A Bit Allocation Scheme for MIMO Equal Gain Precoders

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***Abstract*—We consider the design of the equal gain precoder in flat-fading and limited feedback multiple-input multiple-output (MIMO) systems. The advantage of the equal gain precoder based on the scalar quantization (SQ) is that it is easier to encode the feedback information and no need to construct codebook in advance. In the sort of precoder, the bit allocation methods will dominate the system performance. In this paper, we explore an efficient bit allocation algorithm for the scalar quantized equal gain precoder. The proposed bit allocation scheme can improve the performance of the straightforward scalar quantized equal gain precoder. Compared to the optimal bit allocation scheme with exhaustive search, the proposed algorithm greatly reduces the computational complexity while the performance is near to the optimal scheme.**

Index Terms—MIMO, scalar quantization (SQ), equal gain precoder, limited feedback.

I. Introduction

Multiple-input multiple-output (MIMO) techniques can significantly improve the system performance especially enabling high spectral efficiency and resilience to fading and, therefore, are widely used in current wireless communication standards such as IEEE 802.11n, IEEE 802.16e and 3GPP Long Term Evolution (LTE). Among MIMO techniques, the closed-loop MIMO can provide extra precoding gain compared to the open-loop MIMO because of the provision of the channel state information (CSI) at the transmitter (CSIT). While the full channel information is known to the transmitter, we can jointly design precoders and decoders by optimizing various parameters [8], [9]. These solutions can achieve theoretically optimal performance with different design criteria. However, the huge amount of feedback information will cause the infeasibility in most wireless systems. To overcome the shortcomings of the optimal precoders, the research related to the precoding schemes with limited feedback was introduced. The basic idea is to utilize quantization techniques for the CSI before feedback. The quantization techniques can be categorized to vector quantization (VQ) and scalar quantization (SQ). VQ techniques rely on the predefined codebook maintained at the transmitter and the receiver and employ the encoding function to select the most suitable codeword. SQ techniques quantize the precoding elements independently. In [3], the algorithm to construct the full-diversity codebook was proposed and the quantized equal gain transmission (QEGT) with various combining methods were shown to achieve full diversity order. In [4], a Grassmannian beamforming scheme was proposed. The codebook construction problem was mapped to the Grassmannian line packing problem and the associated encoding function was defined as well. Some constructed codebooks can be found in [5]. In [6], the authors analyzed the capacity loss of equal gain precoder due to both vector and scalar quantization. The performance gap between the optimal precoder and the equal gain precoder and the performance degradation of the equal gain precoder due to scalar quantization were analyzed in [2]. SQ schemes provide the simpler alternatives to VQ schemes, but for small values of feedback bit SQ may perform poorly. SQ schemes can uniquely employ the bit allocation methods to improve the performance. They will dominate the complexity and performance of SQ schemes. In [7], the authors proposed the exhaustive search and the uniform bit allocation methods for the scalar quantized equal gain precoder. The former method searches all possible bit allocations and decides the one with the best gain effect. This method can achieve the optimal performance but encounter huge computational complexity. The latter method uniformly allocates bits to the equal gain precoder. This method can be executed quickly but have to face considerable performance degradation.

In this paper, a simple but efficient bit allocation algorithm for the equal gain precoder is proposed to overcome the previous drawbacks. Assuming B is the total feedback bits, the proposed bit allocation algorithm shows the following advantages: 1). Compared to the exhaustive search algorithm, it can remarkably reduce the computational complexity from the exponential order to the linear order in B. Also, its performance can quite approximate to that of the exhaustive search algorithm with moderate B. 2). It can largely outperform the uniform algorithm at the cost of small additional complexity. 3). From simulation results, assuming the bit allocation table is available in the transmitter we found the proposed scheme can achieve comparable performance with the Grassmannian precoder in the same B. In fact, this assumption will make extra protocol efforts.

