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A Joint Codebook Design for Beamforming Systems with Transmit Antenna Selection

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***Abstract*—This paper proposes a codebook design that considers transmit antenna selection and beamforming simultaneously. Under the same amount of feedback bits, the proposed codebook can reduce the number of RF units more than half while its performance is still maintained closely (within 0.1 dB loss) to the codebook designs that use all transmit antennas. Since RF units are expensive, the proposed codebook can greatly reduce the hardware implementational cost with slight performance degradation. Moreover, we analyze the proposed codebook and derive a lower bound of symbol error rate (SER). The derived SER lower bound generally matches the simulation results. Simulation results show the proposed codebook with around half active transmit antennas performs comparable to the Lloyd codebook design with full active antennas.**

Index Terms— Beamforming, limited feedback, Grassmannian, vector quantization, antenna selection, hybrid selection, MRT.

I. Introduction

Beamforming and antenna selection are two popular techniques to gain diversity in multiple-antenna systems. Both of them can achieve full diversity order to improve system performance. They are introduced as follows.

The optimal beamforming scheme is the maximal ratio transmission (MRT) [1]. In frequency division duplex (FDD) systems, channel reciprocal assumption is not valid; hence feedback of channel state information (CSI) is needed from receiver to transmitter for beamforming. Since only limit feedback is available, quantization of beamforming vectors is required in this case. In general, vector quantization (VQ) is used to quantize the beamforming vectors and form a codebook in advance. Several researches were conducted for quantized MRT (QMRT). For instance, the performance for QMRT was analyzed in [2] for MISO systems. The QMRT codebook design by Grassmannian line packing was proposed in [3], and codebook design by Lloyd vector quantization (LVQ) was studied in [4].

Antenna selection is used to reduce the number of radio frequency (RF) units while the corresponding diversity order is still maintained. Transmit antenna selection and receive antenna selection may be referred as hybrid selection/maximum ratio transmission (H-S/MRT) and hybrid selection/maximum ratio combining (H-S/MRC) [8], respectively. Extensive researches in this field were investigated in [7]-[8]. For transmit antenna selection, again, feedback is needed to indicate which antennas are selected.

Transmit antenna selection and beamforming can actually be used simultaneously to *1)* reduce the number of RF units, and *2)* improve the system performance by exploiting the beamforming gain. However, researches dealt with these two topics separately. To use these two techniques simultaneously, many feedback bits are required for both selected antenna indices and beamforming codebook. This motivates us to explore new codebook designs that jointly consider transmit antenna selection and beamforming. Compared with the conventional QMRT codebook, the advantages of new codebooks are still unknown. If the new codebook performs nearly the same with the QMRT codebook, it has implementational advantage due to the use of fewer RF units than conventional QMRT codebooks.

In this paper, we consider the codebook design for transmit antenna selection as well as beamforming simultaneously, and propose the corresponding codebook constructing algorithm. If the number of total transmit antennas is M_t and that of selected antennas is L_t, the proposed algorithm sets $M_t - L_t = p$ elements in each codeword of the new codebook to be zeros; the locations of zeros in each codeword can be arbitrary. With the proposed codebook design, beamforming and transmit antenna selection are automatically done by selecting one codeword from the proposed codebook. Moreover, we derive a lower bound of symbol error rate (SER) for the proposed codebook using *virtual branch technique* [8], [9]. The derived lower bound generally matches the simulation results. Both simulated and theoretical results show that, under the same amount of feedback bits and with around half of the RF units, the proposed codebook can achieve within 0.1 dB comparable performance of QMRT which uses all transmit antennas. Therefore, we conclude that the proposed codebook design can achieve a very close performance with QMRT while its implementational complexity is greatly reduced.

II. System Model and Codebook Design

A. System Model

We consider a MISO system with M_t transmit antennas and a single receive antenna as shown in Fig. 1. Let L_t be the number of RF units. If transmit antenna selection is used, it leads to $L_t < M_t$. To perform beamforming in FDD systems, codebook is generally needed to quantize the beamforming vectors in both transmitter and receiver.

