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MAI-Free Performance of PMU-OFDM Transceiver in Time-Variant Environment

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ABSTRACT

An approximately multi-user OFDM transceiver was introduced to reduce the multi-access interference (MAI) due to the carrier frequency offset (CFO) to a negligible amount via precoding by Tsai, Lin and Kuo. In this work, we investigate the performance of this precoded multi-user (PMU) OFDM system in a time-variant channel environment. We analyze and compare the MAI effect caused by time-variant channels in the PMU-OFDM and the OFDMA systems. Generally speaking, the MAI effect consists of two parts. The first part is due to the loss of orthogonality among subchannels for all users while the second part is due to the CFO effect caused by the Doppler shift. Simulation results show that, although OFDMA outperforms the PMU-OFDM transceiver in a fast time-variant environment without CFO, PMU-OFDM outperforms OFDMA in a slow time-variant channel via the use of M/2 symmetric or anti-symmetric codewords of M Hadamard-Walsh codes.

Keywords: PMU-OFDM, OFDMA, MAI, multiuser, Doppler shift, CFO, time variant

1. INTRODUCTION

Orthogonal frequency division multiplexing (OFDM) is a promising technique to combat frequency selective fading channels. The OFDM system has been proposed for high data rate wired and wireless communication systems [1],[2]. There are several applications adopting the OFDM technique, *e.g.* the European digital audio broadcast (DAB) standard, the European terrestrial digital video broadcast (DVB-T), the IEEE 802.11a wireless local area networks (WLAN's) standard and the IEEE 802.16a wireless metropolitan area network (WMAN). In OFDM, a broadband channel is shared by overlapping orthogonal subchannels, where the inter symbol interference (ISI) effect can be compensated by several one-tap multiplications in the frequency domain. As a result, OFDM has an advantage over CDMA-based systems that experience ISI in frequency selective channels. However, OFDM is susceptible to the loss of orthogonality among subcarriers, which results in the inter-carrier interference (ICI). ICI arises when the timing and/or the carrier frequency offset (CFO), is caused by the Doppler shift or the oscillator phase noise. In a high-speed environment, a large Doppler shift will cause a large CFO value. Hence, CFO induced by the Doppler shift is one of the main reasons that OFDM is not used in applications such as urban cellular mobile telephony, where high mobility is required.

There has been a large amount of research effort on the estimation and compensation of the timing and the frequency offsets in OFDM systems [5],[6],[7]. The CFO effect is nevertheless exacerbated in multiuser OFDM systems such as

Digital Wireless Communications VII and Space Communication Technologies, edited by Raghuveer M. Rao, Sohail A. Dianat, Michael D. Zoltowski, Rabindra Singh, Susan P. Miller, Proc. of SPIE Vol. 5819 (SPIE, Bellingham, WA, 2005) · 0277-786X/05/\$15 · doi: 10.1117/12.605276 Orthogonal Frequency Division Multiple Access (OFDMA). In OFDMA, each user occupies a subset of carriers. Subchannels are assigned to different users either randomly or according to a certain rule. The CFO of each user not only introduces ICI for this user but also causes multiple access interference (MAI) for other users. Thus, although OFDMA is MAI-free over time-invariant channels when time and frequency are well synchronized, they will experience MAI when either time or frequency fails to synchronize. The CFO estimation for OFDMA is a challenging task, which has attracted a lot of attention since the CFO estimation must be able to combat MAI at the receiver end. Under this situation, the estimation task usually demands a high computational cost. Several CFO estimation algorithms have been proposed in the literature. They all impose additional complexity on the receiver design [6].

A new multiuser OFDM transceiver was introduced in [8], which is called the precoded multi-user (PMU) OFDM. Unlike OFDMA, all users in PMU-OFDM share the whole bandwidth by using a set of orthogonal codes. The PMU-OFDM system is approximately MAI free over frequency selective channels. It was shown in [9] that, by choosing *M*/2 symmetric (or anti symmetric) codewords from *M* Hadamard Walsh codewords in the precoding stage, MAI can be reduced to a negligible amount. The PMU-OFDM system is robust to time and frequency offsets. By choosing the codes properly in the precoding stage, the PMU-OFDM transceiver is able to combat the MAI effect induced by CFO. Moreover, for a fixed CFO, the MAI effect due to CFO decreases when the number of users increases [10]. In the previous work, the PMU-OFDM system was only examined in a quasi-stationary channel environment. That is, the channel is assumed to be time-invariant at least over one OFDM symbol block, where the CFO was resulted from oscillator instabilities or a slowly varying channel.

