An EM-Based Joint Maximum Likelihood Estimation of Carrier Frequency Offset and Channel for Uplink OFDMA Systems

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Abstract— **A maximum likelihood estimator (MLE) based on Expectation Maximization (EM) method is presented to jointly estimate the carrier frequency offset (CFO) and the channel of each user in uplink OFDMA systems. The proposed MLE distinguishes itself from existing methods by its applicability to any arbitrary carrier assignment schemes. The proposed MLE achieves high computational efficiency by transforming a multidimensional maximization problem into a number of substantially smaller separate maximization problems. Computer experiments have been conducted to confirm the effectiveness of the proposed estimators in estimation accuracy and robustness against the near-far effect.**

I. Introduction

Being effective in combating multipath mobile wireless channels, the Orthogonal Frequency Division Multiple Access (OFDMA) technology has attracted much attention recently as one of the most promising techniques for broadband wireless communications. An OFDMA system divides available carriers into groups, called subchannels, and assign one or multiple subchannels to multiple users. Two critical issues in the design of an uplink OFDMA system are investigated in this work. They are carrier frequency offset (CFO) estimation and channel estimation. Similar to OFDM, OFDMA is sensitive to the CFO between the transmitter and the receiver. Inaccurate CFO estimation results in the loss of orthogonality among carriers, thus leading to severe performance degradation. In addition, channel estimation for each user in the system is another indispensable task for achieving high-rate data transmission. These two tasks are especially challenging in uplink OFDMA because of the existence of multiple CFO's and transmission channels.

The CFO estimation for uplink OFDMA has been studied by researchers, *e.g.* [1–4]. However, the methods proposed in [1–3] focus on either the sub-band carrier assignment or the interleaved carrier assignment. Suppose that there are K users in the system. For the sub-band carrier assignment, the system divides carriers into K consecutive subchannels and assigns each subchannel to one of the K users. For the interleaved carrier assignment, carriers j, $K+j$, $2K+j$, $\cdots$ are assigned to user j, where $1 \leq j \leq K$. The recent trend of OFDMA favors a more flexible carrier assignment scheme. An example is given in Fig. 1 [5], where each user can select whatever carriers that are available at a particular time instance. Since there is no rigid association between carriers and users, the generalized carrier assignment scheme provides more flexibility than the sub-band and the interleaved schemes. This will offer a significant advantage, when dynamic resource allocation or adaptive modulation is to be widely used in the near future. Recently, Morelli [4] proposed a synchronization method to handle the generalized assignment scheme by assuming that all users are already synchronized in time and frequency except for one new user. This assumption reduces the complexity of the problem dramatically since the interference from already-synchronized users can be totally eliminated. However, this assumption may not hold in practice.

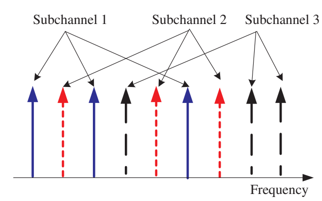

Fig. 1. Illustration of a generalized carrier assignment scheme.

The main contribution of this work is the proposal of a maximum likelihood estimator (MLE) to jointly estimate CFO and channel for all users simultaneously regardless of the underlying carrier assignment scheme. The proposed estimator achieves high computational efficiency by using the Expectation Maximization (EM) algorithm to reduce the complexity of high-dimensional search for the optimal solution. The proposed scheme demands that all users send one pilot FFT block in the time domain in the beginning of the uplink transmission process and, consequently, it is applicable to any subcarrier assignment schemes. Since a similar uplink transmission structure has been specified in IEEE802.16a (Fig.128av of [5]), this requirement should not be a serious constraint in practical OFDMA systems.