Notation: Bold upper and lower case letters denote matrices

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and vectors, respectively. We use $(\cdot)^T$ to denote the transpose, $(\cdot)^*$ to denote the conjugate and $(\cdot)^H$ to denote the conjugate transpose. $\mathcal{C}$ is the complex set and $\mathcal{C}^{M\times N}$ is the complex-valued matrix. $\lfloor\cdot\rfloor$ denotes the floor operation and $\lceil\cdot\rfloor$ denotes the round operation.

II. System Model

Fig.1 presents the block diagram of a limited feedback precoded MIMO system where the vectors **w** and **z** are called the precoder and postcoder respectively. The system considered has N_t transmit and N_r receive antennas. At first, the modulated symbol $x \in \mathcal{C}$ is multiplied by the precoding vector $\mathbf{w} \in \mathcal{C}^{N_t\times 1}$. For equal gain precoding, the precoder can be expressed as

$$\mathbf{w} = \frac{1}{\sqrt{N_t}}\left[\mathrm{e}^{j\theta_1}\ \mathrm{e}^{j\theta_2}\ ...\ \mathrm{e}^{j\theta_{N_t}}\right]^T, \tag{1}$$

where $\frac{1}{\sqrt{N_t}}$ is used to normalize the transmit power so that the total power will be kept the same for different N_t. After

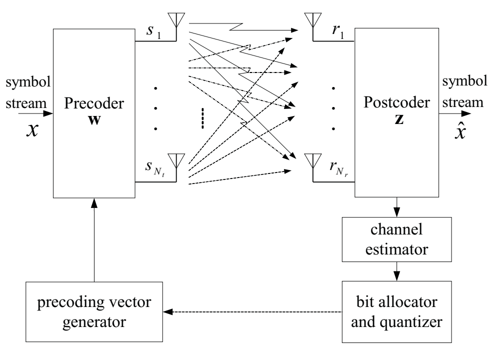

Fig. 1. Block diagram of a limited feedback precoded MIMO system.

the precoding, the symbol vector, $\mathbf{s} = [s_1\ s_2\ ...\ s_{N_t}]^T$, to be transmitted is given by

$$\mathbf{s} = \mathbf{w}x \tag{2}$$

Then, **s** is transmitted to the MIMO channel.

At the receiver, assuming that there are N_r receive antennas, the receive vector $\mathbf{r} = [r_1\ r_2\ ...\ r_{N_r}]^T$ from the MIMO channel is given by

$$\mathbf{r} = \mathbf{Hs} + \mathbf{n} = \mathbf{Hw}x + \mathbf{n}, \tag{3}$$

where $\mathbf{H} \in \mathcal{C}^{N_r\times N_t}$ is a channel matrix with its (i,j)th element equal to h_{ij}, and $\mathbf{n} \in \mathcal{C}^{N_r\times 1}$ is a noise vector with its elements assumed to be complex white Gaussian with variance σ_n^2. **r** is multiplied by a postcoding vector $\mathbf{z} \in \mathcal{C}^{1\times N_r}$ to form $\hat{x}$, *i.e.*

$$\hat{x} = \mathbf{zr} = \mathbf{zHw}x + \mathbf{zn} \tag{4}$$

To achieve the best performance, the MRC technique [3] is applied in the receiver, let the postcoder be $\mathbf{z} = (\mathbf{Hw})^H$. Thus

$$\hat{x} = \gamma x + \mathbf{zn} \tag{5}$$

where $\gamma = \mathbf{w}^H\mathbf{H}^H\mathbf{Hw}$ is the gain effect (including diversity gain and array gain) due to the space-diversity and the precoding.

The channel is assumed to be quasi-static in time. the CSI **H** is assumed to be known perfectly at the receiver, and partially at the transmitter through a limited feedback channel. In addition, we assume that the feedback channel has no delay and is error-free while conveying quantization information to the transmitter.