Let the number of codewords in the proposed codebook be $N = 2^B$, where B is the number of feedback code bits. Since we select L_t branches from M_t antennas, there should be $M_t - L_t = p$ zeros in every codeword of this new codebook.

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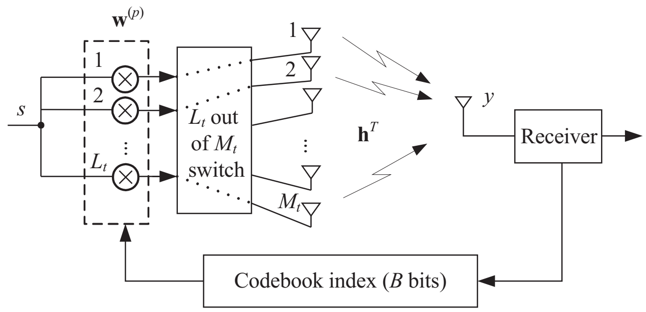

Fig. 1. The proposed system model.

Let the codebook be defined as follows:

$$\mathcal{W}^{(p)} = \left[\begin{array}{cccc} \mathbf{w}_1^{(p)}, & \mathbf{w}_2^{(p)}, & \ldots, & \mathbf{w}_N^{(p)} \end{array} \right] \in \mathbb{C}^{M_t \times N}, \quad (1)$$

where p is used to emphasize that there are p zeros in each codeword $\mathbf{w}_i^{(p)}$. Note that the p zeros can be in arbitrary elements of $\mathbf{w}_i^{(p)}$. Since there are $L_t = M_t - p$ nonzero elements in $\mathbf{w}_i^{(p)}$, we denote $\mathbf{w}_i^{(p)} \in \Omega_{L_t} \subseteq \Omega_{M_t}$, where Ω_{M_t} refers to the set of unit vectors in $\mathbb{C}^{M_t}$. The zero-elements in codeword $\mathbf{w}_i^{(p)}$ corresponds to unselected antennas.

Let the transmit symbol be s, such as a complex QAM symbol, and the receive symbol be y. The relationship between s and y is given by

$$y = \mathbf{h}^T \mathbf{w}^{(p)} s + \eta, \quad (2)$$

where $\mathbf{w}^{(p)}$ is the closest codeword, $\mathbf{h}^T$ is the channel with $\mathbf{h}^T \sim \mathcal{CN}(0, \mathbf{I}_{M_t})$, and $\eta \sim \mathcal{CN}(0, N_0)$ is the noise. Let $\mathbb{E}\{|s|^2\} = E_s$ and assume $\|\mathbf{w}^{(p)}\| = 1$ to keep total transmitted power unchanged before and after beamforming.

B. Codebook Design Preliminary

Before introducing how to design the new codebook, Let us review the basic concept of codebook design. With perfect CSI at the transmitter, the optimal transmit beamforming vector for maximizing the receive SNR is $\mathbf{u} = \left(\mathbf{h}^T\right)^H / \|\mathbf{h}\|,\ \mathbf{u} \in \Omega_{M_t}$, usually called maximum ratio transmission (MRT) [1].

In FDD systems, full CSI may not be available in transmit side. Hence we need to quantize the MRT. Generally codebook is constructed in advance for quantizing MRT. Consider a general codebook as follows:

$$\mathcal{W} = [\begin{array}{cccc} \mathbf{w}_1 & \mathbf{w}_2 & \ldots & \mathbf{w}_N \end{array}] \in \mathbb{C}^{M_t \times N}. \quad (3)$$

For each channel realization, one codeword in the codebook is selected to maximize the instantaneous receive SNR, *i.e.*,

$$\mathbf{w} = \arg \max_{\mathbf{w}_i \in \mathcal{W}} |\mathbf{h}^T \mathbf{w}_i|^2. \quad (4)$$

The resulting instantaneous receive SNR is then given by

$$\gamma = \max_{1 \le i \le N} \{|\mathbf{h}^T \mathbf{w}_i|^2\} \bar{\Gamma}, \quad (5)$$

where $\bar{\Gamma} = E_s / N_0$. The design of codebook has been thoroughly investigated in [2] and [3]. More specifically, [2] focuses on minimizing outage probability and [3] on maximizing the *average* receive SNR. They are consistent with the same criterion to design codebook for MISO systems.

