstract*—This paper investigates transmit antenna selection for linearly precoded multiuser multiple-input–multiple-output (MU-MIMO) systems. First, in some precoded single-user MIMO systems, using all transmit antennas does not always lead to the best performance due to ill-conditioned channel matrices. This condition motivates us to investigate whether a similar result can be obtained in MU-MIMO systems. Based on the derived analytical results, we found that, for a given number of transmit antennas, decreasing the number of active transmit antennas [number of radio frequency (RF) units] always degrades system performance in the linearly precoded MU-MIMO systems. However, in practical systems, RF units are expensive. To reduce the hardware cost, antenna selection is usually used to reduce the number of RF units. Thus, we further analyze the performance loss due to transmit antenna selection (TAS). These analytical results provide good design references for using TAS in practical systems. Moreover, based on the analytical results, we proposed several simple TAS algorithms for linearly precoded MU-MIMO systems. Complexity analysis and simulation results show that the computational complexity of the proposed algorithms can significantly be reduced, whereas the performance is still comparable with the optimal selection scheme. As a result, the analyzed results enable us to better understand how TAS affects the MU-MIMO systems. In addition, the proposed algorithms make TAS more feasible to be used in practical systems.**

***Index Terms*—Minimum mean square error (MMSE), multiuser multiple-input–multiple-output (MU-MIMO) linear precoding zero-forcing (ZF), transmit antenna selection (TAS).**

I. Introduction

MULTIPLE-INPUT–multiple-output (MIMO) techniques are widely used in wireless communications to achieve high throughput [1], [3]. For single-user transmission with M_T transmit antennas at the base station (BS) and M_R receive antennas at the mobile station (MS), the capacity gain is known to be roughly $\min\{M_T, M_R\}$ times of signal-input–single-output (SISO) systems [1], [2]. Recently, multiuser MIMO (MU-MIMO) systems have attracted extensive attention. In downlink MU-MIMO systems, the BS simultaneously transmits signals to several MSs using the same frequency band; the desired signals for one user hence lead to interference to other users. Such interference seriously degrades system performance. Hence, research has been conducted to mitigate interference and maximize transmission capacity for downlink MU-MIMO systems; for example, see [4]–[6].

Dirty paper coding (DPC) was shown to achieve the sum capacity of downlink MU-MIMO channels [7], [9]–[11]. However, implementing DPC may be difficult in practical systems due to its high complexity of coding procedure. Thus, several linear precoding schemes that can nearly achieve the performance of DPC have been proposed for offering reasonable tradeoffs between complexity and performance. Beamforming (BF) is a popular linear precoding [4]–[6], [22], [23]. In [23], the authors designed BF vectors from an orthogonal codebook to maximize the received signal-to-interference-plus-noise ratio (SINR) for each data stream. However, for finite users, orthogonal BF cannot completely eliminate the interference; the residual interference hence limits the capacity. This problem cannot be solved, even if we increase the transmit power. Several solutions were proposed for this issue. For example, zero-forcing (ZF) precoding in [22] chooses the precoding vectors to mitigate interference among users. However, if some users' channel conditions are strongly correlated, the received power of certain users will be small due to the use of ZF precoding [22]. Hence, when the total transmit power is small, some users suffer from low receive signal-to-noise ratio (SNR). Therefore, compensation for transmit power is required to maintain satisfactory performance for these users. In receiver design [12], the minimum-mean-square-error (MMSE) scheme can be used to mitigate noise enhancement. Similarly, MMSE precoding performs similar to transmit matched filter to enlarge the receive signal power in the region of low transmit power [13] and can be used to improve system performance.

In addition to the aforementioned precoding methods, selection diversity is another popular solution for increasing the receive SINR in MIMO communications. Antenna selection is a common form of selection diversity, and much related research has been studied in single-user MIMO systems, either at the transmitter or the receiver [14]–[18]. A comprehensive overview of MIMO antenna selection techniques was provided in [28] and [29]. In [15], antenna selection was proposed that

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The authors are with the Department of Electrical Engineering, National Chiao Tung University, Hsinchu 300, Taiwan (e-mail: oxygen.ece92@nctu.edu.tw; shanghot@mail.nctu.edu.tw).

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minimizes the error rate for spatial multiplexing systems. Optimal and suboptimal receive antenna selection algorithms were proposed in [17] and [18] to maximize capacity for spatial multiplexing systems. However, in downlink MU-MIMO channels, each user may not obtain channel information of other users, and the techniques used in single-user communications cannot directly be applied. Because precoding can be used to maximize the receive SINR or mitigate interference from other users, combining precoding and selection techniques could be a good solution for improving performance in MU-MIMO systems. With a large number of users [21]–[24], the BS can select users with favorable channel conditions to improve system throughput due to multiuser diversity [19], [20]: a selection diversity among users. Moreover, orthogonal BF [23] and ZF precoding [22] were shown to be asymptotically optimal in terms of capacity when the number of users tends to infinity.

Similar to user selection, transmit antenna selection (TAS) can be used in multiuser environments, that is, we can let the number of transmit antennas of a BS be larger than the number of radio frequency (RF) units. Then, an appropriate transmit antenna set is determined to achieve selection diversity; for example, see [26]. In [26], several design criteria for selecting transmit antennas in MU-MIMO systems are proposed. However, these designs demand exhaustive search to find the best antenna set, and the computational complexity is usually extremely high. Moreover, although the performance of TAS has extensively been studied for single-user MIMO systems (for example, see [8] and [28]), little research has been conducted to analyze how TAS affects the MU-MIMO systems. These conditions motivate us to analyze the performance of TAS in MU-MIMO systems and propose low-complexity antenna selection algorithms with only slight performance degradation for MU-MIMO systems.

In this paper, we analyze the performance of TAS for ZF and MMSE precoded MU-MIMO systems, respectively. The theoretical result shows that, if the BS adopts equal transmit power for all users, decreasing the number of active transmit antennas always degrades the performance (sum throughput performance for ZF precoding and MSE performance for MMSE precoding). Note that this result stands contrary to other precoding systems in [14] and [31]. In [14] and [31], respectively, it is shown that using all transmit antennas does not always lead to the maximum throughput [14] in single-user MIMO systems and equal gain BF systems, because some transmit antennas may cause ill-conditioned matrices; in this case, removing these antennas and redistributing the power to the selected antennas result in higher throughput. Moreover, because decreasing the number of active transmit antennas always leads to a performance loss, we further analyze this performance loss and derive an upper bound of the sum throughput loss for selecting transmit antennas in ZF-precoded MU-MIMO systems. The derived upper bound is quite close to the simulation result for a moderately high SNR. The simulation result matches the upper bound when the SNR tends to infinity. As a result, the derived upper bound may provide a good design reference for trading the number of RF units and the system performance in ZF-precoded MU-MIMO systems. For MMSE precoding, directly analyzing the sum throughput is difficult. As an alternative, we use the mean-square-error (MSE) criterion instead of the sum throughput criterion to analyze the system performance. Simulation result shows that using the MSE or sum throughput criterion to select transmit antennas leads to nearly the same performance. Similar to ZF precoding, we show that removing active transmit antennas always increases the MSE. Furthermore, based on the derived theoretical results, we propose several transmit selection algorithms for linearly precoded MU-MIMO systems. Complexity analysis and simulation results show that the computational complexity can greatly be reduced from exponential order to polynomial order, whereas the throughput loss is within only 5% of the overall sum throughput. Finally, simulation results are provided to show the advantages of the proposed TAS algorithm. The simulation results corroborate the theoretical results.

This paper is organized as follows. In Section II, we introduce the system model. In Section III, we review the ZF and MMSE linear precoding approaches in MU-MIMO systems. In Section IV, we give the performance analysis of using TAS in linearly precoded MU-MIMO systems. Several fast TAS algorithms are proposed in Section V. Complexity analysis and comparison are provided in Section VI, and simulation results are demonstrated in Section VII. Conclusions follow in Section VIII.

Notations: All vectors are in lowercase boldface, and matrices are in uppercase boldface. $(\cdot)^T$, $(\cdot)^H$, and $(\cdot)^\dagger$ denote the transpose, conjugate transpose, and pseudoinverse of a matrix, respectively. $\mathbf{X}^{(i,j)}$ denotes the element in the ith row and jth column of a matrix $\mathbf{X}$. Diag$\{x_1, x_2, \ldots, x_n\}$ is an $n \times n$ diagonal matrix, with the elements being $x_1, x_2, \ldots, x_n$. $\mathrm{tr}(\cdot)$ is the trace of a square matrix. $\mathbb{E}\{\cdot\}$ denotes the expectation. $\|\cdot\|$ is the Euclidean vector norm. $|\mathcal{S}|$ is the size of set $\mathcal{S}$.