III. Scalar Quantization and Bit Allocation

Scalar quantization maps the continuous phase angles into the closest discrete values. In this correspondence, we assume the space of quantization levels are equal. That is, if there are eight quantization levels, the available values will be 0, $\pm\pi/4$, $\pm\pi/2$, $\pm 3\pi/4$ and π. From (1), through simple manipulations we obtain $\mathbf{w} = \frac{1}{\sqrt{N_t}}\mathrm{e}^{j\theta_1}\left[1\ \mathrm{e}^{j\tilde{\theta}_2}\ ...\ \mathrm{e}^{j\tilde{\theta}_{N_t}}\right]^T$ where $\tilde{\theta}_i = \theta_i - \theta_1, \tilde{\theta}_i \in [0, 2\pi), i = 2, ...N_t$. Denote

$$\tilde{\mathbf{w}} = \frac{1}{\sqrt{N_t}}\left[1\ \mathrm{e}^{j\tilde{\theta}_2}\ ...\ \mathrm{e}^{j\tilde{\theta}_{N_t}}\right]^T \tag{6}$$

Since $\left|\mathbf{w}^H\mathbf{H}^H\mathbf{Hw}\right|^2 = \left|\tilde{\mathbf{w}}^H\mathbf{H}^H\mathbf{H}\tilde{\mathbf{w}}\right|^2$ [7], we can only quantize $\left(\tilde{\theta}_2, ..., \tilde{\theta}_{N_t}\right)$ instead of $(\theta_1, ..., \theta_{N_t})$ and thus the total feedback information can be reduced. Let the quantized precoder be

$$\hat{\mathbf{w}} = \frac{1}{\sqrt{N_t}}\left[1\ \mathrm{e}^{j\hat{\theta}_2} ...\ \mathrm{e}^{j\hat{\theta}_{N_t}}\right]^T \tag{7}$$

where $\hat{\theta}_i = (2\pi n_i)/N_i, 0 \leqslant n_i \leqslant N_i - 1, i = 2, ..., N_t$, with $N_i = 2^{b_i}$ and n_i denoting the number of quantization levels and feedback index of $\tilde{\theta}_i$, respectively, and b_i is the number of quantization bits for $\tilde{\theta}_i$. For MISO channels, the optimal equal gain precoders have closed-form solutions, which can be derived from the channel vector by reversing the phase bias of each vector element. Moreover, for MIMO channels, the solution for the equal gain precoder can also be obtained through the cyclic method [7]. Then, we quantize the parameters θ_i of the optimal equal gain precoder to the closest available values $\hat{\theta}_i$. At last, we need to send the index set $(n_2, n_3, ..., n_{N_t})$ from the receiver to the transmitter, which totally requires $B = \sum_{i=2}^{N_t} b_i$ feedback bits.

Using bit allocation in θ_i can further improve system performance. Bit allocation is a problem to decide the quantization bit set $\{b_i\}, i = 2, ..., N_t$ for $\hat{\theta}_i$. Prior work on bit allocation schemes [7] are listed below:

1. Exhaustive search bit allocation: This idea searches all the possible bit allocation set to find the optimal sets $\left\{b_i^{opt}\right\}_{i=2}^{N_t}$ that maximizes $\left|\hat{\mathbf{w}}^H\mathbf{H}^H\mathbf{H}\hat{\mathbf{w}}\right|^2$. This method can achieve the best performance. For a given bit budget B, it needs to search total $\begin{pmatrix} B + N_t - 2 \\ B \end{pmatrix} = ((B + N_t - 2)!) / ((B)! (N_t - 2)!)$ possible sets [1] for each channel realization. To clearly see the complexity, we find the lower and upper bounds of the search iterations. The search iterations can be lower bounded by

$$\begin{aligned}\begin{pmatrix} B+N_t-2 \\ B \end{pmatrix} &= \left(\frac{B+N_t-2}{B}\right)\cdots\left(\frac{N_t-1}{1}\right) \\ &\geqslant \left(\frac{B+N_t-2}{B}\right)^{B}\end{aligned} \tag{8}$$

By the inequality $k! \geqslant (k/e)^k$ derived from the Stirling's approximation, the search iterations can be upper bounded by

$$\begin{aligned}\begin{pmatrix} B+N_t-2 \\ B \end{pmatrix} &= \frac{(B+N_t-2)(B+N_t-3)\ldots(N_t-1)}{B(B-1)\ldots 1} \\ &\leqslant \frac{(B+N_t-2)^B}{(B)!} \leqslant \left(\frac{e(B+N_t-2)}{B}\right)^{B}\end{aligned} \tag{9}$$