In this work, we examine the performance of the PMU-OFDM system in a fast time-varying channel, where the CFO is caused by the Doppler shift. In particular, we would like to compare the performance of PMU-OFDM and OFDMA over time-variant channels with CFO. We will show by simulation that, in a fast time-variant channel, the performance of both systems degrade considerably since the CFO induced by the Doppler shift and time-variant channels will add significant MAI to these systems. We observe that, in a slower time-variant environment where the coherent time of the channel is much larger than the OFDM block length, the PMU-OFDM system can take advantage of code selection in the precoding stage and suppress MAI to a negligible amount [9], [10]. As a result, it outperforms OFDMA under this situation

The rest of the paper is organized as follows. The PMU-OFDM system is reviewed in Section 2. The MAI effect due to the Doppler shift for PMU-OFDM is derived in Section 3. We present simulation results and compare the performance of the two systems in Section 4. Finally, some concluding remarks are drawn in Section 5.

2. SYSTEM MODEL

We give a brief review of the PMU-OFDM system in this section. The block diagram of the new multiuser OFDM system is shown in Fig. 1.



Fig. 1. The block diagram of the new multiuser OFDM transceiver

Suppose that there are T users sharing the multiple access channel. Let the input for user *i* be an $N \times 1$ vector denoted by \mathbf{x}_i . Each symbol in \mathbf{x}_i is spread by M times in the frequency domain to form a new vector \mathbf{y}_i , which can be expressed as

$$y_{i}[m+kM] = x_{i}[k],$$
 (1)

where m = 0, 1, ..., M - 1 and k = 0, 1, ..., N - 1.

In the next stage, \mathbf{y}_i is multiplied by a diagonal matrix \mathbf{W}_i to form an $NM \times 1$ vector $\mathbf{z}_i \cdot \mathbf{W}_i$ is constructed from repeating the *i*th column of an $M \times M$ Hadamard matrix, \mathbf{D} by N times along the diagonal given by:

$$\mathbf{W}_{i} = dig\left(\mathbf{d}_{i}^{t}, \mathbf{d}_{i}^{t} \dots \mathbf{d}_{i}^{t}\right), \quad , \qquad (2)$$

where \mathbf{d}_i^{t} is the transpose of the *i*th column of **D**. Let $w_i[l]$ be the *l*th diagonal element of \mathbf{W}_i . Since columns of **D** are orthogonal to each others, the diagonal elements of \mathbf{W}_i , for each two user *i* and *j*, satisfy the following equality:

$$\sum_{m=0}^{M-1} w_i [m+kM] w_j^* [m+kM] = \begin{cases} M, & i=j\\ 0, & i\neq j \end{cases},$$
(3)

where m = 0, 1, K M - 1, k = 0, 1, K N - 1 and w^* is the conjugate of w.

As shown in Fig.1, the *l*th element of vector \mathbf{z}_i is given by:

$$z_{i}[l] = y_{i}[l]w_{i}[l], \quad 0 \le l \le NM - 1.$$
(4)

Then, the frequency domain symbols, $z_i[m]$ (i = 1, 2, ...T, m = 0, 1, ..., NM - 1) are passed through an $NM \times NM$ inverse discrete Fourier transform (IDFT) matrix to transform to time domain symbols, $g_i(n)$, *i.e.*

$$g_{i}(n) = \frac{1}{NM} \sum_{m=0}^{NM-1} z_{i}[m] e^{j \frac{2\pi}{NM}mn} .$$
(5)

Next, following the parallel to serial conversion, L-1 cyclic prefix are inserted at the beginning of each OFDM symbol before transmitting to the time-variant, frequency selective channel of length L. At the receiver end, redundant cyclic

prefix symbols are removed and the serial message symbols are converted to parallel symbols and go through the DFT matrix. The received signals of user *i* will go through matrix \mathbf{W}_{i}^{*} , which corresponds to the de-spreading process. Finally, the average is taken over *M* symbols to reconstruct the originally transmitted symbol, *i.e.*

$$\hat{x}_{i}[k] = \frac{1}{M} \sum_{m=0}^{M-1} \hat{y}_{i}[m+kM]$$

$$= \frac{1}{M} \sum_{m=0}^{M-1} r[m+kM] w_{i}^{*}[m+kM], \quad 0 \le k \le N-1.$$
(6)

Finally, the frequency equalization operation is done. That is, $\hat{x}_i[k]$ is multiplied by a complex coefficient so that the distortion effect caused by the channel will be compensated.