II. Proposed Uplink OFDMA Model

Consider an OFDMA system with N carriers as depicted in Fig. 2. The signal received by the base station (BS) is a superposition of contributions from K active users. The nth block sent by the kth user in the frequency domain is denoted by $\boldsymbol{s}_k(n)$, where $k \in \{1, \cdots, K\}$. $s_{k,j}(n)$ can be non-zero if and only if the jth carrier is assigned to the kth mobile user, for $j \in \{0, \cdots, N-1\}$. The corresponding time-domain output vector is equal to

$$\boldsymbol{x}_k(n) = \boldsymbol{F}^H \boldsymbol{s}_k(n),$$

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where $\mathbf{F}$ is the DFT matrix and $(\cdot)^H$ stands for the Hermitian of a matrix. A cyclic prefix (CP) of length N_g is inserted in $\boldsymbol{x}_k(n)$ to form $\boldsymbol{u}_k(n)$ of length $Q = N + N_g$ to combat the dispersive channel before $\boldsymbol{u}_k(n)$ is transmitted to the channel. Let h_k be the kth user's discrete-time composite channel impulse response (including the shaping filter) of order L_k. We define the corresponding channel impulse response vector as

$$\boldsymbol{h}_k \stackrel{\text{def}}{=} [h_k(0), h_k(1), \cdots, h_k(L_k)]^T .$$

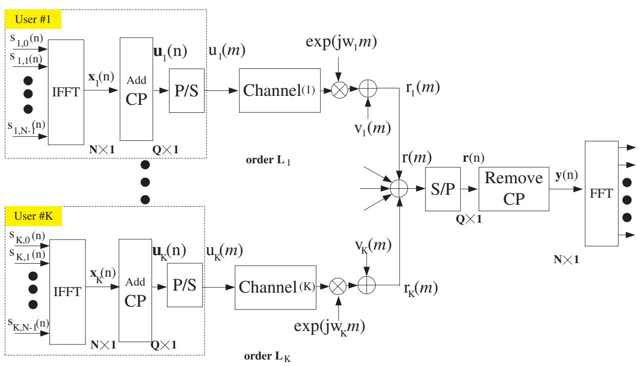

Fig. 2. The OFDMA discrete-time equivalent baseband model.

In the presence of CFO and timing errors, the output of the BS receive filter is given by

$$r(m) = \sum_{k=1}^{K} \left\{ e^{j\omega_k m} \sum_{l=0}^{L_k} h_k(l) u_k(m - l - \mu_k) \right\} + v(m),$$

where $\omega_k = 2\pi\Delta f_k/Q$ is the normalized angular frequency offset with respect to the carrier spacing for the kth user, $v(n)$ is the zero-mean white Gaussian noise and μ_k is the kth user's integer-valued timing error as depicted in Fig. 3. Note that the fractional timing error is indistinguishable from the phase of the channel impulse response and has been incorporated in $\boldsymbol{h}_k$ [4].

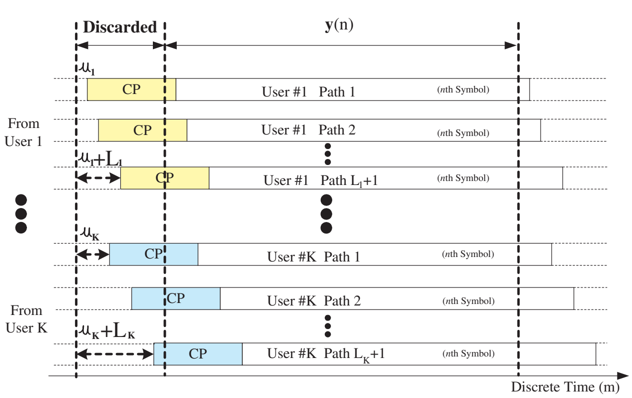

Fig. 3. Illustration of multipath and timing errors.

