A Novel Fast Codebook Search for MIMO Beamforming Systems

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***Abstract*—In this paper, we propose a fast algorithm to quickly search the best codeword in a given codebook for MIMO beamforming systems. The proposed algorithm can effectively reduce the computational complexity and feedback latency for various codebooks. Moreover, we analyze both the complexity and the performance for the proposed algorithm. From the complexity analysis, we found that around half of the computational complexity will be reduced in each iteration. From the performance analysis, we found that as the number of quantization bits increases, the analytical result is close to the simulation result. Simulation results are provided to evaluate the proposed algorithm in several beamforming scenarios.**

I. Introduction

Multiple-input and multiple-output (MIMO) techniques are widely used in current wireless communication systems to enhance the system performance. Among the MIMO techniques, beamforming/precoding and combining can achieve a full diversity gain. Hence, they are adopted in most wideband communication standards, *e.g.* IEEE 802.16x, IEEE 802.11n and LTE. Since transmitter may not be able to know full channel information, researches have been conducted to use beamforming with limited feedback, *e.g.* see [5] and [6]. Most of the beamforming techniques will construct a codebook in advance. Then, the system selects the best codeword and the corresponding codeword index according to channel condition, and sent the index back to the transmitter. Since wireless channels may vary in a short period, the computational time to select the best codeword should be kept reasonably small so that it will not lead to large feedback latency. In general we use the exhaustive search to determine the best codeword for codebook-based beamforming systems. For a codebook with large code size, however, using exhaustive search causes high computational complexity and long feedback latency.

Several algorithms were proposed to quickly search the best codeword in vector quantizers. For instance, the author in [1] proposed a search algorithm using the concept of the eigen space with simplified search criterion to find the codeword. Hierarchy-oriented search method in [2] used the training matrices and the concept of Voronoi region to speed up the search. However, these algorithms may not be used in MIMO beamforming systems directly since they applied the MSE criteria instead of the SNR criteria to find the best codeword. Hence, we are motivated to explore solutions dedicated for MIMO beamforming systems to reduce the search complexity and latency.

In this work, we propose a fast codeword searching algorithm for MIMO beamforming systems. The algorithm can effectively reduce the computational complexity and latency. Moreover, we analyze the performance and complexity of the proposed algorithm. Simulation results show that the proposed algorithm works well in various codebooks including the Grassmannian [5], the random vector quantization (RVQ) [3], and the Lloyd codebooks. The proposed algorithm also works well for non-codebook based beamforming system such as equal gain beamforming [7]. The simulation result show that in a 4T2R MIMO equal gain beamforming system, the performance loss is within 0.3 dB, and in a 4T1R MISO beamforming system with RVQ codebook, the performance loss is within 1 dB while the search complexity can be greatly reduced using the proposed algorithm.

The organization of this paper is as follows: In Sec. II, we introduce the background including the system model, the performance criteria, and the concepts of codebooks, beamforming and combining. We also propose a procedure to construct the codebook based on the Lloyd algorithm in this section. In Sec. III we propose a fast codebook search algorithm. Then we analyze the performance and the complexity in Sec. IV. The simulation results are given in Sec. V. Finally, conclusion is made in Sec. VI.

II. System Model and Codebook Construction

A MIMO system using beamforming and combining techniques with N_t transmitter antennas and N_r receiver antennas is shown in Fig. 1. The modulated signal s with symbol energy $E_s[|s|^2] = \mathcal{E}_t$ is first multiplied by a $N_t \times 1$ beamforming vector $\mathbf{w}$ and then spread in $N_r \times N_t$ channel $\mathbf{H}$ by transmission antennas. A $N_r \times 1$ noise vector $\mathbf{n}$ is added, and then the data from different branches is combined to form x using a $N_r \times 1$ combining vector $\mathbf{z}$. The symbol x to be detected can be represented by

$$x = \mathbf{z}^H \mathbf{H} \mathbf{w} s + \mathbf{z}^H \mathbf{n}. \tag{1}$$

We assume that elements of $\mathbf{H}$ have distribution in $CN(0, 1)$ and the i-th element of $\mathbf{n}$, n_i, is white Gaussian with power $E_s[|n_i|^2] = N_0$, $1 \leq i \leq N_r$. The norms of $\mathbf{w}$ and $\mathbf{z}$ are confined to be unit.