Grassmannian Codebook Design. Since $\mathbf{u}$ is uniformly distributed on Ω_{M_t}, a good codebook design should minimize the maximum correlation between any pair of codewords, *i.e.*,

$$\mathcal{W} = \min_{\mathcal{W} \in \mathbb{C}^{M_t \times N}} \max_{1 \le i < j \le N} |\mathbf{w}_i^H \mathbf{w}_j|. \quad (6)$$

This is related to the Grassmannian line packing problem in [3]. The distance between two lines $\mathbf{w}_i$ and $\mathbf{w}_j$ are defined as

$$d(\mathbf{w}_i, \mathbf{w}_j) = \sqrt{1 - |\mathbf{w}_i^H \mathbf{w}_j|} = \sin(\theta_{i,j}). \quad (7)$$

where $\theta_{i,j}$ is the angle between the lines generated by the two beamforming vector $\mathbf{w}_i$ and $\mathbf{w}_j$. $d(\mathbf{w}_i, \mathbf{w}_j)$ is also called *chordal distance*. This means that the (codebook) set $\mathcal{W}$ is chosen to maximize its minimum chordal distance defined as

$$d(\mathcal{W}) = \sqrt{1 - \max_{1 \le i < j \le N} |\mathbf{w}_i^H \mathbf{w}_j|} = \min_{1 \le i < j \le N} \sin(\theta_{i,j}). \quad (8)$$

Therefore, the codebook design in (6) is equivalent to

$$\mathcal{W} = \max_{\mathcal{W} \in \mathbb{C}^{M_t \times N}} \min_{1 \le i < j \le N} d(\mathbf{w}_i, \mathbf{w}_j), \quad (9)$$

which is usually called *Grassmannian codebook*.

The design of the Grassmannian codebook is a challenging problem, especially in large dimension, *i.e.*, when M_t is large. The authors in [5] applied VQ with the generalized Lloyd algorithm as an alternative design approach.

Generalized Lloyd Algorithm (GLA). VQ design with the GLA efficiently generates near-optimal codebook, which is usually called *Lloyd codebook*. Lloyd codebook design for quantizing MRT was studied in [4] and [5]. The main idea of this design is to obtain the locally optimal solution through iteratively minimizing the SNR loss over the channel statistics. By iterating the following two conditions for several different initial setting, we can obtain the Lloyd codebook. :

- *Condition 1. Nearest neighborhood condition (NNC)*
  Given the codeword $\{\mathbf{w}_i;\ i = 1, \ldots, N\}$, the optimal partition region (Voronoi cell) $\mathcal{A}_i$ of codeword i satisfies

$$\mathcal{A}_i = \{\mathbf{h}_i^T : |\mathbf{h}^T \mathbf{w}_i|^2 \ge |\mathbf{h}^T \mathbf{w}_j|^2, \forall j \ne i\}. \quad (10)$$

- *Condition 2. Centroid condition (CC)*
  Given the partition regions $\{\mathcal{A}_i;\ i = 1, \ldots, N\}$, codeword i are formed for $i = 1, \ldots, N$, such that

$$\begin{aligned} \mathbf{w}_i^{opt} &= \arg \max_{\|\mathbf{w}_i\|=1} \mathbb{E}\{|\mathbf{h}^T \mathbf{w}_i|^2 \big|\ \mathbf{h}^T \in \mathcal{A}_i\} \\ &= \arg \max_{\|\mathbf{w}_i\|=1} \mathbf{w}_i^H \mathbf{R}_i \mathbf{w}_i \\ &= \text{(principal eigenvector) of } \mathbf{R}_i, \end{aligned} \quad (11)$$

  where $\mathbf{R}_i = \mathbb{E}\{\left(\mathbf{h}^T\right)^H \mathbf{h}^T \big|\ \mathbf{h}^T \in \mathcal{A}_i\}$ is the correlation matrix of $\mathcal{A}_i$, and the principal eigenvector is the eigenvector corresponding to the largest eigenvalue of $\mathbf{R}_i$.

■

The above two conditions are repeated until a given criterion, *e.g.*, MSE, converges.