In the BS, the serial-to-parallel (S/P) conversion transforms $r(m)$ into $\boldsymbol{r}(n)$. After removing the CP from $\boldsymbol{r}(n)$, we form $\boldsymbol{y}(n)$. Define $L_{max} \stackrel{\text{def}}{=} \max\{\mu_k + L_k\}$, $\forall k$. We assume $N_g \geq L_{max}$, which generally holds for preamble symbols in OFDMA systems [6]. It implies the following two conditions. First, each received block $\boldsymbol{r}(n)$ contains only intersymbol interference from its immediate previous block. Second, as a consequence of the first condition, $\boldsymbol{y}(n)$ contains *no* intersymbol interference after the CP removal. These two conditions can be easily proved by extending the results about OFDM given in [7].

The received signal $\boldsymbol{y}(n)$ can be written in matrix form as

$$\boldsymbol{y}(n) = \sum_{k=1}^{K} \underbrace{e^{j\bar{\omega}_k}\boldsymbol{\Gamma}(\omega_k)\boldsymbol{A}_k(n)}_{\boldsymbol{G}(\omega_k)}\boldsymbol{h}_{k,\mu_k} + \boldsymbol{v}(n), \tag{1}$$

where

$$\boldsymbol{\Gamma}(\omega_k) = \text{diag}\left(1, e^{j\omega_k}, \cdots, e^{j(N-1)\omega_k}\right), \tag{2}$$

$$[\boldsymbol{A}_k(n)]_{p,q} = [\boldsymbol{u}_k(n)]_{p-q}, 1 \leq p \leq N, 1 \leq q \leq N_g, \tag{3}$$

$$\boldsymbol{h}_{k,\mu_k} \stackrel{\text{def}}{=} \left[\begin{array}{ccc} \mathbf{0}^T_{\mu_k \times 1} & \boldsymbol{h}^T_k & \mathbf{0}^T_{(N_g-\mu_k-L_k)\times 1} \end{array}\right]^T . \tag{4}$$

and where $\bar{\omega}_k = \omega_k(nQ + N_g)$ is the phase term associated with n and $[\boldsymbol{u}_k(n)]_l$, $-N_g+1 \leq l \leq N-1$, is the lth element of $\boldsymbol{u}_k(n)$.

Assume that all users send out pilot symbols on the carriers assigned only to themselves in the nth block simultaneously. Therefore, all $\boldsymbol{A}_k(n)$'s are known to the BS. We omit all functional dependency on n for the following discussion. Also, we show In the appendix that the signal model proposed in Eq. (1) is equivalent to that given in [4] under the assumption of $N_g \geq L_{max}$. The major difference between Eq. (1) and the signal model in [4] lies in the timing error modeling. By implicitly embedding μ_k in $\boldsymbol{h}_{k,\mu_k}$, Eq. (1) has the advantage of decoupling the estimation of ω_k from the estimation of μ_k and $\boldsymbol{h}_k$. This decoupled structure is helpful in simplifying the MLE derivation.

Please note that the position of the first non-zero element in $\boldsymbol{h}_{k,\mu_k}$ indicates the timing error associated with the user. The parameter L_k can be found by counting the number of non-zero elements in $\boldsymbol{h}_{k,\mu_k}$. Also, Eq. (1) is applicable to any underlying transmission channel models and timing errors as long as $N_g \geq L_{max}$.

III. Derivation of EM-Based MLE

Since $\boldsymbol{v}$ is assumed to be the zero-mean additive white Gaussian noise (AWGN), the least-squares solution is also the maximum likelihood (ML) solution. The maximum likelihood estimates of ω_k and $\boldsymbol{h}_{k,\mu_k}$, $k = 1, \cdots, K$, are given by minimizing the following cost function:

$$\min_{\boldsymbol{\omega},\boldsymbol{h}} \left\{ \left| \boldsymbol{y} - \sum_{k=1}^{K} \boldsymbol{G}(\omega_k)\boldsymbol{h}_{k,\mu_k} \right|^2 \right\}, \tag{5}$$

where

$$\boldsymbol{\omega} = [\omega_1, ..., \omega_K]^T ,$$

$$\boldsymbol{h} = [\boldsymbol{h}^T_{1,\mu_1} \cdots \boldsymbol{h}^T_{K,\mu_K}]^T .$$