With the norm constraint of $\mathbf{w}$ and $\mathbf{z}$ and using (1), the SNR is given by

$$\gamma_r = \frac{\mathcal{E}_t |\mathbf{z}^H \mathbf{H} \mathbf{w}|^2}{N_0} = \frac{\mathcal{E}_t \Gamma_r}{N_0}, \tag{2}$$

where $\Gamma_r = |\mathbf{z}^H \mathbf{H} \mathbf{w}|^2$ is the precoding gain. Since both $\mathcal{E}_t$ and N_0 are constant, the receiver will choose the codeword which maximizes the precoding gain Γ_r.

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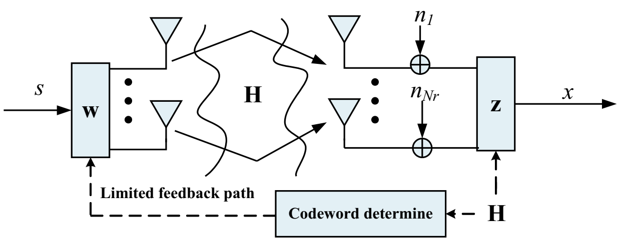

Fig. 1. A beamforming system with limited feedback.

Maximum ratio combining (MRC) is a general combining technique to maximize the SNR value when the channel information is known to the receiver. The MRC vector is given as $\mathbf{z} = \frac{\mathbf{Hw}}{\|\mathbf{Hw}\|_2}$. With the MRC vector $\mathbf{z}$, the precoding gain is

$$\Gamma_r = \| \mathbf{Hw} \|_2^2 . \tag{3}$$

As a result, the best codeword $\mathbf{w}$ is selected by the exhaustive search, *i.e.*

$$\mathbf{w} = arg \max_{\mathbf{v} \in \mathcal{F}} \| \mathbf{Hv} \|_2, \tag{4}$$

where $\mathcal{F}$ is a given codebook with size N, which relates to the number of quantization bits $B = \log_2 N$.

Various algorithms were proposed to construct the codebooks. Grassmannian line packing optimally packs one-dimensional subspace and then forms the Grassmannian codebook [5]. The RVQ codebook generates its elements simply using a white Gaussian source. Equal gain transmission (EGT), which can achieve the full diversity order, has equal power in all its elements. A simple method to construct EGT is via scalar quantization [7], where the phase of each element is quantized uniformly.

The Lloyd codebook can be generated by the Lloyd algorithm. Since we would like to maximize the SNR value, we group the training matrices by (4). Then, the expectation value of the precoding gain in (3) can be represented as

$$E_s[\| \mathbf{Hw} \|_2^2] = E_s[|\mathbf{w}^H \mathbf{H}^H \mathbf{Hw}] = \mathbf{w}^H E_s[\mathbf{H}^H \mathbf{H}]\mathbf{w}. \tag{5}$$

From (5), the best beamforming vector $\mathbf{w}$ is the right singular vector with the largest singular value of $E_s[\mathbf{H}^H \mathbf{H}]$. Define a distortion value d as $d = \frac{\Gamma_C - \Gamma_P}{\Gamma_P}$, where Γ_P and Γ_C are the precoding gains for the previous codebook $\mathcal{F}$ and the current (updated) codebook $\mathcal{F}'$ using the Lloyd algorithm respectively. Assume the codebook size is N. Given a set of training matrices, a Lloyd algorithm to construct the beamforming vector for MIMO systems is as follows.

Proposed Lloyd Algorithm for MIMO Systems

- **Step 0:** Give a predetermined distortion threshold d_s, which relates to d in (II). Generate N codewords as the initial codebook $\mathcal{F}$ arbitrarily.
- **Step 1:** Partition the training matrices into N clusters by (4) according to $\mathcal{F}$.
- **Step 2:** Find the new codewords $\mathbf{w}$ for each cluster by (5). Then, we have a new codebook $\mathcal{F}'$
- **Step 3:** Calculate the distortion value d by (II). Let $\mathcal{F}$=$\mathcal{F}'$. If $d > d_s$, repeat the whole process from Step 1. If $d < d_s$, the algorithm stops.