Therefore, the time complexity is with $\mathcal{O}\left(\left(\frac{e(B+N_t-2)}{B}\right)^{B}\right)$. As the number of transmit antennas N_t and the total feedback bits B grow, the tremendous computational complexity makes the method of exhaustive search somewhat impractical. For example, when $N_t = 8$ and $B = 14$, for each channel realization this approach requires 38760 iterations to compute the channel gains and then decide the most suitable bit allocation.

2. Uniform bit allocation: This is a suboptimal method that makes b_i approximately equal. For example, let $\bar{b} = \lfloor (B)/(N_t-1) \rfloor$, $\tilde{b} = \bar{b} + 1$ and $\bar{N} = B - \bar{b}(N_t-1)$. Starting from the second antenna, we can assign $\{b_i\}_{i=2}^{\bar{N}+1} = \tilde{b}$ for the first $\bar{N}$ parameters $\left\{\tilde{\theta}_i\right\}_{i=2}^{\bar{N}+1}$ and $\{b_i\}_{i=\bar{N}+2}^{N_t} = \bar{b}$ bits for the remaining $(N_t-1)-\bar{N}$ parameters $\left\{\tilde{\theta}_i\right\}_{i=\bar{N}+2}^{N_t}$.

IV. Proposed Algorithm

The criterion of the proposed bit allocation algorithm is to maximize the average receive SNR with acceptable complexity. Considering a MISO system and assuming $\mathbf{h}$ is a $N_t \times 1$ channel vector and its ith coefficient is h_i and $\hat{\theta}_j$ is the quantized phase, the average receive SNR can be expressed as [2]

$$\mathbb{E}_{\mathbf{h}}\{\tilde{\rho}_e\} = \frac{\sigma_x^2}{\sigma_n^2}\begin{pmatrix} \mathbb{E}_{\mathbf{h}}\left\{|h_i|^2\right\} + \\ c_1\mathbb{E}_{\mathbf{h}}\left\{\Re\left\{h_1^* h_j e^{j\hat{\theta}_j}\right\}\right\} + \\ c_2\mathbb{E}_{\mathbf{h}}\left\{\Re\left\{h_i^* h_j e^{j\left(\hat{\theta}_j - \hat{\theta}_i\right)}\right\}\right\} \end{pmatrix} \tag{10}$$

where $c_1 = 2(N_t-1)/N_t$ and $c_2 = (N_t-1)(N_t-2)/N_t$. Let $\varepsilon_i = \hat{\theta}_i - \tilde{\theta}_i$ and then we have $\hat{\theta}_i = \tilde{\theta}_i + \varepsilon_i$. (10) can be rewritten as

$$\mathbb{E}_{\mathbf{h}}\{\tilde{\rho}_e\} = \frac{\sigma_x^2}{\sigma_n^2}\begin{pmatrix} \mathbb{E}_{\mathbf{h}}\left\{|h_i|^2\right\} + \\ c_1\mathbb{E}_{\mathbf{h}}\{|h_1^* h_j|\}\,\mathbb{E}_{\mathbf{h}}\{\cos(\varepsilon_j)\} + \\ c_2\mathbb{E}_{\mathbf{h}}\{|h_i^* h_j|\}\,\mathbb{E}_{\mathbf{h}}\{\cos(\varepsilon_j - \varepsilon_i)\} \end{pmatrix} \tag{11}$$

From (11) we observe that if we can minimize the differences of the quantization errors of all transmit antenna pairs, then the average SNR can be maximized. In addition, we make further observations that ε_1 is zero and when a bit is allocated to the antenna with largest quantization error, the quantization error of this antenna has high probability to be greatly reduced after the assignment and so do the differences of ε_1 and ε_i, $i = 2, ..., N_t$. Let the total feedback bits to represent $\hat{\mathbf{w}}$ be B and define $B = \sum_{i=2}^{N_t} b_i$, where b_i is the number of quantization bits of the ith transmit antenna and b_1 is set to 0 to reduce the total feedback information. A bit allocation algorithm based on the maximum-quantization-error-first criterion is proposed as follows:

Algorithm Bit_Allocation
Input: N_t (the number of transmit antennas), B (a total feedback budget) and $\tilde{\mathbf{w}}$ (an unquantized equal gain precoder in (6))
Output: $\hat{\Theta} = \left[0\ \hat{\theta}_2\ \ldots\ \hat{\theta}_{N_t}\right]^T$ (the quantized phase vector)
$j \leftarrow 0$; $\hat{\Theta} \leftarrow [0\ 0\ \ldots\ 0]^T$; $b_i \leftarrow 0$, for $2 \leqslant i \leqslant N_t$;
$\varepsilon_i \leftarrow \hat{\theta}_i - \tilde{\theta}_i$, for $2 \leqslant i \leqslant N_t$;
while $j < B$ **do**
  $c \leftarrow \arg\max_i(|\varepsilon_i|)$;
  $b_c \leftarrow b_c + 1$;
  $\Delta \leftarrow 2\pi/2^{b_c}$;
  $\tilde{n}_c \leftarrow \tilde{\theta}_c/\Delta$;
  $\varepsilon_c \leftarrow (\lceil \tilde{n}_c \rfloor - \tilde{n}_c)\cdot\Delta$;
  $j \leftarrow j + 1$;
end
$\Delta_i \leftarrow 2\pi/2^{b_i}$, for $2 \leqslant i \leqslant N_t$;
$n_i \leftarrow \tilde{\theta}_i/\Delta_i$, for $2 \leqslant i \leqslant N_t$;
$\hat{\theta}_i \leftarrow \lceil n_i \rfloor \cdot \Delta_i$, for $2 \leqslant i \leqslant N_t$;
return $\hat{\Theta}$;

The proposed algorithm can be completed within at most B iterations. Hence the computational complexity is with $\mathcal{O}(B)$. Fig.2 shows the complexity comparison based on the factors, B and N_t, for the exhaustive search and the proposed method. As B and N_t increase, the complexity of the exhaustive search algorithm eventually becomes unacceptable while that of the proposed algorithm does not change too much.

Example 1: Proposed bit allocation algorithm. Consider $N_t = 4$, $N_r = 2$, $B = 6$ and the channel matrix H is

$$\mathbf{H} = \begin{bmatrix} 0.6926 + 0.6930i & -0.1878 - 0.8427i \\ -0.3878 - 0.3097i & -0.0681 + 0.9662i \\ -0.9763 - 0.6171i & -0.5150 + 0.0632i \\ 1.3336 + 0.1751i & -2.0799 + 0.2878i \end{bmatrix}^T$$

Assume $\hat{\mathbf{w}} = e^{j\hat{\Theta}}/\sqrt{N_t}$ and $\hat{\Theta}^{(i)} = \left[0\ \hat{\theta}_2\ \hat{\theta}_3\ \hat{\theta}_4\right]^T$ presents the quantized phase vector at the ith iteration. Using the proposed algorithm, we can obtain $\hat{\Theta}^{(1)} = 2\pi[0\ 0\ 0\ 0]^T, \hat{\Theta}^{(2)} = 2\pi\left[0\ 0\ {}^1\!/_2\ 0\right]^T$,

$\hat{\Theta}^{(3)} = 2\pi\left[0\ {}^1\!/_2\ {}^1\!/_2\ 0\right]^T, \hat{\Theta}^{(4)} = 2\pi\left[0\ {}^1\!/_2\ {}^1\!/_2\ {}^1\!/_4\right]^T$,

$\hat{\Theta}^{(5)} = 2\pi\left[0\ {}^1\!/_2\ {}^1\!/_2\ {}^2\!/_8\right]^T, \hat{\Theta}^{(6)} = 2\pi\left[0\ {}^1\!/_2\ {}^1\!/_2\ {}^3\!/_{16}\right]^T$