Although Grassmannian codebook and Lloyd codebook are designed using different criteria, they perform almost the same from the simulation point of view. In the following section, we would introduce how to design codebook that jointly considers transmit antenna selection and beamforming.

III. Proposed Algorithm for Codebook Generating

There should be p zeros in every codeword of the proposed codebook. We use the Lloyd codebook generated by the GLA as an initial codebook. First, we set the element with minimum power of $\mathbf{w}_i$ to zero and redistribute the power of this element uniformly to other elements so that the vector power is still unit, denoted it by $\mathbf{w}_i^{(0)}$, for $1 \leq i \leq N$. Assume the q-th element of $\mathbf{w}_i^{(0)}$ is zero. We let $\mathbf{R}_i^{(0)} = \mathbf{R}_i$ except that the q-th row and q-th column of $\mathbf{R}_i^{(0)}$ are set to be zeros. Then, applying the GLA with $\mathbf{w}_i^{(0)}$ and $\mathbf{R}_i^{(0)}$ until it converges, denoted the converged results as $\mathbf{w}_i^{(1)}$ and $\mathbf{R}_i^{(1)}$. $\mathbf{w}_i^{(1)}$, $1 \leq i \leq N$ is the new codebook with one zero, *i.e.*, $M_t - 1$ antennas are selected. Similar procedure can be conducted until there are p-zero elements in the codebook, denoted the results as $\mathbf{w}_i^{(p)}$ and $\mathbf{R}_i^{(p)}$. $\mathbf{w}_i^{(p)}$, $1 \leq i \leq N$ is the new codebook with p zero, which is the proposed codebook that jointly considers transmit antenna selection and beamforming. To summarize, the proposed algorithm for generating p-zero elements in each codeword is described in Algorithm 1 as follows:

Algorithm 1: Generating proposed codebook $\mathcal{W}^{(p)}$.

Initialization: Let $\mathbf{w}_i$ be the codebook obtained by the GLA. For $1 \leq i \leq N$, $\mathbf{w}_i^{(0)}$ is obtained by setting the minimum-power element of $\mathbf{w}_i$ to zero and then redistribute the power of this element uniformly to other elements so that the vector power is still unit. Let the corresponding correlation matrix be $\mathbf{R}_i^{(0)}$ and $n = 0$.

while $n \leq p$ **do**

1) Perform the GLA to $\mathbf{w}_i^{(n)}$ and $\mathbf{R}_i^{(n)}$ until converging. Let the resulting vectors be $\tilde{\mathbf{w}}_i^{(n+1)}$.
2) Obtain $\mathbf{w}_i^{(n+1)}$ by setting the nonzero minimum-power element of $\tilde{\mathbf{w}}_i^{(n+1)}$ to zero and then redistribute the power of this element uniformly to other elements so that the vector power is still unit. Let the corresponding correlation matrix be $\mathbf{R}_i^{(n+1)}$.
3) $n = n + 1$.

The resulting $\mathbf{w}_i^{(p)}$, $1 \leq i \leq N$ is the proposed codebook.

Figs. 2-3 illustrates an example about obtaining $\mathbf{w}_i^{(1)}$ from the initial codeword $\mathbf{w}$. Let $\mathbf{w}_i = [\frac{2}{3}\ \frac{2}{3}\ \frac{1}{3}]^T$. In Fig. 2, the smallest element is forced to zero and the resulting vector is normalized to unit vector as $\mathbf{w}_i^{(0)} = [\frac{1}{\sqrt{2}}\ \frac{1}{\sqrt{2}}\ 0]^T$. Then performing the GLA, $\mathbf{w}_i^{(0)} = [\frac{1}{\sqrt{2}}\ \frac{1}{\sqrt{2}}\ 0]^T$ becomes $\mathbf{w}_i^{(1)} = [\frac{1}{2}\ \frac{\sqrt{3}}{2}\ 0]^T$ shown in Fig. 3. Let the codebook size $N = 8$, the resulting 8 codewords after converging are shown in Fig. 4.