3. PERFORMANCE ANALYSIS IN DOPPLER SHIFT ENVIRONMENT

In this section, we formulate the mathematical model for the PMU-OFDM system in a Doppler shift environment, where the Doppler shift will result in a time-variant channel and the CFO effect. Let $b_i(n)$ be the input of user *i* to the DFT, *i.e.* $b_i(n)$ is obtained by removing cyclic prefix symbols from the convolution result between $g_i(n)$ and the timevariant channel. We have

$$b_i(n) = \sum_{\tau=0}^{L-1} h_i(n,\tau) e^{j\frac{2\pi}{NM}n\varepsilon_i} g_i(n-\tau) , \qquad (7)$$

where $h_i(n, \tau)$ is the impulse response of the time variant channel at time *n* for user *i* and ε_i is the normalized CFO of user *i* due to the Doppler shift. Note that we do not consider the contribution of oscillator phase noise to CFO but only the Doppler shift here. Let r[l] be the frequency domain received signal, *i.e.*

$$r[l] = \sum_{i=1}^{T} s_i[l] + e[l], \qquad (8)$$

where

$$s_{i}[l] = \sum_{n=0}^{NM-1} b_{i}(n) e^{-j\frac{2\pi}{NM}nl}, \qquad (9)$$

and e[l] is the complex Gaussian noise after the DFT.

For symbol detection of the *j*th user, r[l] is multiplied by \mathbf{W}_j^* and then averaged over *M* symbols. Let l = v + kM. We have

$$\hat{x}_{j}[k] = \frac{1}{M} \sum_{\nu=0}^{M-1} r[\nu + kM] w_{j}^{*}[\nu + kM] \quad .$$
⁽¹⁰⁾

Since for each individual user, we use the same Hadamard Walsh codes for all symbols, we have

$$w_i[v+kM] = w_i[v], \quad i=1,2,..T, \ 0 \le k \le N-1$$
 (11)

By substituting r[l] from Eq. (8) in Eq. (10), we are led to

$$\hat{x}_{j}[k] = \varphi_{j}[k] + \gamma[k] + \hat{e}[k], \qquad (12)$$

where

$$\varphi_{j}[k] = \frac{1}{M} \sum_{\nu=0}^{M-1} s_{j} [\nu + kM] w_{j}^{*} [\nu + kM], \qquad (13)$$

$$\gamma[k] = \sum_{\substack{i=1,\\i\neq j}}^{T} \frac{1}{M} \sum_{\nu=0}^{M-1} s_i [\nu + kM] w_j^* [\nu + kM],$$
(14)

and

$$\hat{e}[k] = \frac{1}{M} \sum_{\nu=0}^{M-1} e[\nu + kM] w_{j}^{*}[\nu + kM].$$
(15)

Let us consider $\varphi_j[k]$ first. We can divide $\varphi_j[k]$ into two parts. The first part is the distorted symbol while the second part is the interferences from all other frequencies, $m \neq l$. Let m = u + fM. Then, using Eqs. (1), (4), (5), (7) and (9), $\varphi_j[k]$ can be written as

$$\varphi_j[k] = \eta_j[k] + \mu_j[k], \qquad (16)$$

where

$$\eta_{j}[k] = \frac{1}{NM} \sum_{\nu=0}^{M-1} \frac{1}{M} \sum_{n=0}^{NM-1} \sum_{\tau=0}^{L-1} h_{j}(n,\tau) e^{j\frac{2\pi}{NM}n\varepsilon_{j}} x_{j}[k] e^{-j\frac{2\pi}{NM}(\nu+kM)\tau} w_{j}[\nu] w_{j}^{*}[\nu],$$