Instead of directly solving the multi-dimensional optimization problem as given by Eq. (5), we proposed to adopt an EM-based approach to simplify the optimization problem. The EM algorithm was first studied for parameter estimation of superimposed signals in [8]. The idea has been applied to channel estimation for OFDM in [9]. Based on the same essence of [8,9], EM-based methods are proposed in this section to decompose the computationally intensive multi-dimensional optimization problem in Eq. (5) into K separate ML optimization problems.

A. Conventional EM

Let us first consider the conventional EM method. We view $\boldsymbol{y}$ as the observed data and the "complete" data $\boldsymbol{d}_k$, $k = 1, \cdots, K$, is given by

$$\boldsymbol{d}_k \stackrel{\text{def}}{=} \boldsymbol{G}(\omega_k)\, \boldsymbol{h}_{k,\mu_k} + \boldsymbol{v}_k, \tag{6}$$

where $\boldsymbol{v} = \sum_{k=1}^{K} \boldsymbol{v}_k$. Thus, $\boldsymbol{y} = \sum_{k=1}^{K} \boldsymbol{d}_k$, where $\boldsymbol{d}_k$ is the component in the received signal $\boldsymbol{y}$ contributed by the kth user. The EM algorithm starts with an arbitrary initial guess $\hat{\boldsymbol{\omega}}^{(0)}$ and $\hat{\boldsymbol{h}}_{k,\mu_k}^{(0)}$. For the ith iteration, the parameters are updated by the following procedure.

E-Step: For $k = 1, \cdots, K$, compute

$$\begin{aligned} \hat{\boldsymbol{b}}_k^{(i)} &= \boldsymbol{G}\left(\hat{\omega}_k^{(i)}\right) \hat{\boldsymbol{h}}_{k,\mu_k}^{(i)}, && (7) \\ \hat{\boldsymbol{d}}_k^{(i)} &= \hat{\boldsymbol{b}}_k^{(i)} + \beta_k \left[\boldsymbol{y} - \sum_{k=1}^{K} \hat{\boldsymbol{b}}_k^{(i)}\right], && (8) \end{aligned}$$

where β_k's are chosen such that $\sum_{k=1}^{K} \beta_k = 1$.

M-Step: For $k = 1, \cdots, K$, compute

$$\left[\hat{\omega}_k^{(i+1)}, \hat{\boldsymbol{h}}_{k,\mu_k}^{(i+1)}\right] = \underset{\omega_k, \boldsymbol{h}_{k,\mu_k}}{\arg\min} \left\{ \left| \hat{\boldsymbol{d}}_k^{(i)} - \boldsymbol{G}(\omega_k)\, \boldsymbol{h}_{k,\mu_k} \right|^2 \right\}. \tag{9}$$

Using the least squares solution of $\boldsymbol{h}_{k,\mu_k}$, the optimization problem in Eq.(9) can be achieved in two steps:

$$\begin{aligned} \hat{\omega}_k^{(i+1)} &= \underset{\omega_k}{\arg\min} \left\{ \left| \hat{\boldsymbol{d}}_k^{(i)} - \boldsymbol{G}(\omega_k)\, \boldsymbol{P}(\omega_k)\, \hat{\boldsymbol{d}}_k^{(i)} \right|^2 \right\}, \\ \hat{\boldsymbol{h}}_{k,\mu_k}^{(i+1)} &= \boldsymbol{P}\left(\hat{\omega}_k^{(i+1)}\right) \hat{\boldsymbol{d}}_k^{(i)}, && (10) \end{aligned}$$

where

$$\boldsymbol{P}(\omega_k) = \left(\boldsymbol{G}^H(\omega_k)\, \boldsymbol{G}(\omega_k)\right)^{-1} \boldsymbol{G}^H(\omega_k). \tag{11}$$

The advantage of this algorithm is that its complexity grows linearly with the number of users and the maximization step can be accomplished in parallel for all K users . However, the EM algorithm given above suffers from two drawbacks. The first one is the slow convergence rate since its convergence rate is inversely related to the Fisher information of the complete data space [10]. The second one is the introduction of free variables β_k's. Inappropriate values of β_k's will lead to not only an even slower convergence rate but also possible convergence to a local stationary point [8]. So far, no theoretical analysis on the selection of β_k's has been reported yet.