II. System Model of Downlink Multiuser Multiple-Input–Multiple-Output Channels

The block diagram of a downlink MU-MIMO system with M_T transmit antennas and K users is shown in Fig. 1. The BS is equipped with M_F RF transmission units. Assume $K \leq M_F \leq M_T$. Each user has one receive antenna so that the total number of receive antennas is K. Let x_k be the transmitted symbol of the kth user, with $\mathbb{E}\{|x_k|^2\} = 1$. x_k is first multiplied by $\sqrt{p_k}$, where p_k represents the transmission power allocated to the kth user. Then, it is passed to an $M_F \times 1$ precoding vector $\mathbf{w}_k$. The signals for all K users after precoding are summed to form an $M_F \times 1$ vector $\mathbf{s}$. The vector $\mathbf{s}$ is passed to M_F RF units and then transmitted to a MIMO channel through the selected antennas. The MIMO channel is assumed to be quasistatic and flat with Rayleigh distribution. Let $\mathbf{H} = [\mathbf{h}_1\ \mathbf{h}_2 \cdots \mathbf{h}_{M_T}]$ be the $K \times M_T$ MIMO channel, where the $K \times 1$ vector $\mathbf{h}_i$ is the channel vector that corresponds to the ith transmit antenna. Due to the use of TAS, i.e., selecting M_F from M_T antennas, the effective MIMO channel is a $K \times M_F$ matrix given by $\mathbf{H}_\mathcal{S} = [\tilde{\mathbf{h}}_1^T; \tilde{\mathbf{h}}_2^T; \cdots; \tilde{\mathbf{h}}_K^T]$, where the subscript $\mathcal{S}$ is an antenna set of M_F selected transmit antennas, and $\mathbf{H}_\mathcal{S}$ is a submatrix of $\mathbf{H}$, obtained by selecting M_F columns from $\mathbf{H}$; the $1 \times M_F$ vector $\tilde{\mathbf{h}}_j^T$ is the jth row of $\mathbf{H}_\mathcal{S}$, representing the channel vector

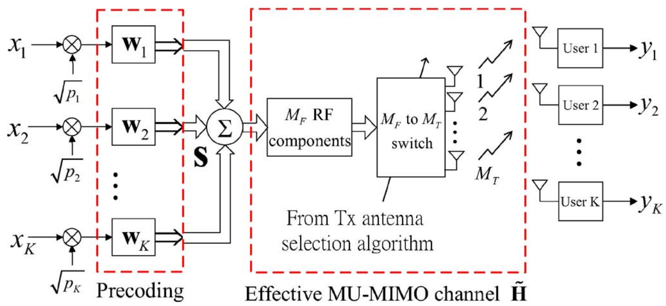

Fig. 1. Downlink MU-MIMO system with TAS.

of the selected transmit antenna set from the BS to the jth user. The received signal y_k for the kth user is given by

$$y_k = \tilde{\mathbf{h}}_k^T \sqrt{p_k}\mathbf{w}_k x_k + \tilde{\mathbf{h}}_k^T \sum_{l=1, l\neq k}^{K} \sqrt{p_l}\mathbf{w}_l x_l + n_k \tag{1}$$

where n_k is the noise of the kth user that has independent and identically distributed (i.i.d.) complex Gaussian distribution with zero mean and variance σ^2 (assume that all users have the same noise statistic). The second term of (1) represents the interference from other users. Let $\mathbf{W}_\mathcal{S} = [\mathbf{w}_1\ \mathbf{w}_2 \cdots \mathbf{w}_K]$ be the $M_F \times K$ precoding matrix for all users. Then, the relationship between the transmitted symbol vector $\mathbf{x} = [x_1\ x_2 \cdots x_K]^T$ and the received symbol vector $\mathbf{y}$ can be expressed as

$$\mathbf{y} = \mathbf{H}_\mathcal{S}\mathbf{W}_\mathcal{S}\mathbf{P}_\mathcal{S}\mathbf{x} + \mathbf{n} \tag{2}$$

where $\mathbf{P}_\mathcal{S} = \mathrm{diag}\{\sqrt{p_1}, \sqrt{p_2}, \ldots, \sqrt{p_K}\}$ is the $K \times K$ power allocation matrix, and $\mathbf{n} = [n_1\ n_2 \cdots n_K]^T$ is the noise vector with covariance matrix $\mathbf{R}_n = \sigma^2\mathbf{I}$. We assume that the channel information of all users is perfectly known to the BS.

III. Review of Linear Precoders in Multiuser Multiple-Input–Multiple-Output Systems

Let us review two popular linear precoders for the precoding matrix $\mathbf{W}_\mathcal{S}$, i.e., the ZF and the MMSE precoders, and their performance.

A. ZF Precoding and Its Performance

In ZF precoding [22], the precoding vectors are designed to completely cancel the interference that arose from other users, i.e., determining $\mathbf{w}_l$ so that $\tilde{\mathbf{h}}_k^T\mathbf{w}_l = 0$ for $\forall l \neq k$. To achieve this interference-free property, $\mathbf{W}_\mathcal{S}$ can be designed as the pseudoinverse of $\mathbf{H}_\mathcal{S}$, i.e.,

$$\mathbf{W}_\mathcal{S} = \mathbf{H}_\mathcal{S}^\dagger = \mathbf{H}_\mathcal{S}^H\left(\mathbf{H}_\mathcal{S}\mathbf{H}_\mathcal{S}^H\right)^{-1}. \tag{3}$$

Based on (3), $\mathbf{H}_\mathcal{S}\mathbf{W}_\mathcal{S} = \mathbf{I}$. The sum throughput using ZF precoding is then given by

$$R_{\mathrm{ZF}} = \log\left(\det\left(\mathbf{I} + \frac{1}{\sigma^2}\mathbf{P}_\mathcal{S}^2\right)\right) \text{ s.t. } \mathrm{tr}\left(\mathbf{W}_\mathcal{S}\mathbf{P}_\mathcal{S}^2\mathbf{W}_\mathcal{S}^H\right) \leq P \tag{4}$$

where P is the constraint of the total transmit power. Assume that the transmit power of all users is equal. That is, $p_k = \beta_{\mathrm{ZF}} = P/\mathrm{tr}(\mathbf{W}_\mathcal{S}^H\mathbf{W}_\mathcal{S})$ for $k = 1, \ldots, K$. Then, the power matrix $\mathbf{P}_\mathcal{S} = \sqrt{\beta_{\mathrm{ZF}}}\mathbf{I}$, and (4) becomes

$$R_{\mathrm{ZF}} = K\log\left(1 + \frac{\beta_{\mathrm{ZF}}}{\sigma^2}\right). \tag{5}$$

ZF precoding makes all individual users enjoy the interference-free property. The transmission for each user can thus be treated as a single-user system in multiple-input–single-output channels. However, if some users are with serious fading channels, i.e., channel gains $\|\mathbf{h}_k\|$ are small for some k, the elements of $\mathbf{W}_\mathcal{S}$ in ZF precoding tend to be large to compensate for the serious channel fading. This condition deteriorates the throughput R_{ZF}, because the scaling factor β_{ZF} is small.

B. MMSE Precoding and Its Performance

MMSE precoding may be used to overcome the drawback of ZF precoding. Similar to the well-known MMSE receiver, we can apply MMSE precoding at the transmit side [13]. Based on (1), the MMSE precoder designs the precoding matrix such that the MSE is minimized, i.e.,

$$\min_{\mathbf{W}_\mathcal{S}} \sum_{k=1}^{K}\left|x_k - y_k/\mathbf{E}_\mathcal{S}^{(k,k)}\right|^2 \tag{6}$$

where $\mathbf{E}_\mathcal{S}^{(k,k)}$ is the kth diagonal element of matrix $\mathbf{H}_\mathcal{S}\mathbf{W}_\mathcal{S}\mathbf{P}_\mathcal{S}$. Based on (1) and (2), $\mathbf{E}_\mathcal{S}^{(k,k)} = \sqrt{p_k}\tilde{\mathbf{h}}_k^T\mathbf{w}_k$. In addition to the channel-state information, to solve (6), the BS is assumed to know the covariance matrix of noise $\mathbf{R}_n$. The MMSE precoding matrix is given by

$$\mathbf{W}_\mathcal{S} = \left(\mathbf{H}_\mathcal{S}^H\mathbf{H}_\mathcal{S} + \frac{\mathrm{tr}(\mathbf{R}_n)}{P}\mathbf{I}\right)^{-1}\mathbf{H}_\mathcal{S}^H. \tag{7}$$

Similar to ZF precoding, equal transmit power for all users is assumed for MMSE precoding in our discussion, i.e., $\mathbf{P}_\mathcal{S} = \sqrt{\beta_{\mathrm{MMSE}}}\mathbf{I}$; β_{MMSE} is a scaling factor that is determined by the power constraint P given by [13] $\beta_{\mathrm{MMSE}} = P/\mathrm{tr}((\mathbf{T}_\mathcal{S}^{-1}\mathbf{H}_\mathcal{S}^H)^H(\mathbf{T}_\mathcal{S}^{-1}\mathbf{H}_\mathcal{S}^H))$, where

$$\mathbf{T}_\mathcal{S} = \mathbf{H}_\mathcal{S}^H\mathbf{H}_\mathcal{S} + \frac{\mathrm{tr}(\mathbf{R}_n)}{P}\mathbf{I}. \tag{8}$$

The throughput of using MMSE precoding with equal user power is expressed as (see [13])

$$R_{\mathrm{MMSE}} = \sum_{k=1}^{K}\log(1 + \mathrm{SINR}_k) \tag{9}$$

where SINR_k represents the SINR of the kth user, which is given by

$$\mathrm{SINR}_k = \frac{\beta_{\mathrm{MMSE}}\left\|\tilde{\mathbf{h}}_k^T\mathbf{w}_k\right\|^2}{\sum_{j=1, j\neq k}^{K}\beta_{\mathrm{MMSE}}\left\|\tilde{\mathbf{h}}_k^T\mathbf{w}_j\right\|^2 + \mathbf{R}_n^{(k,k)}}. \tag{10}$$

IV. Performance Analysis of Using Transmit Antenna Selection in Linearly Precoded Multiuser Multiple-Input–Multiple-Output Systems

In single-user MIMO systems, using all transmit antennas does not always lead to the best performance [14] and [31]. The reason is that some transmitting branches may cause ill-conditioned matrices; in this case, removing these antennas and redistributing the power to the selected antennas result in better performance. In MU-MIMO systems, we are interested to know how TAS affects the performance of linearly precoded MU-MIMO systems. In this section, we theoretically prove that removing transmit antennas from linear precoded MU-MIMO systems always degrades the performance. The upper bound of the performance degradation due to removing transmit antennas is also derived.