$$=\frac{1}{NM}\sum_{n=0}^{NM-1}\sum_{\tau=0}^{L-1}h_{j}(n,\tau) e^{j\frac{2\pi}{NM}n\varepsilon_{j}}e^{-j\frac{2\pi}{N}k\tau}x_{j}[k]\frac{1}{M}\sum_{\nu=0}^{M-1}e^{-j\frac{2\pi}{NM}\nu\tau}w_{j}[\nu]w_{j}^{*}[\nu],$$
(17)

$$\mu_{j}[k] = \frac{1}{NM} \sum_{f=0}^{N-1} x_{j}[f] \frac{1}{M} \sum_{n=0}^{NM-1} \sum_{\tau=0}^{L-1} h_{j}(n,\tau) e^{j\frac{2\pi}{NM}n\varepsilon_{j}} A(n,\tau),$$
(18)

and

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$$A(n,\tau) = \sum_{\substack{u=0,\\u\neq v}+(k-f)M}^{M-1} \sum_{v=0}^{M-1} e^{-j\frac{2\pi}{NM}(u+fM)\tau} w_j[u] w_j^*[v] e^{j\frac{2\pi}{NM}(u-v+(f-k)M)n}.$$
(19)

Since L = N, we can assume $\tau = N$, for $0 \le \tau \le L - 1$. As a result, $e^{-j\frac{2\pi}{NM}v\tau}$ is nearly constant for v = 0, 1, ..., M - 1. Using this property, $\eta_j [k]$ can be written as

$$\eta_{j}[k] \approx \frac{1}{NM} \sum_{n=0}^{NM-1} \sum_{\tau=0}^{L-1} h_{j}(n,\tau) e^{j\frac{2\pi}{NM}n\varepsilon_{j}} e^{-j\frac{2\pi}{NM}(kM)\tau} e^{-j\frac{2\pi}{NM}(v)\tau} x_{j}[k] \frac{1}{M} \sum_{\nu=0}^{M-1} w_{j}[\nu] w_{j}^{*}[\nu] = \frac{1}{NM} \sum_{n=0}^{NM-1} \sum_{\tau=0}^{L-1} h_{j}(n,\tau) e^{j\frac{2\pi}{NM}n\varepsilon_{j}} e^{-j\frac{2\pi}{N}k\tau} e^{-j\frac{2\pi}{NM}(v)\tau} x_{j}[k],$$
(20)

where we substitute $\frac{1}{M} \sum_{v=0}^{M-1} w_j [v] w_j^* [v]$ by 1. In above, since $\eta_j [k]$ is the distortion factor for the *j*th user and

does not depend on other users. It can be compensated at the receiver end by frequency equalization if the *k*th frequency domain channel coefficient for the *j*th user is available. This can be obtained by including preambles symbols in each OFDM block. $\mu_j [k]$ is the ICI from other subchannels of user *j* due to the time-variant channel and the Doppler shift.

Next, let us consider the MAI terms of the *k*th symbol of user *j* due to all other users, *i.e.* $\gamma[k]$ in Eq. (14). Naturally, we would like to reduce MAI as much as possible. Again, we can decompose $\gamma[k]$ into two terms: the interferences form the same subchannel, m = l, and the interferences from different subchanels $m \neq l$. Let us rewrite MAI by using Eqs. (1), (4), (5), (7) and (9) as

$$\gamma[k] = o[k] + \delta[k], \qquad (21)$$

where

$$o[k] = \frac{1}{NM} \sum_{\substack{i=1, \\ i \neq j}}^{T} x_i[k] \sum_{n=0}^{NM} \sum_{\tau=0}^{-1} h_i(n,\tau) e^{j\frac{2\pi}{NM}(n\varepsilon_i - kM\tau)} \frac{1}{M} \sum_{\nu=0}^{M-1} w_i[\nu] w_j^*[\nu] e^{-j\frac{2\pi}{NM}\nu\tau}, \quad (22)$$

$$\delta[k] = \frac{1}{NM} \sum_{f=0}^{N-1} x_i[f] \sum_{\substack{i=1, \ i \neq j}}^{T} \frac{1}{M} \sum_{n=0}^{NM-1} \sum_{\tau=0}^{L-1} h_i(n,\tau) e^{j\frac{2\pi}{N}n\varepsilon_i} B(n,\tau) , \qquad (23)$$

and

$$B(n,\tau) = \sum_{\substack{u=0,\\u\neq v+(k-f)M}}^{M-1} \sum_{v=0}^{M-1} e^{-j\frac{2\pi}{NM}(u+fM)\tau} w_i[u] w_j^*[v] e^{j\frac{2\pi}{NM}(u-v+(f-k)M)n}.$$
 (24)