B. SAGE Algorithm

The SAGE algorithm [10] was proposed to overcome the drawbacks of the conventional EM algorithm. Instead of optimizing over all "complete" data $\boldsymbol{d}_k$'s simultaneously in each iteration, we consider one $\boldsymbol{d}_k$ per iteration, for $k = 1, \cdots, K$ sequentially, and associate all noise with the current $\boldsymbol{d}_k$ [9]. The complete SAGE algorithm in the ith iteration is given below. For $i \geq 1$, we have the following two steps.

E-Step: For $k = 1, \cdots, K$, compute

$$\begin{aligned} \hat{\boldsymbol{b}}_k^{(i)} &= \boldsymbol{G}\left(\hat{\omega}_k^{(i)}\right) \hat{\boldsymbol{h}}_{k,\mu_k}^{(i)}, && (12) \\ \hat{\boldsymbol{d}}_m^{(i)} &= \hat{\boldsymbol{b}}_m^{(i)} + \left[\boldsymbol{y} - \sum_{k=1}^{K} \hat{\boldsymbol{b}}_k^{(i)}\right], && (13) \end{aligned}$$

where $m = (i \mod K)$.

M-Step: Compute

$$\left[\hat{\omega}_m^{(i+1)}, \hat{\boldsymbol{h}}_{m,\mu_m}^{(i+1)}\right] = \underset{\omega_m, \boldsymbol{h}_{m,\mu_m}}{\arg\min} \left\{ \left| \hat{\boldsymbol{d}}_m^{(i)} - \boldsymbol{G}(\omega_m)\, \boldsymbol{h}_{m,\mu_m} \right|^2 \right\}. \tag{14}$$

For $k \neq m$,

$$\left[\hat{\omega}_k^{(i+1)}, \hat{\boldsymbol{h}}_{k,\mu_k}^{(i+1)}\right] = \left[\hat{\omega}_k^{(i)}, \hat{\boldsymbol{h}}_{k,\mu_k}^{(i)}\right]. \tag{15}$$

Eq. (14) can be optimized in two steps as shown in Eq. (10). For the M-step, we see that SAGE is more computationally efficient since it updates only one user's parameters at each iteration.

C. Initialization Strategies

It is a well-known fact that the EM-based algorithms do not guarantee to converge to the global maximum point if there exist multiple stationary points. In that case, the EM-based algorithms may converge to different maxima depending on the initial conditions. Three strategies are proposed to initialize $\hat{\boldsymbol{\omega}}^{(0)}$ and $\hat{\boldsymbol{h}}_{k,\mu_k}^{(0)}$ for the proposed EM-based ML estimator as given below.

1. Each $\omega_k^{(0)}$ is initialized to its expected value, $\omega_k^{(0)} = 0$, since we assume that ω_k is a zero-mean uniformly distributed random variable. Then, we can set $\hat{\boldsymbol{h}}_{k,\mu_k}^{(0)}$ by substituting $\omega_k^{(0)}$ to Eq. (10).
2. We use the MLE proposed for the single-user OFDM in [11]. By regarding all other users' signals as noise, we make a rough guess on $\omega_k^{(0)}$ and $\hat{\boldsymbol{h}}_{k,\mu_k}^{(0)}$ using Morelli's method sequentially, for $k = 1, \cdots, K$. However, since Morellis' method was originally developed for the single-user OFDM system, the estimate provided by this method is not accurate due to the interference from other users.
3. We can use the genetic algorithm (GA) to examine the whole multi-dimensional likelihood function given by Eq. (5). Since GA starts with different initial values and produces generations with larger likelihood values, the initial values obtained via GA are more likely to be close to the global optimal point. However, GA is often computationally demanding, which depends on its structure.