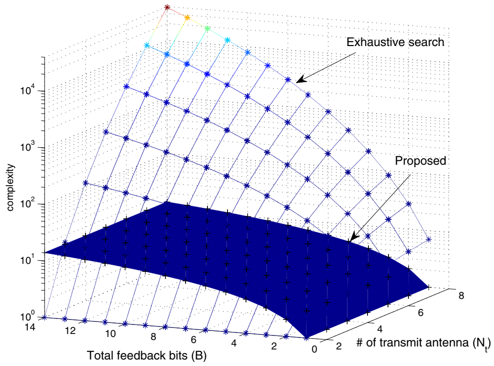

Fig. 2. Bit allocation complexity between the exhaustive search and the proposed methods.

Since $B = 6$, the solution is $\hat{\Theta}^{(6)}$.

V. Simulation Results

In the simulations below, we assume the elements of channel matrix $\mathbf{H}$ are independent and identically distributed (i.i.d.) complex Gaussian with zero mean and unit variance. The default modulation is BPSK except those especially specified.

Example 2: Comparison of optimal, uniform and proposed bit allocation algorithms. We compare the performance among the optimal, the uniform and the proposed algorithms used in the equal gain precoder. The modulation order is 16-QAM. Fig. 3 shows the bit error probability (BEP) comparison of these three bit allocation algorithms. In a 4T1R (4-transmit and 1-receive) channel environment, the proposed algorithm outperforms the uniform bit allocation by 1.1 dB, 1 dB and 0.8 dB for $B = 3, 4$ and 5 respectively. Similarly, in a 4T2R channel environment, the proposed algorithm outperforms the uniform bit allocation by 0.9 dB, 0.9 dB and 0.7 dB for $B = 3, 4$ and 5 respectively. Moreover, when a moderate B is given, for example $B = 5$, we observe that the proposed algorithm can achieve nearly the same performance with the optimal bit allocation. Also, the computational complexity can be reduced from 21 iterations to 5 iterations per channel realization.

Example 3: Comparison of various precoders. We compare the performance of the following precoders: antenna selection (AS) precoder, Grassmannian (GS) precoder [4], equal gain (EG) precoder with the proposed bit allocation algorithm, and the optimal precoder. We assume extra protocol is designed so that the transmitter knows the variable allocated bits for the precoding value of each antenna. Fig. 4 shows that the performance comparison in 4T1R and 4T2R channel environments. Please note that the Grassmannian precoding scheme belongs to VQ techniques, which are not capable of employing bit allocation schemes for extra performance gain. Therefore, we find that the EG precoder with the proposed bit allocation algorithm ($B = 4$ and 6) slightly outperforms the Grassmannian precoder by around 0.05 dB in 4T1R and around 0.05 dB in 4T2R. Moreover, when $B = 6$, it

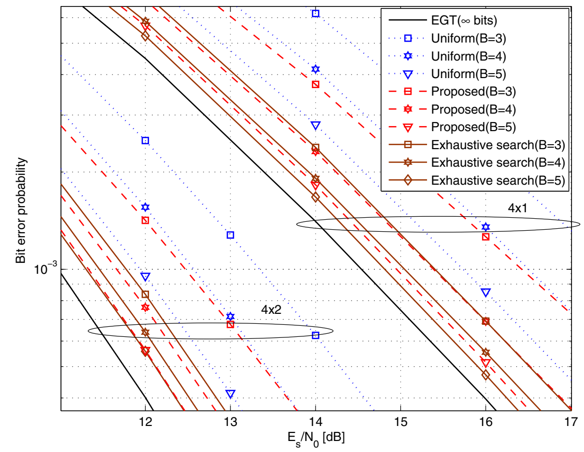

Fig. 3. BEP performance for the equal gain precoders with the optimal, the uniform and the proposed algorithms.