If L = N, then $\tau = N$ for $0 \le \tau \le L - 1$. Again, we can assume that $e^{-j\frac{2\pi}{NM}v\tau}$ is nearly constant for v = 0, 1, ..., M - 1. Using this property and Eq. (1), o[k] can be derived as

$$o[k] \approx \frac{1}{NM} \sum_{\substack{i=1,\\i\neq j}}^{T} x_i[k] \sum_{n=0}^{NM-1} \sum_{\tau=0}^{L-1} h_i(n,\tau) e^{j\frac{2\pi}{NM}n\varepsilon_i} e^{-j\frac{2\pi}{NM}(kM)\tau} e^{-j\frac{2\pi}{NM}v\tau} \frac{1}{M} \sum_{v=0}^{M-1} w_i[v] w_j^*[v]$$

= 0 (25)

where we used this property of Hadamard Walsh codes: $\sum_{m=0}^{M-1} w_i [v] w_j^* [v] = \begin{cases} M, & i=j \\ 0, & i\neq j \end{cases}$

Thus, MAI reduces to $\delta[k]$, which includes the interferences from other users (summation over *i*), other subchannels (summation over *v*). In other words, $\delta[k]$ has ICI from other users due to channel variation along time and MAI due to CFO from other users.

Let us examine how the PMU-OFDM system operates in a slow time-varying channel. In a slowly fading channel, we can assume that the channel remains stationary during one OFDM block, *i.e.* $h_j(n, \tau) = h_j(\tau)$. Thus, Eqs. (18)- (20) and (23) can be written as

$$\eta_{j}[k] = \frac{1}{NM} \sum_{n=0}^{NM-1} e^{j\frac{2\pi}{NM}n\varepsilon_{j}} \sum_{\tau=0}^{L-1} h_{j}(\tau) e^{-j\frac{2\pi}{N}k\tau} e^{-j\frac{2\pi}{NM}v\tau} = \frac{1}{NM} \sum_{n=0}^{NM-1} e^{j\frac{2\pi}{NM}n\varepsilon_{j}} \lambda_{j}[k], \quad (26)$$

$$\mu_{j}[k] = \frac{1}{NM} \sum_{f=0}^{N-1} x_{j}[f] \frac{1}{M} \sum_{n=0}^{NM-1} \sum_{\tau=0}^{L-1} h_{j}(\tau) e^{j\frac{2\pi}{NM}n\varepsilon_{j}} A(n,\tau),$$
(27)

$$\delta[k] = \frac{1}{NM} \sum_{f=0}^{N-1} x_i[f] \sum_{i=1}^{T} \frac{1}{M} \sum_{n=0}^{NM-1} \sum_{\tau=0}^{L-1} h_i(n,\tau) e^{j\frac{2\pi}{N}n\varepsilon_i} B(n,\tau),$$
(28)

where

$$\lambda_j[k] = \kappa \sum_{\tau=0}^{L-1} h_j(\tau) e^{-j\frac{2\pi}{N}k\tau} , \qquad (29)$$

 $A(n,\tau)$ and $B(n,\tau)$ are defined in Eqs. (19) and (24) and $\kappa \approx e^{-j\frac{2\pi}{NM}v\tau}$ is a constant. Note that although the channel does not change over one symbol duration, we still have the MAI term caused by CFO.

It was shown in [10] that a proper code selection using M/2 symmetric (or anti-symmetric) codewords of the M Hadamard Walsh code, $\delta[k]$ will be reduced to a negligible amount when N is sufficiently large. Also, under this situation, $\mu_j[k] \approx 0$ [10]. Therefore, only the CFO for each individual user must be compensated. In contrast, each receiver in OFDMA has to estimate and remove MAI. The disadvantage of using M/2 symmetric codewords is that it reduces the load of users to one half.