III. Proposed Fast Codebook Algorithm

The precoding gain in (3) can be further represented as

$$\begin{aligned} \Gamma_r &= \sum_{i=1}^{N_t} \| \mathbf{h_i} \|_2^2 \, |w_i|^2 + \sum_{j=1}^{N_t - 1} \sum_{k=j+1}^{N_t} \\ &\cdot 2 \, \| \mathbf{h_j} \|_2 \| \mathbf{h_k} \|_2 \, |w_j||w_k| \cos(\phi_{kj} + \theta_j - \theta_k), \end{aligned} \tag{6}$$

where $\mathbf{h_i}$ is the i-th column vector of $\mathbf{H}$, θ_i is the phase of w_i (the i-th element of $\mathbf{w}$), and ϕ_{kj} is defined as $e^{j\phi_{kj}} = \frac{\mathbf{h_k}^H \mathbf{h_j}}{\|\mathbf{h_k}\|_2 \|\mathbf{h_j}\|_2}$. From (6), we see the precoding gain decreases due to negative cosine values. Therefore, the proposed algorithm will prefilter the codewords that will result in negative cosine values. Since there are $\frac{N_t(N_t-1)}{2}$ cosine values in (6), we define the number of stages $\mathcal{S}$, where $\mathcal{S} \leq \frac{N_t(N_t-1)}{2}$, as the number of cosine values to be evaluated. Given an $\mathcal{S}$, we need to decide the order of the cosine values to be evaluated. From (6), we know that if $\| \mathbf{h_i} \|_2 > \| \mathbf{h_j} \|_2 > \| \mathbf{h_k} \|_2$, $\| \mathbf{h_i} \|_2 \| \mathbf{h_j} \|_2 \, |w_i||w_j|$ may have greater influence in precoding gain than $\| \mathbf{h_i} \|_2 \| \mathbf{h_k} \|_2 \, |w_i||w_k|$ does. Hence, the order of evaluation can be determined by using the following procedure.

The Procedure to Determine the Order of Evaluation:

- **Step 0:** Given initial values $k = 1$ and $n = 1$, a $1 \times \frac{N_t(N_t-1)}{2}$ sequence vector $\mathbf{p}$, a $1 \times N_t$ norm-value sequence $\mathbf{t}$, and one channel realization $\mathbf{H}$.
- **Step 1:** Calculate and sort the norms of the column vectors $\mathbf{h_i}$, and save the index of the norm value from the highest to the lowest in $\mathbf{t}$. For example, if $\| \mathbf{h_3} \|_2 > \| \mathbf{h_1} \|_2 > \| \mathbf{h_2} \|_2 > \| \mathbf{h_4} \|_2$, we save the sequence in $\mathbf{t}$ as $[\; 3 \quad 1 \quad 2 \quad 4 \;]$.
- **Step 2:** Set $m = k + 1$.
- **Step 3:** Let $\mathbf{p}(n)$=$(\mathbf{t}(k), \mathbf{t}(m))$, $n = n$+1 and $m = m$+1. Repeat Step 3 until $m > N_t$.
- **Step 4:** Let $k = k + 1$. Repeat Step 2 until $k = N_t$.

After constructing the order of evaluation, the proposed algorithm to eliminate the codewords in advance can be interpreted as follows.

Proposed Fast Algorithm for Codeword Search:

- **Step 0:** Give an $\mathcal{S}$, an initial value $k = 1$, a $1 \times \frac{N_t(N_t-1)}{2}$ vector $\mathbf{p}$ to save the evaluation sequence, and a candidate set $\mathcal{U}$ which is equivalent to the given codebook initially.
- **Step 1:** Calculating and evaluating the cosine value $\cos(\phi_{mn} + \theta_n - \theta_m)$ in (6) for all codewords in $\mathcal{U}$ corresponding to $\mathbf{p}(k) = (m, n)$. If the cosine value of the codeword is negative, drop the codeword out off $\mathcal{U}$.
- **Step 2:** Having $k = k + 1$. Repeat from Step 2 until $k > \mathcal{S}$.
- **Step 3:** The best codeword is obtained by exhaustively searching the residual codewords.