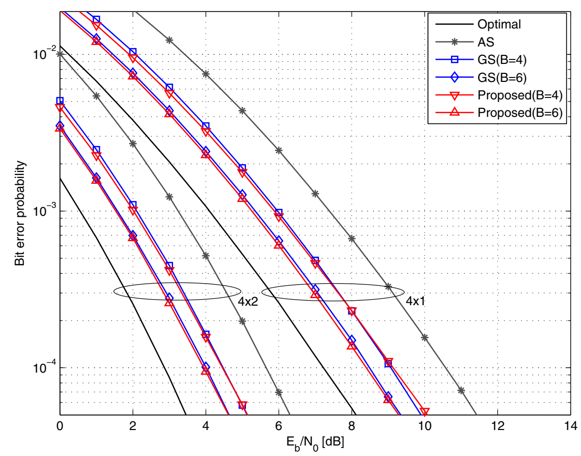

Fig. 4. BEP performance for different precoders in 4T1R and 4T2R channel environments.

outperforms the antenna selection precoder by around 2 dB in a 4T1R environment and 1.8 dB in a 4T2R environment. Compared with the optimal precoder without quantization effect, it degrades 1 dB in both 4T1R and 4T2R cases. Fig. 5 shows the performance comparison from the capacity viewpoint in a 4T1R channel environment. Similar to the BEP results, it outperforms the antenna selection and provides comparable performance as the Grassmannian precoder for $B = 4$ and 6. By increasing the receiver antenna to three, Fig. 6 shows the performance comparison in a 4T3R channel. We again observe that with the same feedback bits, it provides comparable performance with the Grassmannian precoder.

From Figs. 4, 5 and 6, the implementation superiority of the proposed bit allocation algorithm can be explained as follows: Let us consider the computational complexity to generate the feedback information. In a MISO channel, the closed-form optimal precoder can be obtained directly from the channel.

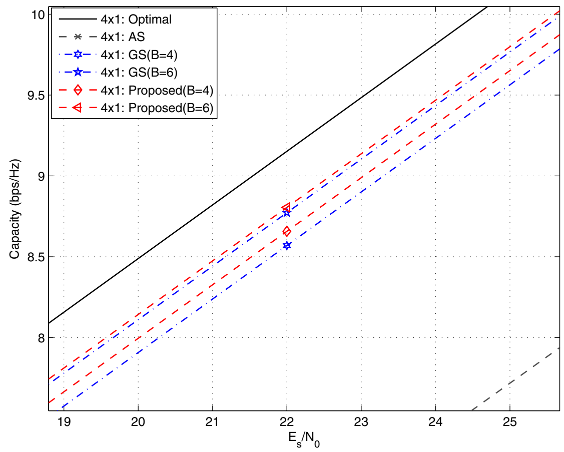

Fig. 5. Capacity performance for different precoders in a 4T1R channel environment.

Although the EG precoder with the proposed bit allocation algorithm and Grassmannian precoder have nearly the same performance in the same B, the former needs only B iterations while the latter needs 2^B iterations to determine the feedback information. Therefore, when B increases, the iteration number grows linearly for the proposed EG precoder while that grows exponentially for the Grassmannian precoder. For example, letting $B = 6$ and $N_t = 4$, the proposed EG precoder needs 6 iterations while the Grassmannian precoder needs 64 iterations (64-codeword codebook is used). Moreover, as shown in [2], the proposed EG precoder also outperforms the Grassmannian precoder in the codebook construction part.

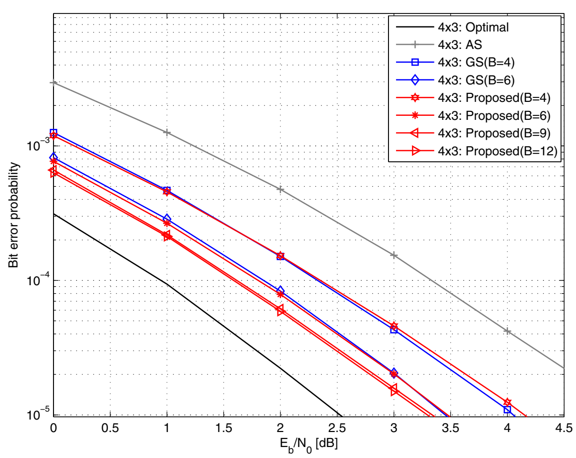

Fig. 6. BEP performance for equal gain, Grassmannian and the optimal precoders in a 4T3R channel environment.