Next, we analyze the performance of OFDMA over a time-variant channel with CFO. A more detailed description of OFDMA in a time-variant environment can be found in [3]. In the OFDMA system, the whole available spectrum is partitioned among users into non-overlapping subgroups. Suppose that there are *NM* available subcarriers and *M* users, then every user utilizes *N* subchannels. Let S_j denote the set of subchannels belonging to the *j*th user and $a_j[k]$ be the input symbol of the *j*th user. The detected symbol of the *k*th sub-channel of the *j*th user is given by

$$\hat{a}_{j}[k] = \alpha_{j}(k) a_{j}[k] + \beta_{j}[k] + \xi[k] + \hat{\nu}(k), \qquad (30)$$

where

$$\alpha_{j}[k] = \frac{1}{NM} \sum_{n=0}^{NM-1} \sum_{\tau=0}^{L-1} h_{j}(n,\tau) e^{j\frac{2\pi}{NM}n\varepsilon_{j}} e^{-j\frac{2\pi}{NM}k\tau}, \qquad (31)$$

$$\beta_{j}[k] = \frac{1}{NM} \sum_{\substack{m \in S_{j} \\ m \neq k}} \sum_{n=0}^{NM-1} \sum_{\tau=0}^{L-1} h_{j}(n,\tau) e^{j\frac{2\pi}{NM}n(m-k)} e^{j\frac{2\pi}{NM}n\varepsilon_{j}} e^{-j\frac{2\pi}{NM}\tau m}, \qquad (32)$$

$$\xi[k] = \frac{1}{NM} \sum_{\substack{i=1, \ i \neq j}}^{T} \sum_{\substack{m \in S_{j} \\ m \neq k}} \sum_{n=0}^{NM-1} \sum_{\tau=0}^{L-1} h_{i}(n,\tau) e^{j\frac{2\pi}{NM}n(m-k)} e^{j\frac{2\pi}{NM}n\varepsilon_{i}} e^{-j\frac{2\pi}{NM}\tau m}, \qquad (33)$$

and $\hat{v}(k)$ is the frequency domain AWGN. Similar to the PMU-OFDM transceiver, we observe that OFDMA in a timevarying channel with CFO has the distortion factor, $\alpha_j[k]$, the ICI term, $\beta_j[k]$, due to channel variation and CFO from the *j*th user, and the MAI term, $\xi[k]$, caused by the time-variant channel and CFO from all other users. Also, note that the self-distortion factor, $\alpha[k]$, in OFDMA was obtained without making any approximations as opposed to the PMU-OFDM system.

4. SIMULATION RESULTS

In this section, we will compare the performance of the PMU-OFDM and the OFDMA systems via computer simulation. It is assumed that the channel coefficients at the same path but different time indices have Rayleigh distributions. The

time-varying fading channel is assumed to have the Clark Doppler spectrum and a C++ program was written to simulate the correlated complex Gaussian channel coefficients [4].

The CFO effect induced by Doppler shift is modeled by multiplying the received vector of the *j*th user by $J(\varepsilon_i)$, where

 $J(\varepsilon_j) \in \pounds^{NM \times NM}$ is a diagonal matrix with its *n*th diagonal element given by $e^{j\frac{2\pi}{NM}n\varepsilon_j}$. The CFO value of each user is randomly assigned to be either $+\varepsilon$ or $-\varepsilon$. The relation between the Doppler frequency and the CFO value is given by [6]

$$\varepsilon = NM \frac{f_d}{f_s} = NM \frac{f_c v}{f_s c} , \qquad (34)$$

where f_d is the Doppler frequency in Hz, f_s is the sampling frequency in Hz, v is the speed of the user in km/hr, c is the speed of light in km/hr and NM is the total number of subcarriers. Let us consider a practical example in wireless OFDM systems with the following parameters:

- Total number of subchannels NM = 256; (N = 32, M = 8);
- Total available bandwidth $f_s = 1$ MHz;
- Length of channel L = 2;
- CP time $T_{cp} = 1 \ \mu s$;
- Useful OFDM symbol time $T_{sym} = 256 \ \mu s$;
- Total available bandwidth $f_s = 1$ MHz;
- Symbol rate $f_{sym} = \frac{1}{T_{sym} + T_{cp}} = 3.89$ KHz;
- Carrier frequency $f_c = 1.0$ GHz;
- $\Delta f = 1/T_{sym} = 3.90$ KHz;

Thus, the maximum Doppler frequency for a mobile moving at a speed v = 100 km/hr is: $f_d = f_c \frac{v}{c} \approx 185$ Hz. According to (34), the corresponding CFO is $\varepsilon = 0.047$.