IV. Performance Analysis

The symbol error rate (SER) for MPSK modulation is theoretically analyzed and a lower bound is obtained in this section. The analysis is conducted by applying the virtual branch technique for antenna selection [8] and the quantization concept in [6]. Then the SNR loss is derived by using similar derivation for H-S/MRT in Case 3 of [9]. For notation convenience, we use the following notations: $\bar{\Gamma}=E_s/N_0$, $c_{\text{MPSK}} = \sin^2(\pi/\mathcal{M})$, and $\Theta = \pi(\mathcal{M}-1)/\mathcal{M}$, where $\mathcal{M}$ corresponds to $\mathcal{M}$-PSK.

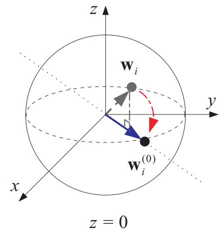

Fig. 2. Forcing the minimum-power element to zero and normalizing the resulting vector to unit vector.

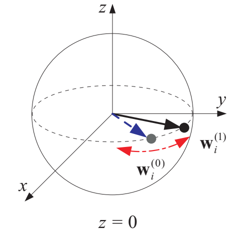

Fig. 3. Performing the GLA.

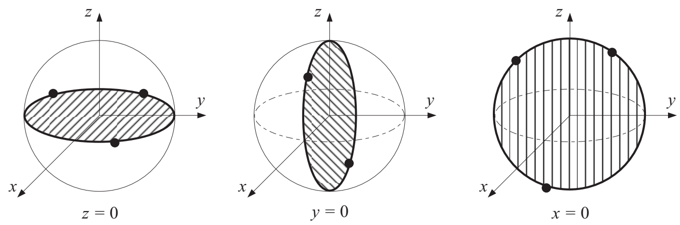

Fig. 4. $\mathbb{R}^3$ illustration of the proposed codebook with codebook size $N = 8$ and the number of active antennas $L_t = 2$.

A. Transmit Antenna Selection

The SER for H-S/MRC in SIMO channels was derived in [9] by using virtual branch technique. We apply these results to H-S/MRT in MISO channels with the same number of channel branches M. Similar to the received SNR analysis in [7, Eq.16], we define the *power normalized coefficient* as follows:

$$\zeta = \frac{M_t}{L_t\left(1 + \displaystyle\sum_{k=L_t+1}^{M_t} \frac{1}{k}\right)}, \quad \zeta \geq 1, \tag{12}$$

where the minimum of ζ is achieved when $L_t = M_t$; this reveals that the performance for MRC ($M_r = M$) in SIMO channels is equivalent to MRT ($M_t = M$) in MISO channels if the transmitter has full CSI. ζ reflects that the power of unselected antennas is "shared" by others at the transmitter (H-S/MRT) rather than "dropped out" at the receiver (H-S/MRC).

The SER of H-S/MRT modified from [8, Eq.33] becomes

$$P_{e,\,\text{H-S/MRT}}(\bar{\Gamma}) = \frac{1}{\pi}\int_0^{\Theta}\left[1 + \frac{c_{\text{MPSK}}\zeta\bar{\Gamma}}{\sin^2\theta}\right]^{-L_t} \cdot \prod_{k=L_t+1}^{M_t}\left[1 + \frac{c_{\text{MPSK}}\zeta\bar{\Gamma}(\frac{L_t}{k})}{\sin^2\theta}\right]^{-1} d\theta. \tag{13}$$

When $L_t = M_t$, the SER of MRT with full CSI is given by

$$P_{e,\,\text{MRT}}(\bar{\Gamma}) = \frac{1}{\pi}\int_0^{\Theta}\left[1 + \frac{c_{\text{MPSK}}\zeta\bar{\Gamma}}{\sin^2\theta}\right]^{-M_t} d\theta. \tag{14}$$

The SNR loss β_{Rx}, equivalent to the power loss in performance between the H-S/MRC and MRC, was explicitly studied in [9]. β_{Rx} was defined to satisfy the following equation:

$$P_{e,\,\text{H-S/MRC}}(\beta_{\text{Rx}}\,\bar{\Gamma}) = P_{e,\,\text{MRC}}(\bar{\Gamma}). \tag{15}$$

The lower bound (LB) and upper bound (UB) of the SNR loss β_{Rx} were derived in [9, Theorem 2] and given respectively in the following expressions:

$$\beta_{\text{Rx}}^{\text{LB}}=\frac{M}{L\left(1+\sum_{k=L+1}^{M}\frac{1}{k}\right)} \text{ and } \beta_{\text{Rx}}^{\text{UB}}=\left(\frac{M!}{L!L^{(M-L)}}\right)^{1/M}, \tag{16}$$

with chosen channel branches L from M at the receiver. In SIMO channels, $M = M_r$, and $L = L_r$, we will apply these results to MISO channels shortly in the section.