A. Performance Analysis for ZF Precoding

In MU-MIMO systems with ZF precoding, the sum throughput of all users is usually used as the design criterion. Thus, we analyze how the sum throughput varies due to TAS. The following three lemmas are needed before reaching the main conclusion.

Lemma 1: Let $\mathbf{H} = [\mathbf{H}_1\ \mathbf{H}_2]$, where $\mathbf{H}\mathbf{H}^H$ and $\mathbf{H}_1\mathbf{H}_1^H$ are both nonsingular. We have

$$\left(\mathbf{H}_1\mathbf{H}_1^H\right)^{-1} = \mathbf{Q} + \mathbf{Q}\mathbf{H}_2\left(\mathbf{I} - \mathbf{H}_2^H\mathbf{Q}\mathbf{H}_2\right)^{-1}\mathbf{H}_2^H\mathbf{Q} \quad (11)$$

where $\mathbf{Q} = (\mathbf{H}\mathbf{H}^H)^{-1}$.

Proof: Because $\mathbf{H} = [\mathbf{H}_1\quad \mathbf{H}_2]$, $\mathbf{H}\mathbf{H}^H = \mathbf{H}_1\mathbf{H}_1^H + \mathbf{H}_2\mathbf{H}_2^H$. Using (12) [30], we have

$$(\mathbf{Z}+\mathbf{U}\mathbf{V}^H)^{-1} = \mathbf{Z}^{-1} - \mathbf{Z}^{-1}\mathbf{U}(\mathbf{I}+\mathbf{V}^H\mathbf{Z}^{-1}\mathbf{U})^{-1}\mathbf{V}^H\mathbf{Z}^{-1}. \quad (12)$$

The inverse of $(\mathbf{H}_1\mathbf{H}_1^H)$ can be rewritten as

$$\begin{aligned}\left(\mathbf{H}_1\mathbf{H}_1^H\right)^{-1} &= \left(\mathbf{H}\mathbf{H}^H - \mathbf{H}_2\mathbf{H}_2^H\right)^{-1}\\ &= \mathbf{Q} + \mathbf{Q}\mathbf{H}_2\left(\mathbf{I} - \mathbf{H}_2^H\mathbf{Q}\mathbf{H}_2\right)^{-1}\mathbf{H}_2^H\mathbf{Q}.\end{aligned}$$

■

Lemma 2: For any matrix $\mathbf{A} \in \mathbb{C}^{m\times n}$ with $\mathrm{rank}(\mathbf{A}) = m$, it satisfies

$$tr\left(\mathbf{Q}\mathbf{A}_c\left(\mathbf{I} - \mathbf{A}_c^H\mathbf{Q}\mathbf{A}_c\right)^{-1}\mathbf{A}_c^H\mathbf{Q}\right) > 0 \quad (13)$$

where $\mathbf{Q} = (\alpha\mathbf{I} + \mathbf{A}\mathbf{A}^H)^{-1}$, $\alpha \geq 0$ is a constant, and $\mathbf{A}_c$ is an $m \times k$ matrix, with its columns chosen from the columns of $\mathbf{A}$, for $1 \leq k \leq (n - m)$.

Proof: See Appendix A. ■

Lemma 3: Let $\mathcal{S}$ and $\mathcal{S}'$ be two transmit antenna sets in ZF-precoded MU-MIMO systems, where $\mathcal{S} \subset \mathcal{S}' \subseteq \{1, 2, \ldots, M_T\}$. Let $\bar{\mathcal{S}} = \mathcal{S}' - \mathcal{S}$. Then, the difference of the sum throughput between these two sets, i.e., $R_D(\bar{\mathcal{S}}) = R(\mathcal{S}') - R(\mathcal{S})$, is given by

$$R_D(\bar{\mathcal{S}}) = K\log \times\left(1 + \frac{SNR\cdot tr(\mathbf{\Lambda}_{\bar{\mathcal{S}}})}{(tr(\mathbf{Q}_{\mathcal{S}'}))^2 + tr(\mathbf{Q}_{\mathcal{S}'})(SNR + tr(\mathbf{\Lambda}_{\bar{\mathcal{S}}}))}\right) > 0 \quad (14)$$

where $\mathbf{Q}_{\mathcal{S}'} = (\mathbf{H}_{\mathcal{S}'}\mathbf{H}_{\mathcal{S}'}^H)^{-1}$, $SNR = P/\sigma^2$, and $\mathbf{\Lambda}_{\bar{\mathcal{S}}} = \mathbf{Q}_{\mathcal{S}'}\mathbf{H}_{\bar{\mathcal{S}}}(\mathbf{I} - \mathbf{H}_{\bar{\mathcal{S}}}^H\mathbf{Q}_{\mathcal{S}'}\mathbf{H}_{\bar{\mathcal{S}}})^{-1}\mathbf{H}_{\bar{\mathcal{S}}}^H\mathbf{Q}_{\mathcal{S}'}$.

Proof: See Appendix B. ■

Now, we are ready to introduce how TAS affects the system performance in the following theorem.

Theorem 1: Let the transmit power constraint and the number M_T of transmit antennas be fixed. Then, the maximum achievable sum throughput is monotonically increasing with the number M_F of active transmit antennas in ZF precoded MU-MIMO systems, where $M_F \leq M_T$. That is, let the two transmit antenna sets $\mathcal{S}_{opt1}$ and $\mathcal{S}_{opt2}$ be two subsets of $\{1, 2, \ldots, M_T\}$ so that they lead to the maximum sum throughput for $M_F = |\mathcal{S}_{opt1}|$ and $M_F = |\mathcal{S}_{opt2}|$, respectively. We have

$$R(\mathcal{S}_{opt1}) < R(\mathcal{S}_{opt2}), \quad \text{if } |\mathcal{S}_{opt1}| < |\mathcal{S}_{opt2}|. \quad (15)$$

Proof: See Appendix C. ■

Theorem 1 shows the fact that the maximum sum throughput always decreases after removing transmit antennas. Thus, we may regard $R_D(\bar{\mathcal{S}})$ as *throughput loss* due to removing antennas.

Now, we would like to derive an upper bound for the throughput loss in the following lemma and theorem.

Lemma 4: If a function f of x is given by

$$f(x) = \log\left(1 + \frac{ax}{b^2 + b(a + x)}\right), \quad \text{for } a,\ b \in \mathbb{R}^+ \quad (16)$$

then $f(x)$ is monotonically increasing with x when $x > 0$.

Proof: The first derivative of $f(x)$ is

$$\frac{df(x)}{dx} = \frac{1}{1 + \frac{ax}{b^2+b(a+x)}}\left(\frac{ab(a + b)}{(b^2 + b(a + x))^2}\right) > 0, \text{ for } x > 0$$

which completes the proof. ■

Theorem 2: For the antenna set $\bar{\mathcal{S}} = \mathcal{S}' - \mathcal{S}$, *where* $\mathcal{S} \subset \mathcal{S}' \subseteq \{1, 2, \ldots, M_T\}$, the throughput loss $R_D(\bar{\mathcal{S}})$ in Lemma 3 is monotonically increasing with the SNR and can be upper bounded by

$$R_D(\bar{\mathcal{S}}) \leq K\log\left(1 + \frac{\mathrm{tr}(\mathbf{\Lambda}_{\bar{\mathcal{S}}})}{\mathrm{tr}\left(\mathbf{H}_{\mathcal{S}'}\mathbf{H}_{\mathcal{S}'}^H\right)^{-1}}\right). \quad (17)$$

Moreover, $R_D(\bar{\mathcal{S}})$ approaches this bound when the SNR tends to ∞.

Proof: See Appendix D. ■

Note that the bound does not depend on the SNR. According to Theorem 2, for any given transmit antenna set, we can upper bound the corresponding sum throughput loss. Later simulation result shows that, when SNR > 20 dB, the simulated throughput loss matches this bound very closely. As a result, this proposed bound provides a useful design reference for TAS in ZF-precoded MU-MIMO systems. For example, the designers could use this bound to determine the number of RF elements M_F and the corresponding performance loss; then, a good tradeoff can be made between hardware complexity and system performance.

B. Performance Analysis for MMSE Precoding

Similar to ZF-precoded systems, we would like to investigate how the TAS affects MMSE-precoded MU-MIMO systems. However, it is difficult to obtain a closed-form solution of the sum throughput loss caused by removing transmit antennas in MMSE precoding systems. Fortunately, based on the simulation results, we know that the resulting sum throughput that was obtained by the maximum sum throughput criterion in (9) is almost the same as the one obtained by the MMSE criterion in (6); similar observation was also found in [25]. Thus, we use the MSE criterion instead of the sum throughput criterion to analyze the performance of TAS. More specifically, referring to (6), TAS attempts to find the transmit antenna set $\mathcal{S}$ such that the MSE of MMSE-precoded MU-MIMO systems is minimized, i.e.,

$$\min_{\mathcal{S}} \min_{\mathbf{W}_{\mathcal{S}}} \sum_{k=1}^{K} \left| x_k - y_k / \mathbf{E}_{\mathcal{S}}^{(k,k)} \right|^2. \tag{18}$$

For a channel matrix $\mathbf{H}_{\mathcal{S}}$, the MSE of the MMSE precoder has been shown to be [13]

$$\delta(\mathcal{S}) = \mathrm{tr}\left[\left(\alpha \mathbf{I} + \mathbf{H}_{\mathcal{S}} \mathbf{H}_{\mathcal{S}}^H \right)^{-1} \right] \tag{19}$$

where $\alpha = \mathrm{tr}(\mathbf{R}_n)/P$. Therefore, if the design criterion is MSE, the desired transmit antenna set $\mathcal{S}_{\mathrm{opt}}$ is determined by

$$\mathcal{S}_{\mathrm{opt}} = \arg\min_{\mathcal{S}} \delta(\mathcal{S}). \tag{20}$$

Now, we show that the MSE in (19) is monotonically decreasing with the number of active transmit antennas in the following theorem.