If all the resulting cosine values are negative after several eliminations, we keep the residual codewords in the previous stage and stop the program. Then, the best codeword is obtained by exhaustively searching the residual codewords.

IV. Complexity and Performance Analysis

As the phases of the channel coefficients are uniformly distributed in $(-\pi, \pi)$, given θ_i and θ_j, the probability density function of $(\phi_{ji} + \theta_i - \theta_j)$ is symmetric about zero. Thus, around half of the codewords in $\mathcal{U}$ will result in negative cosine value in each elimination. Therefore, about half of the codewords will be dropped out in each elimination. Consequently, the number of the residual codewords is around $\frac{N}{2^{\mathcal{S}}}$.

The complexity of the proposed algorithm can be divided by two parts: the evaluation of the cosine values and the exhaustive search. The evaluation of the cosine values requires $N_t(2N_r - 1) + \mathcal{S}(5N_r - 2) + 4N[1 - (\frac{1}{2})^{\mathcal{S}}]$ real-value additions, $2N_r(N_t + 2\mathcal{S})$ real-value multiplications, $\frac{N_t(N_t-1)}{2} - 1$ comparisons and $2N[1 - (\frac{1}{2})^{\mathcal{S}}]$ cosine value inspections. The residual $\frac{N}{2^{\mathcal{S}}}$ codewords require $\frac{N}{2^{\mathcal{S}}}(5N_tN_r - 1)$ real-value additions, $\frac{N}{2^{\mathcal{S}}}[2N_r(2N_t + 1)]$ real-value multiplications and $\frac{N}{2^{\mathcal{S}}} - 1$ comparisons in the exhaustive search.

For the SNR loss of the proposed algorithm, we define a parameter Γ_L to analyze the loss of precoding gain as follows.

$$\Gamma_L = \frac{\mathcal{P}_d\Gamma_{max} + (1 - \mathcal{P}_d)\Gamma_{max}R_L}{\Gamma_{max}} = \mathcal{P}_d + (1-\mathcal{P}_d)R_L, \quad (7)$$

where Γ_{max} is the maximum precoding gain in each channel realization corresponding to the best codeword, detection rate $\mathcal{P}_d$ is the probability that the best codeword is going to stay in $\mathcal{U}$ after eliminations, and R_L is the loss ratio defined as that $R_L\%$ of the best gain Γ_{max} will be achieved if we miss the best codeword.

From (7), it is obvious that the performance loss depends on the detection rate $\mathcal{P}_d$ and the loss ratio R_L. Next, let us focus on how to obtain $\mathcal{P}_d$ and R_L. To find $\mathcal{P}_d$, let us define a probability value $\mathcal{P}_i$ as

$$\mathcal{P}_i = \frac{\mathcal{P}[\cos(\phi_{jk} + \theta_k - \theta_j) > 0 | \mathbf{H} \in \mathbf{w_i}]}{\mathcal{P}[\mathbf{H} \in \mathbf{w_i}]}. \quad (8)$$

$\mathcal{P}_i$ is the probability that the elected cosine value is positive when we select $\mathbf{w_i}$ as our beamforming vector by the exhaustive search. Assume the probability that $\mathbf{H} \in \mathbf{w_i}$ is equal to $\frac{1}{N}$ for individual codewords. After calculating all the $\mathcal{P}_i$ for all codewords, the detection rate can be approximated as follows:

$$\mathcal{P}_d \approx \frac{1}{N}\sum_{i=1}^{N}\mathcal{P}_i. \quad (9)$$

Before deriving the lower bound for the loss ratio in (7), we noticed that the codeword with the second maximum precoding gain has great chance to stay in the residual codewords when the best codeword is eliminated by the proposed algorithm. Let us take a 4T1R system using RVQ codebook with $N = 1024$ and $\mathcal{S} = 1$ for instance. Fig. 2 shows that when the best codeword is filtered out, the probability to select the codeword with the second maximum Γ_r is apparently much higher than others.