B. Quantization for Beamforming

The SER for QMRT in MISO systems was studied in [6]. Here, when $M_t = L_t$, and $\zeta = 1$, the squared value of the minimum chordal distance is a random variable defined as [6, Eq. (19)]

$$Z = \min_i d^2(\mathbf{w}_i, \mathbf{u}), \tag{17}$$

where $\mathbf{u}$ is the optimal beamforming vector and the operator $d(\cdot,\cdot)$ is defined in (7). Therefore, the SER is

$$P_{e,\,\text{QMRT}}(\bar{\Gamma}, N) = \frac{1}{\pi}\int_0^{\Theta}\int_0^1 \left[1+\frac{c_{\text{MPSK}}\zeta\bar{\Gamma}}{\sin^2\theta}(1-z)\right]^{-M_t} \cdot dF_Z(z)d\theta, \tag{18}$$

where $F_Z(z)$ is the cumulative distribution function (CDF) of Z. Notice that, (18) is equivalent to (14) as $N \to \infty$. Since the computation for (18) is difficult to accomplish, [6, Eq. (24)] provided a tight CDF upper bound $\tilde{F}_Z(z)$ as follows:

$$F_Z(z) \le \tilde{F}_Z(z), \quad 0 \le z \le 1, \tag{19}$$

where

$$\tilde{F}_Z(z) = \begin{cases} Nz^{M_t-1}, & 0 \le z < \left(\frac{1}{N}\right)^{\frac{1}{M_t-1}} \\ 1, & z \ge \left(\frac{1}{N}\right)^{\frac{1}{M_t-1}}. \end{cases} \tag{20}$$

Therefore, $F_Z(z)$ in (20) was taken into (18) to provide an SER lower bound of the QMRT as [6, Eq. (25)]

$$P_{e,\,\text{QMRT}}^{\text{LB}}=P_{e,\,\text{QMRT}}, \quad \text{as } \tilde{F}_Z(z) = F_Z(z). \tag{21}$$

The calculated form of (21) is also derived in [6, Eq. (32)].

C. Analysis of Proposed Codebook

Let us call the proposed system as H-S/QMRT for short, since it involves transmit antenna selection and MRT quantization. Assume that the distribution of the minimum chordal distance for the proposed codebook is the same as Z. The SER lower bound for the proposed codebook is expressed by

$$P_{e,\,\text{H-S/QMRT}}^{\text{LB}}(\bar{\Gamma}, N) = \frac{1}{\pi}\int_0^{\Theta}\int_0^1 \left[1+\frac{c_{\text{MPSK}}\zeta\bar{\Gamma}}{\sin^2\theta}(1-z)\right]^{-L_t} \cdot \prod_{k=L_t+1}^{M_t}\left[1+\frac{c_{\text{MPSK}}\zeta\bar{\Gamma}(\frac{L_t}{k})}{\sin^2\theta}(1-z)\right]^{-1} d\tilde{F}_Z(z)d\theta. \tag{22}$$

Notice that, when $M_t = L_t$, (22) becomes (21). Also, when $N \to \infty$, *i.e.*, random variable $Z \to 0$ defined in (17) and thus $\tilde{F}_Z(z) = 1$, (22) becomes (13).