Lemma 5: In MMSE-precoded MU-MIMO systems, for two transmit antenna sets $\mathcal{S}$ and $\mathcal{S}'$, where $\mathcal{S} \subset \mathcal{S}' \subseteq \{1, 2, \ldots, M_T\}$, the MSE difference between these two sets is

$$\begin{aligned} \delta_D(\bar{\mathcal{S}}) &= \delta(\mathcal{S}') - \delta(\mathcal{S}) \\ &= -\,\mathrm{tr}\left(\mathbf{A}_{\mathcal{S}'} \mathbf{H}_{\bar{\mathcal{S}}} \left(\mathbf{I} - \mathbf{H}_{\bar{\mathcal{S}}}^H \mathbf{A}_{\mathcal{S}'} \mathbf{H}_{\bar{\mathcal{S}}} \right)^{-1} \mathbf{H}_{\bar{\mathcal{S}}}^H \mathbf{A}_{\mathcal{S}'} \right) \\ &< 0 \end{aligned} \tag{21}$$

where we define $\mathbf{A}_{\mathcal{S}'} = (\alpha \mathbf{I} + \mathbf{H}_{\mathcal{S}'} \mathbf{H}_{\mathcal{S}'}^H)^{-1}$ and $\bar{\mathcal{S}} = \mathcal{S}' - \mathcal{S}$.

Proof: See Appendix E. ■

Theorem 3: Let the transmit power constraint and the number M_T of transmit antennas be fixed. Then, the MMSE is monotonically decreasing with the number M_F of active transmit antennas in MMSE-precoded MU-MIMO systems, where $M_F \leq M_T$. That is, let the two transmit antenna sets $\mathcal{S}_{opt1}$ and $\mathcal{S}_{opt2}$ be the subsets of $\{1, 2, \ldots, M_T\}$ so that they lead to the MMSE for $M_F = |\mathcal{S}_{opt1}|$ and $M_F = |\mathcal{S}_{opt2}|$, respectively. We have

$$\delta(\mathcal{S}_{opt1}) > \delta(\mathcal{S}_{opt2}), \quad \text{if } |\mathcal{S}_{opt1}| < |\mathcal{S}_{opt2}|. \tag{22}$$

Proof: See Appendix F. ■

V. Proposed Transmit Antenna Selection Algorithms

Based on the analytical results, we propose TAS algorithms for linearly precoded MU-MIMO systems in this section.

A. TAS for ZF Precoding

According to Lemma 3 and Theorem 1, the sum throughput always decreases if TAS is used. In addition, we regard this decrease as throughput loss. Hence, the optimal TAS that maximizes the sum throughput is equivalent to TAS that minimizes the throughput loss. That is, the optimal transmit antenna set $\mathcal{S}_{\mathrm{opt}}$ can be obtained by

$$\mathcal{S}_{\mathrm{opt}} = \arg\max_{\mathcal{S}} R(\mathcal{S}) \equiv \arg\min_{\mathcal{S}} R_D(\bar{\mathcal{S}}) \tag{23}$$

where $\bar{\mathcal{S}} = \mathcal{S}' - \mathcal{S}$. Using the following theorem, the computations in (23) can further be simplified.

Proposition 1: In ZF-precoded MU-MIMO systems, the computational complexity of the optimal TAS that minimizes the throughput loss can be reduced, because (23) can be replaced by

$$\mathcal{S}_{opt} = \arg\min_{\mathcal{S}} tr(\mathbf{\Lambda}_{\bar{\mathcal{S}}}) \tag{24}$$

where $\bar{\mathcal{S}} = \mathcal{S}' - \mathcal{S}$, $|\mathcal{S}| = M_F$, and $\mathcal{S}' = \{1, 2, \ldots, M_T\}$.

Proof: Based on (23), the selection rule that maximizes the sum throughput can be replaced by the approach that minimizes (34). By Lemma 4, the throughput loss is an increasing function with $\mathrm{tr}(\mathbf{\Lambda}_{\bar{\mathcal{S}}})$ if both $\mathrm{tr}((\mathbf{H}_{\mathcal{S}'} \mathbf{H}_{\mathcal{S}'}^H)^{-1})$ and SNR (P/σ^2) are fixed. Because $\mathrm{tr}((\mathbf{H}_{\mathcal{S}'} \mathbf{H}_{\mathcal{S}'}^H)^{-1})$ and the SNR are, indeed, fixed, (23) can be rewritten as

$$\mathcal{S}_{\mathrm{opt}} = \arg\min_{\mathcal{S}} R_D(\bar{\mathcal{S}}) \equiv \arg\min_{\mathcal{S}} \left(\mathrm{tr}(\mathbf{\Lambda}_{\bar{\mathcal{S}}}) \right). \qquad ■$$

Although the computational complexity of the optimal TAS has been reduced, it still demands exhaustive search and, thus, huge computational effort. More specifically, the whole search space is $C_{M_F}^{M_T}$; the computational complexity soon becomes prohibitively high as M_T grows. For example, let $M_T = 20$ and $M_F = 6$. There are $C_6^{20} \approx 3.9 \times 10^4$ possible antenna sets. The huge search complexity may make the optimal solution impractical in real systems. To overcome this problem, we find that a greedy search algorithm may be a good solution, which not only has low complexity but achieves a performance close to the optimal scheme as well. Many different types of TAS algorithms that apply the concept of greedy search have extensively been studied, e.g., antenna selection in single-user MIMO systems [17] and user selection in downlink MU-MIMO systems [22], [27]. However, TAS for downlink MU-MIMO systems has not yet been widely discussed. Here, we apply the greedy concept to select transmit antennas for linearly precoded MU-MIMO systems. This algorithm always removes one transmit antenna from the current antenna set so that the resulting sum throughput is maximized until the number of elements in the set is equal to M_F. The greedy TAS for linearly precoded MU-MIMO systems is described in Algorithm 1.

Algorithm 1: Greedy TAS for linearly precoded MU-MIMO systems.

Let $\mathcal{S} = \{1, 2, \ldots, M_T\}$ and $|\mathcal{S}| = M_T$.
while $|\mathcal{S}| > M_F$ **do**
  $m = \arg\max_r R(\mathcal{S}_r)$ for the ZF precoder
  $m = \arg\min_r \delta(\mathcal{S}_r)$ for the MMSE precoder
  s.t. $\mathrm{tr}(\mathbf{P}_{\mathcal{S}_r}) \le P$, where $\mathcal{S}_r = \mathcal{S} - \{r\}$, and $r \in \mathcal{S}$.
  $\mathcal{S} = \mathcal{S} - \{m\}$.
end while
The resulting set $\mathcal{S}$ is the desired transmit antenna set.

Proposition 2: By applying the greedy method, each time only one transmit antenna instead of an antenna set $\bar{\mathcal{S}}$ is removed, let the index of the removed antenna be m. Then, the computational complexity in (24) can further be reduced to

$$m = \arg\min_r \frac{\left\| \mathbf{h}_r^H \left(\mathbf{H}_{\mathcal{S}}\mathbf{H}_{\mathcal{S}}^H\right)^{-1} \right\|^2}{1 - \mathbf{h}_r^H \left(\mathbf{H}_{\mathcal{S}}\mathbf{H}_{\mathcal{S}}^H\right)^{-1} \mathbf{h}_r} \tag{25}$$

where $\mathcal{S}$ is the current antenna set, and $r \in \mathcal{S}$.

Proof: Because only one transmit antenna is removed in each iteration, $\mathrm{tr}(\mathbf{\Lambda}(\bar{\mathcal{S}}))$ reduces to $\|\mathbf{h}_r^H(\mathbf{H}_{\mathcal{S}}\mathbf{H}_{\mathcal{S}}^H)^{-1}\|^2/1 - \mathbf{h}_r^H(\mathbf{H}_{\mathcal{S}}\mathbf{H}_{\mathcal{S}}^H)^{-1}\mathbf{h}_r$, which is a scalar that reflects that the rth antenna is removed. Therefore, the selection problem becomes

$$m = \arg\min_r R_D(r) \equiv \arg\min_r \mathrm{tr}(\mathbf{\Lambda}_r) \equiv \arg\min_r \frac{\left\| \mathbf{h}_r^H \left(\mathbf{H}_{\mathcal{S}}\mathbf{H}_{\mathcal{S}}^H\right)^{-1} \right\|^2}{1 - \mathbf{h}_r^H \left(\mathbf{H}_{\mathcal{S}}\mathbf{H}_{\mathcal{S}}^H\right)^{-1} \mathbf{h}_r}. \qquad \blacksquare$$

Applying Proposition 2, the fast algorithm of the greedy TAS for ZF-precoded MU-MIMO systems is summarized in Algorithm 2.

Algorithm 2: Fast algorithm of the greedy TAS for ZF-precoded MU-MIMO systems.