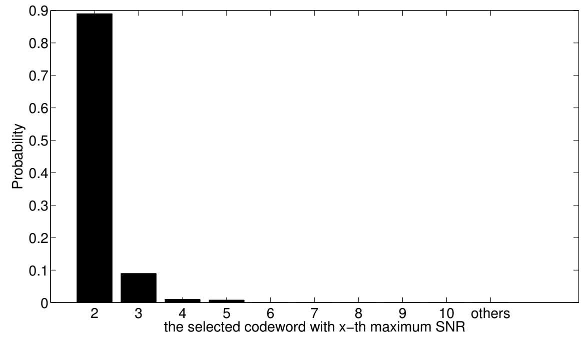

Fig. 2. The probability that the selected codeword is with the x-th maximum SNR when the best codeword is filtered out (4T1R MISO system using RVQ codebok with $N = 1024$ for $\mathcal{S} = 1$).

When the best beamforming vector $\mathbf{w_o}$ of a given codebook is applied, the precoding gain in (3) has the following inequality

$$v_1^2|\mathbf{d_1}^H\mathbf{w_o}|^2 \leq \sum_{i=1}^{k} v_i^2|\mathbf{d_i}^H\mathbf{w_o}|^2 = \|\mathbf{H}\mathbf{w_o}\|_2^2, \quad (10)$$

where k is the rank of $\mathbf{H}$, and $\mathbf{d_i}$ is the right singular vector corresponding to the i-th largest singular value v_i of $\mathbf{H}$. As we mentioned earlier that the algorithm has great chance to keep the second-best codeword $\mathbf{w_s}$ when filtering out the best codeword, the greatest loss occurs when $\mathbf{d_1} = \mathbf{w_o}$ but we take $\mathbf{w_s}$ as the beamforming vector. Therefore, (10) becomes

$$v_1^2|\mathbf{w_o}^H\mathbf{w_s}|^2 \leq \|\mathbf{H}\mathbf{w_o}\|_2^2 . \quad (11)$$

Assume the probability that $\mathbf{H} \in \mathbf{w_i}$ is equal to $\frac{1}{N}$. (11) suggests that we can build a lower bound by defining the loss ratio as the average maximum correlation between a target codeword $\mathbf{w_i}$ and the rest codewords $\mathbf{w_j}$, *i.e.*

$$R_L \geq \frac{1}{N}\sum_{i=1}^{N}\left(\max_{j\neq i, 1\leq j\leq N}|\mathbf{w}_j^H\mathbf{w}_i|\right). \quad (12)$$

Fig. 3 shows the simulation and the analytical SNR loss for a 6T1R MIMO system using the RVQ codebook. When $B > 4$, the derived lower bound is close to the real SNR loss.

V. Simulation Results

BPSK modulation and Rayleigh channels are used. All the codebooks used here can be found in [8] and [9].

Example 1: Various numbers of stages. Fig. 4 shows the simulation results for a 4T1R MISO system using the RVQ codebook [3] with $B = 6$ for various numbers of stages. We see that the performance loss increases with the stage number $\mathcal{S}$, since the larger the stage number is, the higher the probability that we eliminate the best codeword during the elimination process. For $\mathcal{S} = 1$, the performance loss is minor. Even for $\mathcal{S} = 5$, where only two residual codewords are left in general, the performance loss is within 1 dB.

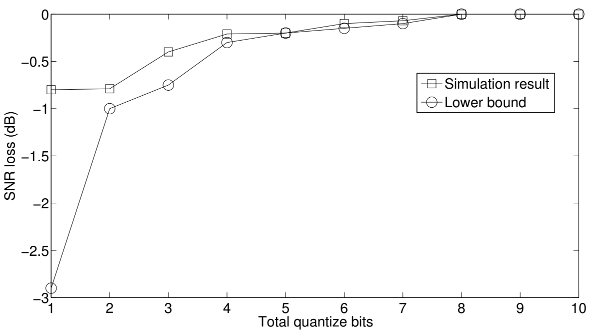

Fig. 3. SNR loss for a 6T2R MIMO system using the RVQ codebook with $N = 1024$ for $\mathcal{S} = 1$.