IV. Simulation Results

The OFDMA system used in our simulation is a scaled-down version of IEEE802.11a. We consider an OFDMA system with $N = 64$, $N_g = 16$ and $K = 2, 3$. Each user has 20 subcarriers randomly assigned. The channel of each user is different but all are modeled as the typical urban channel of the European GSM system, which is the same as that given in [11, 12]. The channel model has six paths and has the following channel response

$$h(k) = \sum_{i=0}^{5} A_i g_T(kT_s - \tau_i - t_0),$$

where $t_0 = 3T_s$ is the timing phase, g_T is a raised cosine rolloff filter with a rolloff factor 0.5, A_i and τ_i are the attenuation and the delay parameters of the ith path. A_i's are independent zero-mean Gaussian random variables with variances $\{-3, 0, -2, -6, -8, -10\}$ in dB. The normalized delays τ_n/T_s are given by $\{0, 0.054, 0.135, 0.432, 0.621, 1.351\}$. Under these channel parameters, $h(k)$ only has significant values for $i \leq 7$, which means $L_k = 8$ in our signal model. Also, we introduce a random timing error, $0 \leq \mu_k \leq 8$, for each user. In each simulation realization, a random Δf_k is generated from the interval $(-0.1, 0.1]$.

We evaluate the performance of the MLE based on the mean squared errors (MSE) of the parameter estimation for user #1. We define SNR as $SNR = \frac{\sigma_1^2}{\sigma_v^2}$, where σ_1^2 is the mean power of the received signal from user #1 and σ_v^2 is the mean noise power. For this SNR definition, the asymptotic CRB of Δf_1 for a single-user OFDM system is given by [12]

$$\text{asCRB}(\Delta f_1) = \frac{1}{4\pi^2 N^3} \frac{6\sigma_v^2}{\boldsymbol{h}_{1,\mu_1}^H \boldsymbol{R}_1 \boldsymbol{h}_{1,\mu_1}}, \quad (16)$$

$$\text{asCRB}(\boldsymbol{h}_{1,\mu_1}) = \frac{\sigma_v^2}{N} \left[\boldsymbol{R}_1^{-1} + \frac{3\boldsymbol{h}_{1,\mu_1} \boldsymbol{h}_{1,\mu_1}^H}{\boldsymbol{h}_{1,\mu_1}^H \boldsymbol{R}_1 \boldsymbol{h}_{1,\mu_1}} \right], \quad (17)$$

where $\boldsymbol{R}_1$ is the covariance matrix of the received signal from user #1. It can be shown that the asymptotic CRB of Δf_1 for multi-user OFDMA can be approximated by Eq. (16) if we assume all the cross-terms of any two users' signals are zero. Therefore, Eq. (16) presents the lower bound in the ideal case of OFDMA systems. Details of the proof will be given in our future report.

For comparison, we show the performance of another iterative MLE recently proposed in [13]. This MLE utilizes the alternative maximization (AM) technique to reduce the complexity of the multi-dimensional optimization problem. While the conventional EM method updates each user's parameters in a parallel fashion in one iteration, the AM MLE sequentially updates one user's parameters based on the most recent updates of other users' parameters.

A. Example 1 (the near-far effect)

In this example, we consider the two-user case ($K = 2$), where user #1's signal is lower than user #2 by 3dB. We performed 200 Monte-Carlo runs. The first initialization strategy in Sec. III-C is used for all proposed MLE's. The results are shown in Fig. 4. The black line with asterisks shows the performance of Morelli's MLE proposed for the single-user OFDM [11]. It is evident that Morelli's method is overwhelmed by the strong interference from user #2 that decreases the effective SNR to less than −3dB. On the other hand, all the EM-based and the AM MLE's performed well in this case. They all give similar performance, which is about 2 dB worse as compared with the asymptotic CRB given by Eq. (16).