Let $\mathcal{S} = \{1, 2, \ldots, M_T\}$, $|\mathcal{S}| = M_T$, and $\mathbf{H}_{\mathcal{S}} = [\mathbf{h}_1\ \mathbf{h}_2 \cdots \mathbf{h}_{M_T}]$. Define $\mathbf{A}_{\mathcal{S}} = (\mathbf{H}_{\mathcal{S}}\mathbf{H}_{\mathcal{S}}^H)^{-1}$.
while $|\mathcal{S}| > M_F$ **do**
  $m = \arg\min_r \frac{\|\mathbf{h}_r^H \mathbf{A}_{\mathcal{S}}\|^2}{1 - \mathbf{h}_r^H \mathbf{A}_{\mathcal{S}} \mathbf{h}_r}$, where $r \in \mathcal{S}$.
  $\mathbf{A}_{\mathcal{S}} = \mathbf{A}_{\mathcal{S}} + \frac{\mathbf{A}_{\mathcal{S}}\mathbf{h}_m\mathbf{h}_m^H\mathbf{A}_{\mathcal{S}}}{1 - \mathbf{h}_m^H\mathbf{A}_{\mathcal{S}}\mathbf{h}_m}$.
  $\mathcal{S} = \mathcal{S} - \{m\}$.
end while
The resulting set $\mathcal{S}$ is the desired transmit antenna set.

B. TAS for MMSE Precoding

For the MMSE precoder, according to Theorem 3, the MSE increases as the number of active antennas decreases. The selection problem of minimizing the MSE in (20) is equivalent to minimizing the increased amount of MSE after removing antennas, i.e.,

$$\mathcal{S}_{\text{opt}} = \arg\min_{\mathcal{S}} \delta(\mathcal{S}) \equiv \arg\min_{\mathcal{S}} \left|\delta_D(\bar{\mathcal{S}})\right| \tag{26}$$

where $\bar{\mathcal{S}} = \mathcal{S}' - \mathcal{S}$, $|\mathcal{S}| = M_F$, and $\mathcal{S}' = \{1, 2, \ldots, M_T\}$.

Proposition 3: By applying the greedy method, each time only one transmit antenna instead of an antenna set $\bar{\mathcal{S}}$ is removed, let the index of the removed antenna be m. Then, the computational complexity in (26) can further be reduced to

$$m = \arg\min_r \frac{\left\|\mathbf{h}_r^H \mathbf{A}_{\mathcal{S}}\right\|^2}{1 - \mathbf{h}_r^H \mathbf{A}_{\mathcal{S}} \mathbf{h}_r} \tag{27}$$

where $\mathcal{S}$ is the current antenna, $r \in \mathcal{S}$, and $\mathbf{A}_{\mathcal{S}} = (\alpha\mathbf{I} + \mathbf{H}_{\mathcal{S}}\mathbf{H}_{\mathcal{S}}^H)^{-1}$.

Proof: Because only one transmit antenna is removed in one iteration, $|\delta_D(\bar{\mathcal{S}})|$ reduces to $(\|\mathbf{h}_r^H\mathbf{A}_{\mathcal{S}}\|^2/1 - \mathbf{h}_r^H\mathbf{A}_{\mathcal{S}}\mathbf{h}_r)$, which is a scalar that reflects that the rth antenna is removed. Therefore, the selection problem becomes

$$\arg\min_r \delta_D(r) \equiv \arg\min_r \frac{\left\|\mathbf{h}_r^H \mathbf{A}_{\mathcal{S}}\right\|^2}{1 - \mathbf{h}_r^H \mathbf{A}_{\mathcal{S}} \mathbf{h}_r}. \qquad \blacksquare$$

Using Proposition 3, the fast algorithm of the greedy TAS for MMSE-precoded MU-MIMO systems is summarized in Algorithm 3.

Algorithm 3: Fast algorithm of the greedy TAS for MMSE-precoded MU-MIMO systems.

Let $\mathcal{S} = \{1, 2, \ldots, M_T\}$, $|\mathcal{S}| = M_T$, and $\mathbf{H}_{\mathcal{S}} = [\mathbf{h}_1\ \mathbf{h}_2 \cdots \mathbf{h}_{M_T}]$. Define $\mathbf{A}_{\mathcal{S}} = (\alpha\mathbf{I} + \mathbf{H}_{\mathcal{S}}\mathbf{H}_{\mathcal{S}}^H)^{-1}$.
while $|\mathcal{S}| > M_F$ **do**
  $m = \arg\min_r \frac{\|\mathbf{h}_r^H \mathbf{A}_{\mathcal{S}}\|^2}{1 - \mathbf{h}_r^H \mathbf{A}_{\mathcal{S}} \mathbf{h}_r}$, where $r \in \mathcal{S}$.
  $\mathbf{A}_{\mathcal{S}} = \mathbf{A}_{\mathcal{S}} + \frac{\mathbf{A}_{\mathcal{S}}\mathbf{h}_m\mathbf{h}_m^H\mathbf{A}_{\mathcal{S}}}{1 - \mathbf{h}_m^H\mathbf{A}_{\mathcal{S}}\mathbf{h}_m}$.
  $\mathcal{S} = \mathcal{S} - \{m\}$.
end while
The resulting set $\mathcal{S}$ is the desired transmit antenna set.

VI. Complexity Analysis

In this section, let us analyze the computational complexity for the proposed algorithms in Section V. A floating-point operation (flop) [30] is used to quantify the complexity. A real addition, multiplication, or division requires a flop. To simplify the procedure of analysis, let us first give the complexity for some matrix operations. Suppose that $\mathbf{X}$ is a $q \times p$ matrix, $\mathbf{Y}$ is a $p \times r$ matrix, and $\mathbf{Z}$ is an $n \times n$ nonsingular matrix. Let complexity order $\mathcal{O}(\cdot)$ represent the arithmetic order of flops. The complexity orders for matrix multiplication $\mathbf{X} \cdot \mathbf{Y}$ and matrix inversion $\mathbf{Z}^{-1}$ are of $\mathcal{O}(pqr)$ and $\mathcal{O}(n^3)$, respectively.

A. Complexity Analysis for ZF Precoding

For the optimal TAS algorithm in Section V-A, the total number of possible combination sets is

$$C_{M_F}^{M_T} = \frac{M_T!}{M_F!(M_T - M_F)!} = \frac{M_T \times (M_T - 1) \times \cdots \times (M_T - M_F + 1)}{M_F!} = \frac{M_T^{M_F} + \cdots}{M_F!} = \mathcal{O}\left(\frac{M_T^{M_F}}{M_F!}\right). \quad (28)$$

In each set, we need to calculate $\mathbf{\Lambda}_{\bar{\mathcal{S}}}$ in (24). Because $\mathcal{S}'$ is the total antenna set, the calculation of $(\mathbf{H}_{\mathcal{S}'}\mathbf{H}_{\mathcal{S}'}^H)^{-1}$, which contains one matrix multiplication and one matrix inversion, only needs to be calculated for one time. Because the dimension of $\mathbf{H}_{\mathcal{S}'}$ is $K \times M_T$, the complexity order is on the order of $\mathcal{O}(K^2 M_T)$ for two multiplications and $\mathcal{O}(K^3)$ for matrix inversion. Then, to obtain $\mathbf{\Lambda}_{\bar{\mathcal{S}}}$, four matrix multiplications and one matrix inversion are needed, and the total computational complexity for each set is on the order of $\mathcal{O}(2K^2(M_T - M_F) + 2K(M_T - M_F)^2 + (M_T - M_F)^3)$. Therefore, based on (28), the overall complexity to obtain the optimal antenna set is on the order (29), shown at the bottom of the page.

Now, considering the computational complexity of the greedy search method in Algorithm 2, the total number of transmit antenna sets is

$$\sum_{l=M_F+1}^{M_T} l. \quad (30)$$

Because the selection rule is replaced by (25), the term $\mathbf{A}_{\mathcal{S}}$ is initially needed, and the corresponding computations contain one matrix multiplication, i.e., $\mathcal{O}(K^2 M_T)$, and one inversion, i.e., $\mathcal{O}(K^3)$. Then, to determine which transmit antenna to remove, the complexity order of the selection rule in (25) is $\mathcal{O}(2K^2)$, because the orders of both the numerator and denominator are $\mathcal{O}(K^2)$. $\mathbf{A}_{\mathcal{S}}$ can be updated when the antenna to be removed is determined. We need to update $\mathbf{A}_{\mathcal{S}}$ for $M_T - M_F$ times, and each time is on the order of $\mathcal{O}(K^2)$ for matrix multiplications in (25). Therefore, the complexity order of Algorithm 2 is

$$\mathcal{O}\left(K^2 M_T + K^3 + 2K^2 \sum_{l=M_F+1}^{M_T} l + K^2(M_T - M_F)\right) \approx \mathcal{O}\left(K^2\left(M_T^2 - M_F^2\right)\right).$$

B. Complexity Analysis for MMSE Precoding

The number of possible antenna sets using the exhaustive search is shown in (28). As the selection rule in (21), the overall computational complexity is almost the same as the exhaustive search in ZF precoding, except that some additions can be ignored, i.e., $(\alpha\mathbf{I} + \mathbf{H}_{\mathcal{S}'}\mathbf{H}_{\mathcal{S}'}^H)$. Hence, we can conclude that the overall complexity for obtaining the optimal antenna set in the sense of minimizing the MSE is also on the order shown in (29).