Moreover, the SNR loss, between this proposed H-S/QMRT system and QMRT, increases as N increases. This result should be bounded by

$$\beta_{\text{Tx}} = \frac{1}{\zeta}\beta_{\text{Rx}}, \tag{23}$$

which is between the performance of H-S/MRT and MRT (as $N \to \infty$). Using similar derivation for (16), β_{Tx} could be bounded by

$$\beta_{\text{Tx}}^{\text{UB}}=\frac{1}{\zeta}\beta_{\text{Rx}}^{\text{UB}} = \frac{1}{\zeta}\left(\frac{M_t!}{L_t!L_t^{(M_t-L_t)}}\right)^{1/M_t}. \tag{24}$$

From (24), the SNR loss using L_t transmit antennas should be approximated by (24) as N increases. We demonstrate the derived bound is generally tight in Sec. V.

V. Simulation Results

In this section, we demonstrate the proposed codebook, which jointly considers transmit antenna selection and beamforming, can achieve comparable performance with the QMRT systems. The QMRT codebook is the Lloyd codebook obtained by using the GLA. We also compare these results to the derived lower bound in (22) and the derived SNR penalty between H-S/MRT and MRT. The simulations were conducted using the following settings: The modulation is QPSK for 4T1R systems and 8-PSK for 8T1R systems. The channel coefficients are with Rayleigh distribution.

Case 1: Performance of the proposed codebook. Let the channel be 4T1R. The SER performance using the proposed codebook with $B = 4,\ 6$ for L_t from 2 to M_t is shown in Fig. 5. From the figure, the performance of the proposed codebook is very close to QMRT and can achieve full diversity order. This shows the advantage of the proposed codebook with fewer RF units.

Case 2: Evaluation of derived lower bound. Fig. 6 depicts the SER lower bound in (22) and compares it with the simulated SER for $B = 4$, $M_t = 4$ and $L_t = 2,\ 3,\ 4$. The SER lower bound for $M_t = L_t$ was derived in (21) [6, Eq. (25)]. The derived lower bound generally matches the simulation result.

Case 3: SNR loss related to antenna selection. Here, we define the SNR loss through the simulated results between the proposed codebook and Lloyd codebook under the same amounts of feedback bits. We would like to see how the number of quantization bits affects the SNR loss. Fig. 7 shows the SER for an 8T1R system with $B = 6,\ 8,\ 10$ and $L_t = 3,\ 4,\ 6,\ 8$. As tabulated in Tab. I, we observe that the proposed codebook with $L_t = 4$ can achieve within 0.08 dB performance loss of Lloyd codebook while only half of the RF units are required. When $L_t \ge 3$, the SNR loss is within 0.2 dB. We also observed that the SNR loss increases as B increases. As B approaches infinity, the SNR loss is approximately close to (24) due to antenna selection.

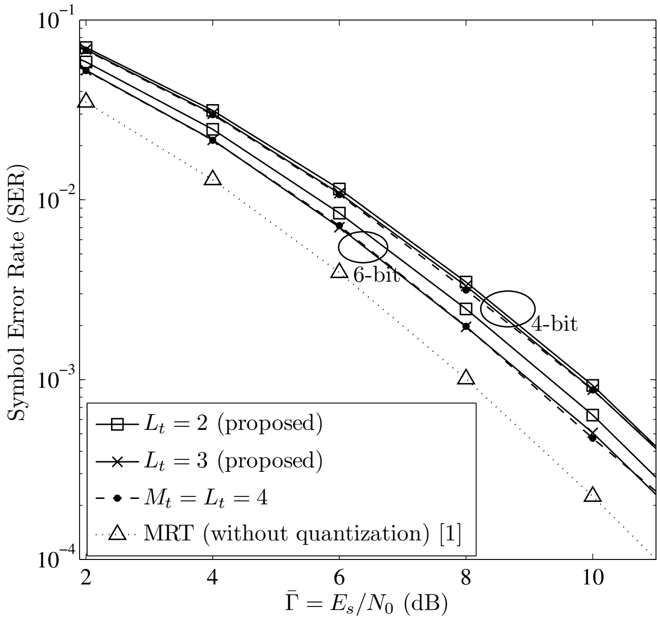

Fig. 5. Performance comparison between the proposed codebook and Lloyd codebook. SER of various codebooks in a 4T1R system with $B = 4,\ 6$ and $L_t = 2,\ 3,\ 4$ are depicted.