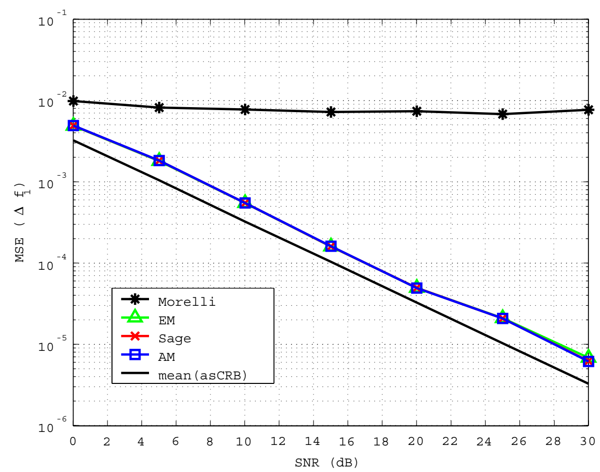

Fig. 4. The CFO estimation performance in the near-far environment.

Fig. 5 shows the convergence behavior of three MLE's for SNR =30dB. The AM MLE has the fastest convergence rate due to the sequential and accumulative updates in each iteration. The stair-like convergence curve of SAGE is due to the fact that user #1's parameters are updated for every $K = 2$ iterations. As expected, the conventional EM method suffers from slow convergence.

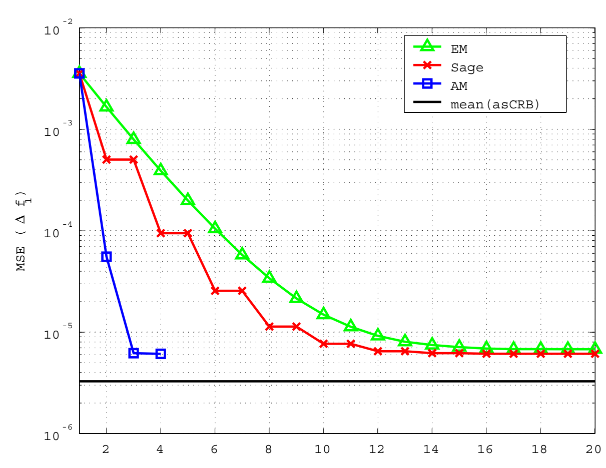

Fig. 5. The convergence behavior of three MLE's for SNR=30dB.

Due to the limit of space, channel estimation results are not shown here. However, since $\hat{\boldsymbol{h}}_{k,\mu_k}$ is solely determined by $\hat{\boldsymbol{\omega}}$, it is to conclude that the proposed estimators should achieve more accurate channel estimation based on $\hat{\boldsymbol{\omega}}$ obtained above.

B. Example 2 (effect of K and N)

In this example, we examine the influence of the user number (K) and the number of carriers N on the CFO estimation performance. The results are shown in Fig. 6. We simulated the three cases with $(N,k) = (64,2)$, $(64,3)$ and $(128,2)$. In this example, all users have the same signal power level. The amount of interference from other users increases as the number of users increases. As a result, the performance of CFO estimation decreases when K increases from 2 to 3 for $N = 64$. On the other hand, when the total amount of interference is a constant ($K = 2$), a larger N will enable the estimator to spread the interference to a higher-dimensional space. As a result, the performance of the CFO estimator improves as N increases from 64 to 128 for $K = 2$.