TABLE I
COMPLEXITY COMPARISON OF VARIOUS ANTENNA SELECTION ALGORITHMS WITH LINEAR PRECODING

Complexity analysis for ZF & MMSE precoding		
Search method	Exhaustive search	Proposed algorithms (Algorithm 2 / Algorithm 3)
Complexity order	$O(K^2 M_T + \frac{M_T^{M_S}}{M_S!}(2K^2(M_T - M_S) + 2K(M_T - M_S)^2 + (M_T - M_S)^3)$	$O(K^2(M_T^2 - M_S^2))$

In Algorithm 3, instead of calculating the sum throughput of each transmit antenna set, the MSE criterion in (20) is used to remove the antennas. Not unlike exhaustive search, the search is similar to Algorithm 2, and the overall complexity is also almost the same as Algorithm 2, i.e., $\mathcal{O}(K^2(M_T^2 - M_F^2))$. Table I summarizes the analyzed computational complexity for all proposed antenna selection algorithms introduced in Section IV.

VII. SIMULATION RESULTS

In this section, simulations are given to show that the derived upper bound in Section IV is close to the simulation results in high-SNR regions and the performance of the proposed TAS algorithms in Section V. In all simulations, 16-QAM signaling and 5000 MU-MIMO channel realizations are used. Without loss of generality, we assume $\sigma^2 = 1$.

Example 1: Derived Upper Bound for Throughput Loss: Let $M_T = 16$ and $K = 4$. Fig. 2 shows the simulated sum throughput loss and the theoretical upper bound derived in (17) as functions of SNR values for $M_F = 4, 6,$ and 8. Observe from the simulated result that, for a fixed M_F, the sum throughput loss is monotonically increasing with the SNR and eventually reaches the derived upper bound when the SNR is sufficiently high (SNR from 10 dB to 18 dB for M_F from 8 to 4). These observations corroborate the theoretical results in Theorem 2. Therefore, the derived upper bound provides a good design reference to trade off between the number of RF units and the sum throughput for ZF-precoded MU-MIMO systems.

Example 2: Performance Comparison of Exhaustive Search and Greedy Search: Let $M_F = 4$ and $K = 4$. The average sum throughput as a function of M_T for ZF- and MMSE-precoded MU-MIMO systems is shown in Figs. 3 and 4, respectively. For ZF-precoded systems, we consider equal power (EP) allocation and water-filling (WF) power allocation. For the MMSE scheme, we consider EP allocation only, because WF power allocation needs an iterative algorithm and is not easy to be applied due to the interference from other users [32]. Based on the figures, the proposed greedy transmit selection algorithms with a throughput criterion can achieve

$$\mathcal{O}\left(\frac{K^2 M_T + \left(2K^2(M_T - M_F) + 2K(M_T - M_F)^2 + (M_T - M_F)^3\right) M_T^{M_F}}{M_F!}\right) \quad (29)$$

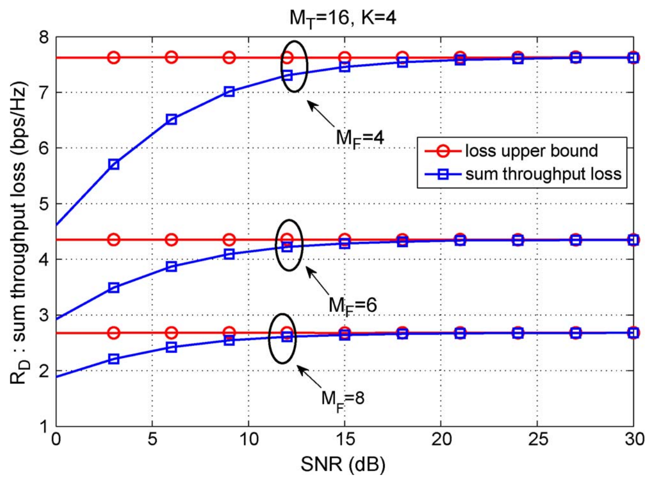

Fig. 2. Sum throughput loss and the derived theoretical upper bound for using TAS in ZF-precoded MU-MIMO systems $(M_T = 16, K = 4)$.

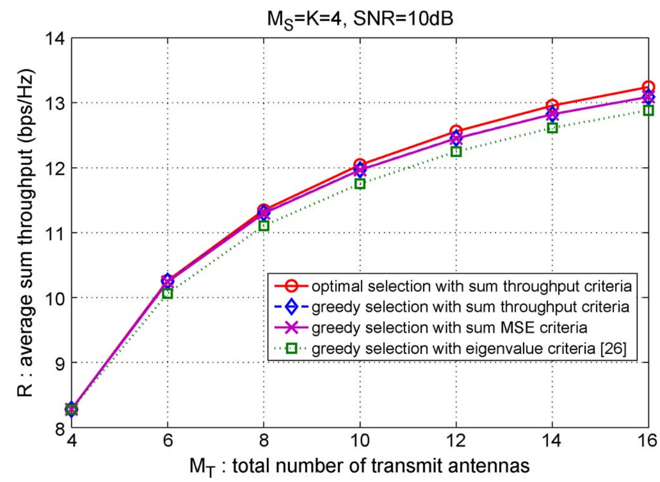

Fig. 4. Sum throughput of various TAS algorithms for $M_F = K = 4$ and SNR $= 10$ dB in MMSE-precoded MU-MIMO systems.

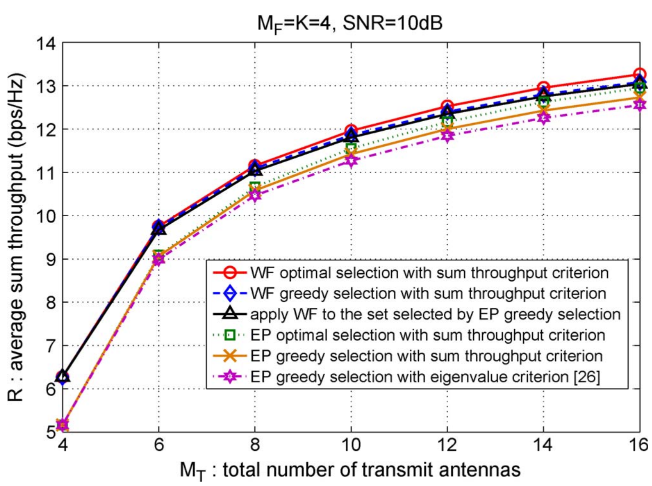

Fig. 3. Sum throughput of various TAS algorithms for $M_F = K = 4$ and SNR $= 10$ dB in ZF-precoded MU-MIMO systems.

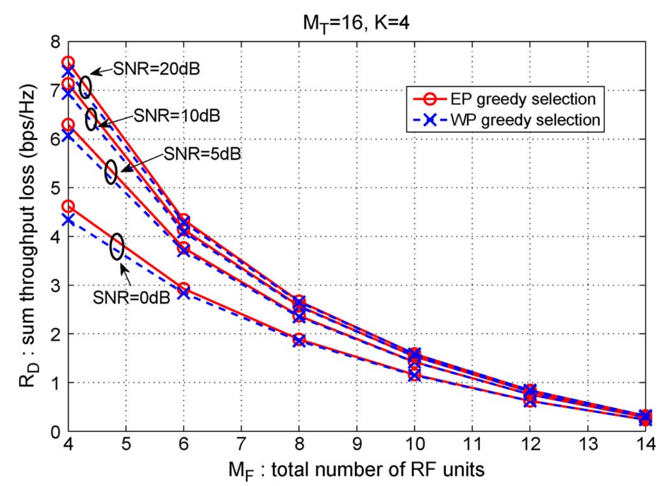

Fig. 5. Sum throughput loss for $M_T = 16$ and $K = 4$ in ZF-precoded MU-MIMO systems.

above 95% sum throughput of the optimal exhaustive search, with much lower computational complexity (see the circled and diamond curves). Moreover, if we apply the optimal WF power allocation in the ZF precoder, the WF procedure is needed to determine the power allocation matrix for every iteration of sum throughput calculation, and this may lead to high computational complexity. To overcome this issue, we can apply the equal gain power allocation for every iteration of sum throughput calculation; whenever the equal gain precoder is determined, we simply apply the WF procedure for one time. This approach can significantly reduce the computational complexity due to the WF procedure, and the corresponding performance is shown in the triangular curve. We see that the triangular and the diamond curves are almost the same. Furthermore, we compare the performance between the proposed algorithm and the algorithm proposed in [26], which maximizes the minimum eigenvalue of the channel matrix instead of the sum throughput. It is observed that the performance using the sum throughput criterion outperforms the performance using the eigenvalue criterion in [26]. Note that the computational complexity of the greedy algorithms can further be simplified with no performance loss using the fast algorithms in Algorithms 2 and 3 for the ZF and MMSE precoders, respectively. As a result, the proposed TAS algorithms can achieve a performance comparable to the optimal TAS algorithm, with much lower computational complexity. Finally, we see that, in the MMSE precoded systems, using the MSE criterion in (18) to select transmit antennas performs almost the same as using the sum throughput criterion (see the crossed and the diamond curves in Fig. 4).

Figs. 5 and 6 show the sum throughput loss for ZF precoding and MSE increase for MMSE precoding as functions of the number of RF units M_F. The two figures show that the performance loss is, indeed, monotonically decreasing with M_F; that is, using more RF units always improves the performance in linearly precoded MU-MIMO systems, which corroborates the theoretical results in Theorems 1 and 3. Moreover, in Fig. 5, we can observe that the performance gap between EP and WF power allocation in the ZF precoder also decreases as M_F increases. This is reasonable, because the smaller the number of M_F, the smaller the equivalent channel gain of each user becomes, and thus, the more pronounced the advantage of WF is in this case.