Example 4: Proposed algorithm with N_t increased. We make the observation while letting N_t increase from 4 to 8. Let the modulation order be 16-QAM. Fig. 7 shows that when N_t and B increase, the performance gap between EG precoders with the exhaustive search algorithm and uniform algorithm becomes larger, but that between EG precoders with the exhaustive search algorithm and proposed algorithm maintains nearly constant. Take $B = 5$ in a 4T2R channel and $B = 11$ in a 8T2R channel for instance, the fixed gap between the exhaustive search algorithm and proposed algorithm is about 0.08 dB while the performance gap between the exhaustive search algorithm and uniform algorithm increases from 0.6 dB to 1.2 dB.

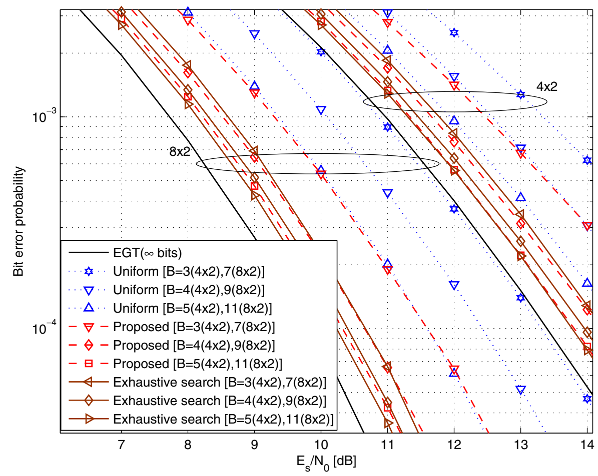

Fig. 7. BEP performance for the equal gain precoders with the optimal, the uniform and the proposed algorithms in 4T2R and 8T2R channel environments.

VI. Conclusion

In this paper, we proposed an efficient bit allocation method for the scalar quantized equal gain precoder without the need of codebook construction and exhaustive search. Compared to the prior work, the proposed equal gain precoder achieves large performance improvement with affordable implementation complexity.

References

[1] G. Casella and R. L. Berger, "*Statistical Inference.*" Pacific Grove, CA: Duxbury, 2001.
[2] Shang-Ho Tsai, "Transmit Equal Gain Precoding in Rayleigh Fading Channels," *IEEE Trans. Signal Processing*, Sep 2009.
[3] D. J. Love and R. W. Heath Jr., "Equal gain transmission in multiple-input multiple-output wireless systems," *IEEE Trans. Commun.*, vol. 51, pp. 1102-1110, July 2003.
[4] D. J. Love, R. W. Heath, Jr. and T. Strohmer, "Grassmannian Beamforming for Multiple-Input Multiple-Output Wireless Systems," *IEEE Trans. Info. Theory*, vol. 49, Oct. 2003, pp. 2735-45.
[5] http://cobweb.ecn.purdue.edu/∼djlove/grass
[6] C. R. Murthy and B. D. Rao, "Quantization Methods for Equal Gain Transmission with Finite Rate Feedback," *IEEE Trans. Signal Processing*, vol. 55, no. 1, pp. 233-245, Jan 2007.
[7] X. Zheng, Y. Xie, J. Li and P. Stoica, "MIMO transmit beamforming under uniform elemental power constraint," *IEEE Trans. Signal Processing*, vol. 55, pp.5395-5406, Nov. 2007.
[8] H. Sampath and P. Stoica and A. Paulraj, "Generalized linear precoder and decoder design for MIMO channels using the weighted MMSE criterion," *IEEE Trans. Communications.*, vol. 49, pp.2198-2206, Dec. 2001.
[9] A. Scaglione, P. Stoica, S. Barbarossa, G. B. Giannaks, and H. Sampath, "Optimal designs for space-time linear precoders and decoders," *IEEE Trans. Signal Processing*, vol. 50, pp. 1051-1064, May 2002.