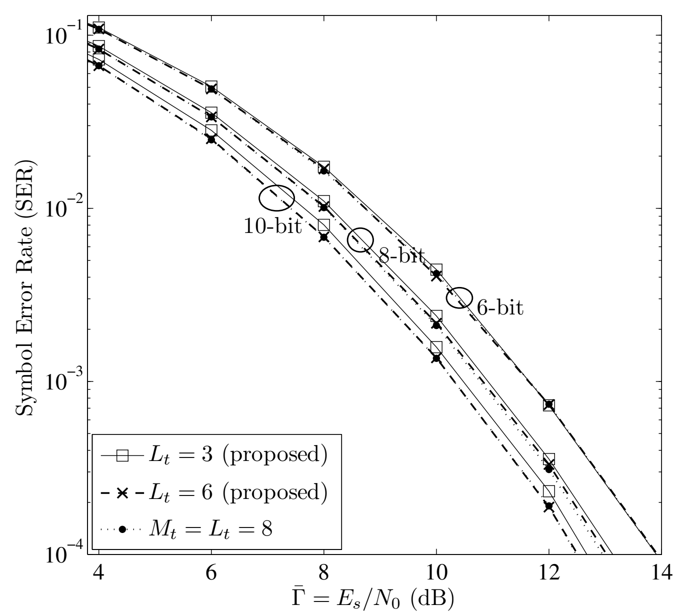

Fig. 7. SER of various codebooks in a 8T1R system. Comparison of the SNR loss with $B = 6,\ 8,\ 10$.

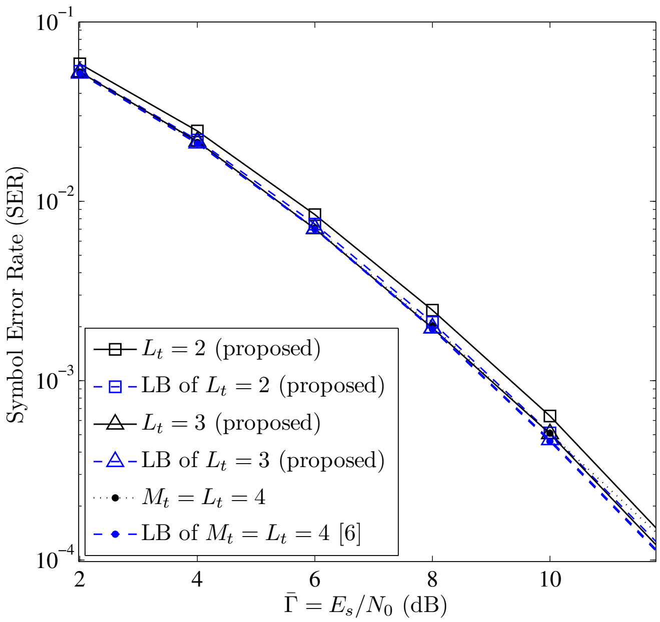

Fig. 6. Comparison of derived SER lower bound (LB) and simulated results in a 4T1R system with $B = 6$ and $L_t = 2,\ 3,\ 4$.

TABLE I
SNR LOSS (IN *dB*) IN AN 8T1R SYSTEM FOR DIFFERENT L_t AND B.

$N = 2^B$ (related to vector-quantization SNR loss)	L_t (related to antenna-selection SNR loss)				
	3	4	5	6	7
64	0.1041	0.0414	0.0217	0.0153	0.0016
256	0.1430	0.0597	0.0253	0.0159	0.0023
1024	**0.1990**	**0.0770**	0.0346	0.0112	0.0059
$\beta_{\text{Tx}}^{\text{UB}}$ from (24) (without quantization)	0.2945	0.1449	0.0627	0.0212	0.0041

VI. Conclusion

We proposed a new codebook design that jointly considers transmit antenna selection and beamforming. With much fewer active transmit antennas and thus RF units, the proposed codebook can still achieve within 0.1 dB comparable performance of the Lloyd codebook design which uses all transmit antennas. We also derived a lower bound of SER for the proposed codebook. Simulation results corroborate the theoretical results. We conclude the proposed codebook design that combines transmit antenna selection and beamforming greatly reduces the hardware complexity with minor performance degradation, and thus makes transceiver design much simpler.

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