Example 3: Sum Throughput Comparison for Different Numbers of RF Units: When the hardware complexity is considered, using fewer RF units to achieve satisfactory sum throughput is generally preferred. This example evaluates how

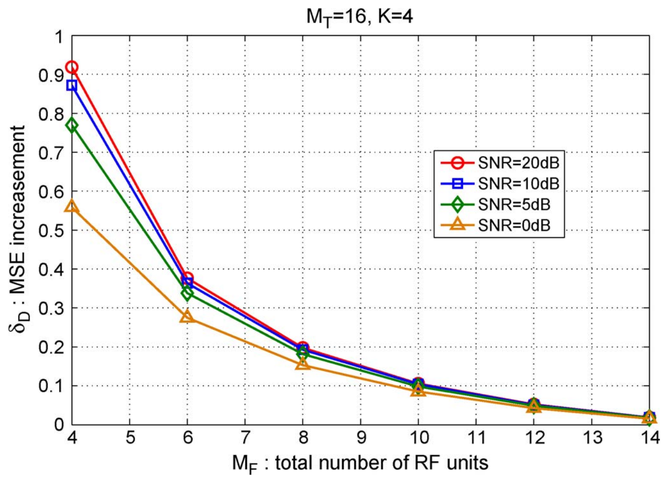

Fig. 6. MSE increase for $M_T = 16$ and $K = 4$ with MMSE-precoded MU-MIMO systems.

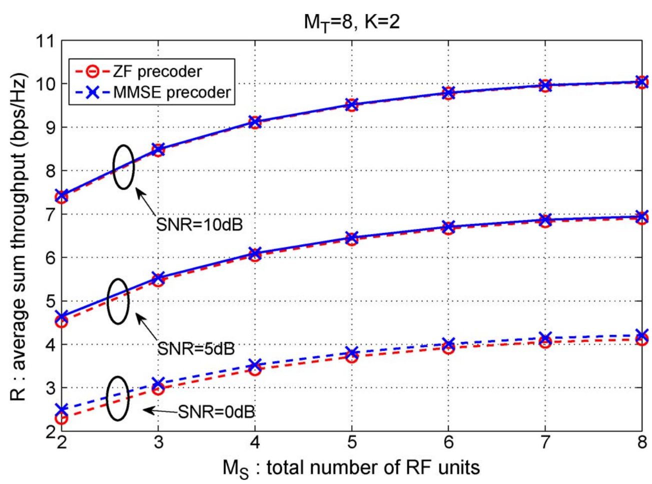

Fig. 7. Sum throughput as a function of M_F for the proposed TAS in Algorithms 2 and 3, with $M_T = 8$ and $K = 2$.

the number M_F of RF units affects the sum throughput. Let $M_T = 8$, which is the number of users, be $K = 2$. SNR $= 0, 5,$ and 10 dB. Fig. 7 shows the sum throughput as a function of M_F for the ZF precoding with Algorithm 2 and the MMSE precoding with Algorithm 3. In general, letting M_F be around two times of K can achieve 85% of the sum throughput obtained by letting $M_F = M_T$. This figure can be used as a design reference to determine a reasonable M_F value to reduce hardware complexity. Moreover, we observe that, when the SNR is low and the number of RF units is small, the sum throughput of the MMSE precoder outperforms the sum throughput of the ZF precoder whereas the performance gap decreases as the SNR value or the number of M_F increases.

Example 4: BER Comparison for Different Numbers of RF Units: Let $K = 2$. The bit-error-rate (BER) performance for the proposed ZF- and MMSE-precoded TASs is given in Fig. 8 for different numbers M_F and M_T. According to [33], the diversity order of a signal-user system with the ZF or MMSE receiver or precoder is $\max(M, N) - \min(M, N) + 1$ at high-SNR regions, where M and N are the numbers of active transmit and receive antennas, respectively. Now, $K = 2$; thus, for $M_T = M_F = 2$, the system does not have selection diversity,

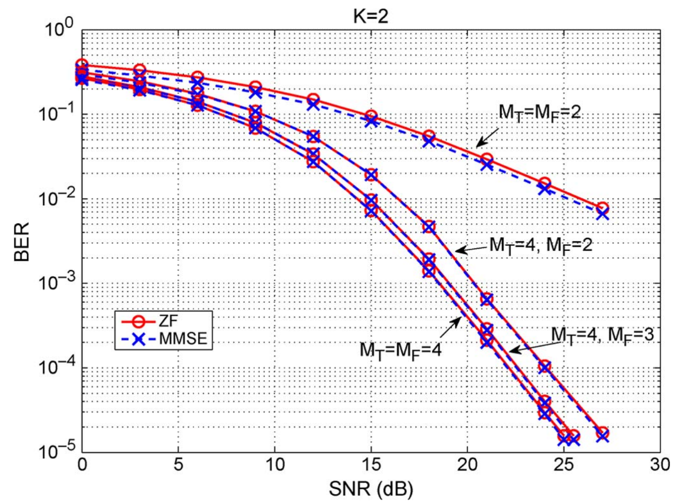

Fig. 8. BER performance for the proposed TAS in Algorithms 2 and 3.

and therefore, the diversity order is 1. For $M_T = 4$ and $M_F = 4$, the system has a diversity order of 3. For $M_T = 4$ and $M_F = 2$ or 3, the systems still have a diversity order of 3. Therefore, we can conclude that, by using TAS $(M_F < M_T)$, the system can achieve full diversity order, which can be obtained using all transmit antennas $(M_F = M_T)$. Moreover, increasing M_F only provides an antenna gain that shifts the BER curve from right to left at high-SNR regions [34]. Finally, we observe that, when the diversity order is 1, i.e., when $M_T = 2$, the MMSE precoder outperforms the ZF precoder. However, when the diversity is greater than 1, i.e., $M_T > 2$, the performance gap between the ZF and MMSE precoders becomes less pronounced, because TAS can significantly improve the MIMO channel condition.

VIII. Conclusion

Antenna selection is an effective method of reducing the use of RF units. In this paper, we have theoretically analyzed how performance is affected by TAS in linearly precoded MU-MIMO systems. Unlike the results in the work of Gore *et al.* and Tsai for single-user MIMO systems that removing active transmit antennas sometimes leads to the best performance if these antennas result in ill-conditioned channel matrices, the analytical results in this paper have shown that removing active transmit antennas always degrades the system performance in linearly precoded MU-MIMO systems. In addition, we have defined the performance loss due to removing active antennas and derived an upper bound of the performance loss for ZF-precoded MU-MIMO systems. This performance bound is tight when the SNR is moderately high. Consequently, this derived performance bound can provide a good reference for making a tradeoff between the number of RF components and the system performance. Moreover, it is known that the optimal TAS exhaustively demands search, and in general, its computational complexity is extremely large. To overcome this issue, we have formulated the problem based on the analytical results and proposed several TAS algorithms that can significantly reduce the computational complexity. The complexity analysis showed that the proposed algorithms can reduce the computational complexity from exponential order to polynomial order, whereas the system performance can still be kept comparable with the optimal algorithm. Simulation results were provided to show the accuracy of the analytical results and the advantages of the proposed algorithms. We concluded that the analyzed results enable us to better understand how TAS affects the MU-MIMO systems. In addition, the proposed algorithms make TAS more feasible to be used in practical systems.

APPENDIX A
PROOF OF LEMMA 2

Applying the singular value decomposition to $\mathbf{A}$ yields $\mathbf{A} = \mathbf{U\Sigma V}^H$, where $\mathbf{\Sigma}$ is an $m \times n$ matrix, with its diagonal elements $\{\sigma_1, \sigma_2, \ldots, \sigma_m\}$ being the singular values of $\mathbf{A}$. Then, we can write $\mathbf{A}_c = \mathbf{U\Sigma V}_c^H$, where $\mathbf{V}_c^H$ is an $n \times k$ matrix, with the columns chosen from the corresponding indexed columns of $\mathbf{V}$. Recall that $\mathbf{Q} = (\alpha\mathbf{I} + \mathbf{AA}^H)^{-1}$. Based on the aforementioned conditions, we have

$$\begin{aligned}\mathrm{tr}&\left(\left(\mathbf{QA}_c\left(\mathbf{I} - \mathbf{A}_c^H\mathbf{QA}_c\right)^{-1}\mathbf{A}_c^H\mathbf{Q}\right)\right.\\ &= \mathrm{tr}\left(\mathbf{\Sigma}^H\left((\alpha\mathbf{I} + \mathbf{\Sigma\Sigma}^H)^{-1}\right)^2\mathbf{\Sigma V}_c^H\mathbf{G}^{-1}\mathbf{V}_c\right) \quad (31)\end{aligned}$$

where we define $\mathbf{G} = \mathbf{V}_c(\mathbf{I} - \mathbf{\Sigma}^H(\alpha\mathbf{I} + \mathbf{\Sigma\Sigma}^H)^{-1}\mathbf{\Sigma})\mathbf{V}_c^H$ Let $\mathbf{V}_c = [\mathbf{V}_{c_1}\ \mathbf{V}_{c_2}]$, where $\mathbf{V}_{c_1}$ and $\mathbf{V}_{c_2}$ are $k \times m$ and $k \times (n - m)$ matrices, respectively. Then

$$\begin{aligned}\mathbf{G} &= [\,\mathbf{V}_{c_1} \quad \mathbf{V}_{c_2}\,]\begin{bmatrix}\mathbf{D} & \mathbf{0}\\ \mathbf{0} & \mathbf{I}\end{bmatrix}\begin{bmatrix}\mathbf{V}_{c_1}^H\\ \mathbf{V}_{c_2}^H\end{bmatrix}\\ &= \mathbf{V}_{c_1}\mathbf{DV}_{c_1}^H + \mathbf{V}_{c_2}\mathbf{V}_{c_2}^H \quad (32)\end{aligned}$$

where

$$\mathbf{D} = \begin{bmatrix}\frac{\alpha}{\alpha+\sigma_1^2} & \cdots & 0\\ \vdots & \ddots & \vdots\\ 0 & \cdots & \frac{\alpha}{\alpha+\sigma_m^2}\end{bmatrix}.$$

Using (32), for $\alpha \geq 0$, the last equation in (31) can further be manipulated as

$$\mathrm{tr}\left(\mathbf{\Sigma}^H\left((\alpha\mathbf{I} + \mathbf{\Sigma\Sigma}^H)^{-1}\right)^2\mathbf{\Sigma V}_c^H\mathbf{G}^{-1}\mathbf{V}_c\right) = \sum_{j=1}^{m}\frac{\nu_{jj}}{\sigma_j^2} > 0$$

where ν_{jj} is defined as the jth diagonal element of $\mathbf{V}_c^H\mathbf{G}^{-1}\mathbf{V}_c$. This case completes the proof of Lemma 2.