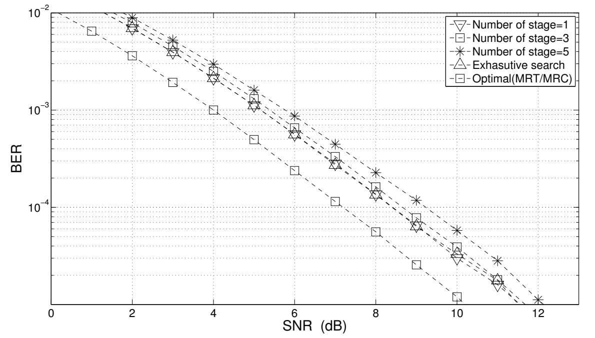

Fig. 4. BER comparison in a 4T1R MISO system using the RVQ codebook with $B = 6$ for various numbers of stages $\mathcal{S}$.

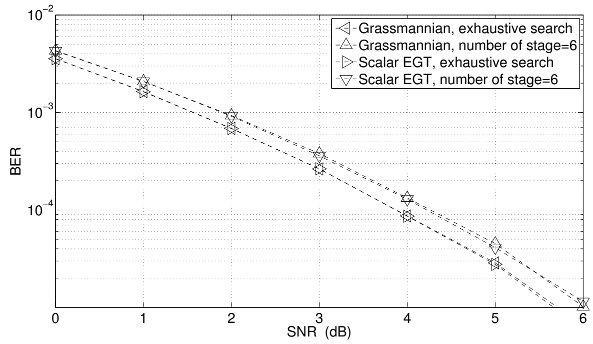

Fig. 5. BER comparison in a 4T2R system using EGT quantization with $B = 6$.

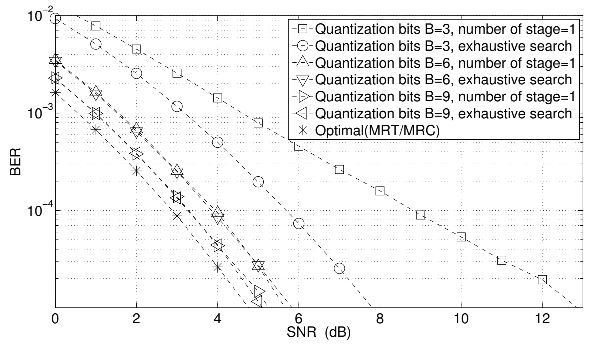

Fig. 6. BER comparison in a 4T2R MIMO system using the RVQ codebook with $\mathcal{S} = 1$ for various codebook sizes.

Example 2: EGT codebooks. For $B = 6$ and $\mathcal{S} = 6$, Fig. 5 shows the BER performance for 4T2R systems using the EGT and the vector quantization respectively. It is worthy to emphasize that the 6-bit Grassmannian codebook in [8] is an EGT codebook. Since the proposed algorithm uses phase information to eliminate codewords and the beamforming codewords of EGT contain only phase information, we expect that the SNR loss for a well designed EGT codebooks is smaller than the vector quantization codebook.

Example 3: Various codebook sizes. Fig. 6 shows the BER in a 4T2R MIMO system using the RVQ codebook with $\mathcal{S} = 1$ for various codebook sizes. As the code size increases, the performance gap of the proposed algorithm and the exhaustive search decreases. The reason is as follows. As mentioned in Section IV, if the best codeword is eliminated during iterations, the probability that the second-best codeword is still kept is high. With large code size, the resulting precoding gain between the best and the second-best codewords is close. Thus, we expect the SNR loss is small for large codebook size.

VI. Conclusion

We proposed a fast codebook search algorithm to reduce the computational complexity and feedback latency for MIMO beamforming systems. Using the proposed algorithm, the resulting codewords to perform the exhaustive search is greatly reduced. The simulation results showed that the proposed algorithm in general achieved a good trade-off between the complexity and performance.

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