Fig.2. The BER comparison among PMU-OFDM and OFDMA over time-varying channel environment with CFO

In other words, for a normalized Doppler frequency $f_d / f_s = 1.85e-04$, the CFO is 0.047. We assume that users are equipped with the channel estimator at the receiver side and are able to compensate the self-CFO effect and channel distortions. That is, for user *j*, we multiply $b_j(n)$ by $e^{-j\frac{2\pi}{NM}n\varepsilon_j}$, (n = 0, 1, ..., NM - 1) before sending it to DFT. By this way, we only consider the CFO effect of other users. In the equalizer, we multiply the *k*th detected symbol by $\eta_j^{-1}[k]$ or $\alpha_j^{-1}[k]$ (with $\varepsilon_j = 0$) which are given in Eqs. (26) and (31) to compensate for the effects of time-varying channel distortions.

We compare the performance of OFDMA and PMU-OFDM systems below. To have a fair comparison, we keep the size of IDFT/DFT of these two systems the same, *i.e.* NM. We focus on the half-loaded case, where M/2 symmetric codewords of the M Hadamard Walsh codes are used for PMU-OFDM. For OFDMA, the *i*th user is assigned subchannels with indices 2(i-1)+kM, $0 \le k \le N-1$, $0 \le i \le M/2-1$. The residual subchannels are left as the guard band.



Fig.3. The BER comparison among PMU-OFDM and OFDMA over slowly time-variant channel with CFO

Fig.2. shows the bit error rate (BER) comparison between PMU-OFDM and OFDMA with normalized Doppler rates 2e-04 and 6e-04, which corresponding to the CFO levels of 0.0512 and 0.15, respectively. Note that the CFO values are obtained from f_d according to Eq. (34). These Doppler shift values approximately correspond to vehicle speeds of 100 km/hr and 300 km/hr, respectively. The channel coherence time can be obtained by $0.45/f_d$, which is approximately equal to $9T_{sym}$ for $f_d/f_s = 2e-04$ and $3T_{sym}$ for $f_d/f_s = 6e-04$. The Monte Carlo method was used to run more than 200,000 samples for every user. We see from Fig. 2 that the PMU-OFDM with code selection outperforms OFDMA when CFO is present.

However, when there is no CFO, OFDMA performs slightly better than PMU-OFDM. This case corresponds to a situation where the relative distance between the transceiver remains the same over one OFDM block length but the channel environment changes over one symbol time. As a result, the channel is time-variant but there is no CFO.

Next, we consider a slow time-varying channel with a normalized Doppler frequency 8e-05 and CFO set to 0.02. This value corresponds to a speed of about 43 km/hr. For this case, the channel time coherence is about $22T_{sym}$. The simulation result is shown in Fig. 3. When the Doppler frequency is 8e-05 and CFO=0, the BER of PMU-OFDM is less than 2e-04 for SNR=30 dB. However, when the channel changes faster along time, say, $f_d = 6e-04$ as the curves shown

in Fig. 2, BER degrades to about 4e-03 at the same SNR value. Again, we observe that PMU-OFDM performs better than OFDMA when there is a CFO. Furthermore, the BER curves of both systems are overlapping when there is no CFO. These results are consistent with derived theoretical results. That is, PMU-OFDM outperforms OFDMA in a very slow time-variant channel since PMU-OFDM can significantly reduce the MAI effect due to CFO [10].

5. CONCLUSION

We examined the performance of the PMU-OFDM transceiver in a Doppler shift environment and compared it with OFDMA. It was shown by simulation results that the PMU-OFDM transceiver can greatly reduce MAI due to the Doppler shift to a negligible amount via the use of M/2 symmetric (or anti-symmetric) codewords of the M Hadamard Walsh code and outperforms OFDMA in a slow time-varying environment with CFO.

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