APPENDIX B
PROOF OF LEMMA 3

Based on (3)–(5), we have

$$\begin{aligned}R(\mathcal{S}') - R(\mathcal{S}) &= K\log\left(1 + \frac{P}{\sigma^2\mathrm{tr}\left(\mathbf{W}_{\mathcal{S}'}^H\mathbf{W}_{\mathcal{S}'}\right)}\right)\\ &\quad - K\log\left(1 + \frac{P}{\sigma^2\mathrm{tr}\left(\mathbf{W}_{\mathcal{S}}^H\mathbf{W}_{\mathcal{S}}\right)}\right)\\ &= K\log\left(\frac{1 + \frac{\mathrm{SNR}}{\mathrm{tr}\left(\left(\mathbf{H}_{\mathcal{S}'}\mathbf{H}_{\mathcal{S}'}^H\right)^{-1}\right)}}{1 + \frac{\mathrm{SNR}}{\mathrm{tr}\left(\left(\mathbf{H}_{\mathcal{S}}\mathbf{H}_{\mathcal{S}}^H\right)^{-1}\right)}}\right). \quad (33)\end{aligned}$$

Applying Lemma 1 into (33) yields

$$R_D(\bar{\mathcal{S}}) = K\log\left(\frac{1 + \frac{\mathrm{SNR}}{\mathrm{tr}\left(\left(\mathbf{H}_{\mathcal{S}'}\mathbf{H}_{\mathcal{S}'}^H\right)^{-1}\right)}}{1 + \frac{\mathrm{SNR}}{\mathrm{tr}\left(\left(\mathbf{H}_{\mathcal{S}'}\mathbf{H}_{\mathcal{S}'}^H\right)^{-1}\right) + \mathrm{tr}(\mathbf{\Lambda}_{\bar{\mathcal{S}}})}}\right). \quad (34)$$

Manipulating (34) leads to the first equation in (14). Moreover, using Lemma 2 and setting $\alpha = 0$, we find the fact that $\mathrm{tr}(\mathbf{\Lambda}_{\bar{\mathcal{S}}})$ is nonnegative. As a result $R_D(\bar{\mathcal{S}}) > 0$.

APPENDIX C
PROOF OF THEOREM 1

Using the result in Lemma 3 yields

$$R(\mathcal{S}_{opt1}) < R(\mathcal{S}_2), \quad \text{if } \mathcal{S}_{opt1} \subset \mathcal{S}_2.$$

Because we assume that $\mathcal{S}_{opt1}$ and $\mathcal{S}_{opt2}$ are the optimal transmit antenna sets that achieve the maximum throughput for $M_F = |\mathcal{S}_{opt1}|$ and $M_F = |\mathcal{S}_{opt2}|$, respectively, this results in

$$R(\mathcal{S}_{opt1}) < R(\mathcal{S}_2) \leq R(\mathcal{S}_{opt2}) \text{ for } |\mathcal{S}_{opt1}| < |\mathcal{S}_2| = |\mathcal{S}_{opt2}|$$

which completes the proof of Theorem 1.

APPENDIX D
PROOF OF THEOREM 2

By Lemma 4, it is obvious that the throughput loss $R_D(\bar{\mathcal{S}})$ in (14) is a monotonically increasing function of the SNR. In addition, based on (14), we have

$$\begin{aligned}R_D(\bar{\mathcal{S}}) &= K\log\left(1 + \frac{\mathrm{tr}(\mathbf{\Lambda}_{\bar{\mathcal{S}}})}{\frac{(\mathrm{tr}(\mathbf{Q}_{\mathcal{S}'}))^2}{\mathrm{SNR}} + \mathrm{tr}(\mathbf{Q}_{\mathcal{S}'})\left(1 + \frac{\mathrm{tr}(\mathbf{\Lambda}_{\bar{\mathcal{S}}})}{\mathrm{SNR}}\right)}\right)\\ &\leq K\log\left(1 + \frac{\mathrm{tr}(\mathbf{\Lambda}_{\bar{\mathcal{S}}})}{\mathrm{tr}(\mathbf{Q}_{\mathcal{S}'})}\right) \quad (35)\end{aligned}$$

where $\mathbf{Q}_{\mathcal{S}'} = (\mathbf{H}_{\mathcal{S}'}\mathbf{H}_{\mathcal{S}'}^H)^{-1}$. Furthermore, by letting the SNR tend to ∞ for the second equation in (35), we have

$$\lim_{\mathrm{SNR}\to\infty} R_D(\bar{\mathcal{S}}) = K\log\left(1 + \frac{\mathrm{tr}(\mathbf{\Lambda}_{\bar{\mathcal{S}}})}{\mathrm{tr}\left(\mathbf{H}_{\mathcal{S}'}\mathbf{H}_{\mathcal{S}'}^H\right)^{-1}}\right).$$

APPENDIX E
PROOF OF LEMMA 5

Based on (19), the MSE difference is

$$\begin{aligned}\delta(\mathcal{S}') - \delta(\mathcal{S}) &= \mathrm{tr}\left(\left(\alpha\mathbf{I} + \mathbf{H}_{\mathcal{S}'}\mathbf{H}_{\mathcal{S}'}^H\right)^{-1}\right)\\ &\quad - \mathrm{tr}\left(\left(\alpha\mathbf{I} + \mathbf{H}_{\mathcal{S}}\mathbf{H}_{\mathcal{S}}^H\right)^{-1}\right)\\ &= \mathrm{tr}\left(\mathbf{A}_{\mathcal{S}'} - \left(\alpha\mathbf{I} + \mathbf{H}_{\mathcal{S}'}\mathbf{H}_{\mathcal{S}'}^H - \mathbf{H}_{\bar{\mathcal{S}}}\mathbf{H}_{\bar{\mathcal{S}}}^H\right)^{-1}\right). \quad (36)\end{aligned}$$

By applying the result of Lemmas 1 and 2, (36) becomes $-\mathrm{tr}(\mathbf{A}_{\mathcal{S}'}\mathbf{H}_{\bar{\mathcal{S}}}(\mathbf{I} - \mathbf{H}_{\bar{\mathcal{S}}}^H\mathbf{A}_{\mathcal{S}'}\mathbf{H}_{\bar{\mathcal{S}}})^{-1}\mathbf{H}_{\bar{\mathcal{S}}}^H\mathbf{A}_{\mathcal{S}'}) < 0$.

APPENDIX F
PROOF OF THEOREM 3

Using the result in Lemma 5, we can conclude the following:

$$\delta(\mathcal{S}_{opt1}) > \delta(\mathcal{S}_2), \quad \text{if } \mathcal{S}_{opt1} \subset \mathcal{S}_2.$$

Because we assume that $\mathcal{S}_{opt1}$ and $\mathcal{S}_{opt2}$ are the optimal transmit antenna sets with the MMSE when $M_F = |\mathcal{S}_{opt1}|$ and $M_F = |\mathcal{S}_{opt2}|$, respectively, we have

$$\delta(\mathcal{S}_{opt1}) > \delta(\mathcal{S}_2) \geq \delta(\mathcal{S}_{opt2}) \text{ for } |\mathcal{S}_{opt1}| < |\mathcal{S}_2| = |\mathcal{S}_{opt2}|$$

which completes the proof of Theorem 3.

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Pu-Hsuan Lin was born in Tainan, Taiwan, in 1985. He received the M.S. degree in electrical control engineering from the National Chiao Tung University (NCTU), Hsinchu, Taiwan, in August 2008. He is currently pursuing the Ph.D. degree with the Department of Electrical Engineering, NCTU.

His research interests include multiple-input–multiple-output–multiuser (MIMO-MU) precoding systems, MIMO decoding, and compressive sensing.

Shang-Ho (Lawrence) Tsai (SM'12) was born in Kaohsiung, Taiwan, in 1973. He received the Ph.D. degree in electrical engineering from the University of Southern California (USC), Los Angeles, in August 2005.

From June 1999 to July 2002, he was with the Silicon Integrated Systems Corporation, where he participated in the very-large-scale-integration (VLSI) design for Discrete Multi-tone Asymmetric Digital Subscriber Line systems. From September 2005 to January 2007, he was with the MediaTek Inc., where he participated in the VLSI design for multiple-input–multiple-output-orthogonal frequency-division multiplexing (MIMO-OFDM) systems. Since February 2007, he has been with the Department of Electrical and Control Engineering (now the Department of Electrical Engineering), National Chiao Tung University, Hsinchu, Taiwan, where he is currently an Associate Professor. His research interests include signal processing for communications, statistical signal processing, and signal processing for VLSI designs.

Dr. Tsai received of a government scholarship for overseas study from the Ministry of Education, Taiwan, in 2002–2005.