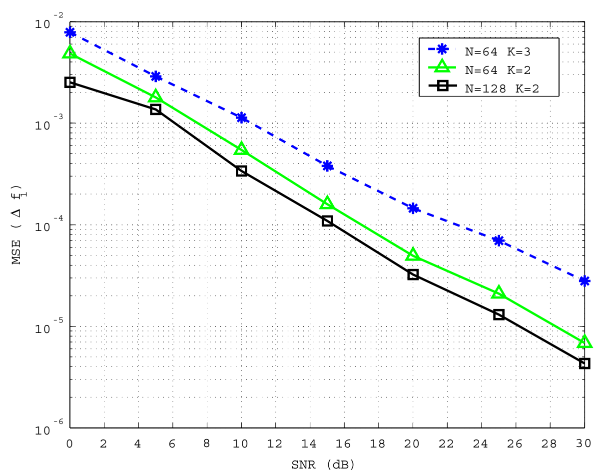

Fig. 6. Influence of K and N on the CFO estimation

V. Conclusion

An EM-based maximum likelihood estimator (MLE) that is capable of jointly estimating CFO and channel for each user in uplink OFDMA was presented in this work. The proposed algorithm is attractive owing to its low computational complexity and general applicability to flexible subcarrier assignment schemes. The robustness of the proposed MLE against the near-far effect was demonstrated by computer simulation results.

VI. Appendix

In this section, we show that Eq. (1) is equivalent to the signal model reported in [4] under the assumption of $N_g \geq L_{\max}$. We start from the received signal from the kth user by BS. From Eq. (1), we have

$$\boldsymbol{y}_k = \sum_{k=1}^{K} e^{j\bar{\omega}_k} \boldsymbol{\Gamma}(\omega_k) \boldsymbol{A}_k \boldsymbol{h}_{k,\mu_k} + \boldsymbol{v}.$$

By expanding $\boldsymbol{A}_k$ and $\boldsymbol{h}_{k,\mu_k}$ according to their definitions in Eq. (3) and Eq. (4), respectively, we have

$$\begin{aligned}
&\boldsymbol{A}_k \boldsymbol{h}_{k,\mu_k} \\
&= \begin{bmatrix} u_0 & u_{-1} & \cdots & u_{-Ng+1} \\ u_1 & u_0 & \cdots & u_{-Ng+2} \\ \vdots & \vdots & \ddots & \vdots \\ u_{N-1} & u_{N-2} & \cdots & u_{N-Ng} \end{bmatrix} \begin{bmatrix} \boldsymbol{0}_{\mu_k \times 1} \\ \boldsymbol{h}_k \\ \boldsymbol{0}_{N_g-\mu_k-L_k \times 1} \end{bmatrix} \\
&= \begin{bmatrix} u_{-\mu_k} & u_{-1-\mu_k} & \cdots & u_{1-L_k-\mu_k} \\ u_{1-\mu_k} & u_{-\mu_k} & \cdots & u_{2-L_k-\mu_k} \\ \vdots & \vdots & \ddots & \vdots \\ u_{N-1-\mu_k} & u_{N-2-\mu_k} & \cdots & u_{N-L_k-\mu_k} \end{bmatrix} \boldsymbol{h}_k \\
&= \boldsymbol{A}_k(\mu_k) \boldsymbol{h}_k,
\end{aligned}$$

where

$$[\boldsymbol{A}_k(\mu_k)]_{p,q} = [\boldsymbol{u}_k]_{p-q-\mu_k}, \quad 1 \leq p \leq N, 1 \leq q \leq N_g,$$

Therefore, we have

$$\boldsymbol{A}_k \boldsymbol{h}_{k,\mu_k} = \boldsymbol{A}_k(\mu_k) \boldsymbol{h}_k,$$

and consequently,

$$\boldsymbol{y} = \sum_{k=1}^{K} e^{j\bar{\omega}_k} \boldsymbol{\Gamma}(\omega_k) \boldsymbol{A}_k(\mu_k) \boldsymbol{h}_k + \boldsymbol{v}. \qquad (18)$$

The above expression is the same as the signal model derived in [4] when $N_g \geq L_{\max}$.

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