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An Equal Gain Transmission in MIMO Wireless Communications

Shang-Ho (Lawrence) Tsai
Department of Electrical Engineering National Chiao Tung University, Hsinchu, Taiwan
E-mail: shanghot@mail.nctu.edu.tw

***Abstract*—The maximum SNR loss between equal gain transmission (EGT) and the optimal scheme, *i.e.* maximum ratio transmission (MRT) is 1.05 dB in MISO channels. However, little is known about the performance loss of EGT in MIMO channels, since there is no simple closed-form solution available for the best EGT in MIMO channels. In this study, an EGT design for MIMO channels is proposed and its performance is analyzed theoretically. Interestingly, the SNR loss between the proposed EGT and the MRT in MIMO channels is shown to be at most 1.05 dB as well. Unlike the optimal MRT, which requires both magnitude and phase information of channels, EGT only needs phase information and hence has several implementional advantages compared to the MRT; that is, it can greatly relax the effort of power amplifier and computational complexity. Consequently, simple transceiver design can be realized using the proposed EGT with small performance degradation.**

Index Terms— Equal gain transmission, EGT, beamforming, precoding, MRT, MIMO.

I. Introduction

MIMO beamforming/precoding techniques are widely used in current wireless communication standards, *e.g.* IEEE 802.11n, IEEE 802.16 family and LTE. Achieving a full diversity order as well as a large array gain in MIMO channels are the main benefits to use the beamforming techniques; these benefits consequently increase transmission range and link robustness.

If all the elements of a beamforming vector have equal magnitude, it is usually called the equal gain transmission (EGT) [6],[7]. Research related to the EGT have been done recently. For instance: The EGT with different combining methods were shown to achieve full diversity order in [6]; moreover, an EGT codebook was proposed. The capacity loss of the EGT due to both vector and scalar quantization in MISO channel was analyzed in [7]. An iterative EGT design for MIMO channels was proposed in [9]. The SNR loss of the EGT in MISO channels and the corresponding performance loss due to scalar quantization were analyzed in [8].

The solution for the best EGT in MISO channels can be easily obtained from the channel phase (see e.g. [8],[9]). Unfortunately, no simple closed-form solution is currently available for the best EGT in MIMO channels [6],[9]; hence, little knowledge is known about how EGT performs in MIMO channels. In this paper, an EGT design for MIMO channels is proposed and its performance is theoretically analyzed. Interestingly, in MIMO channels, the SNR loss between the proposed EGT and the optimal scheme, *i.e.* maximum ratio transmission (MRT) is shown to be at most 1.05 dB, which is also the maximum SNR loss for EGT in MISO channels [8]; that is, with the proposed MIMO EGT and its performance analysis, the SNR loss of a well designed EGT is thus known to be at most 1.05 dB in both MISO and MIMO channels, no matter how the number of transmit antennas increases.

The rest of this paper is organized as follows: The system model and the background of beamforming is presented in Sec. II. The proposed EGT in MIMO channels is introduced in Sec. III; moreover, the corresponding performance is analyzed in this section. Simulation results are provided in Sec. IV. Finally, concluding remarks are given in Sec. V.

Notations: Boldfaced lowercases and **Boldfaced** uppercases denote vectors and matrices, respectively. $\mathbb{E}\{x\}$ is the expectation of random variable x. $\mathbf{A}^*$ and $\mathbf{A}^t$ are the conjugate and transpose of $\mathbf{A}$, respectively. $\mathbf{A}^\dagger$ is the Hermitian of $\mathbf{A}$.

II. System Model and Preliminaries

The block diagram of a beamforming system is shown in Fig. 1. Let the number of transmit antennas be N_t. At the first stage, one transmit symbol (can be complex such as QPSK and M-QAM) is multiplied by a $N_t \times 1$ beamforming vector $\mathbf{f}$. The square of the L_2 norm for the beamforming vector is usually normalized to unit, *i.e.* $\|\mathbf{f}\|_2^2 = 1$ so as to have the same average power before and after the beamforming processing, where the square of the L_2 norm for $\mathbf{x}$ is defined as $\|\mathbf{x}\|_2^2 = |x_1|^2 + |x_2|^2 + \cdots |x_n|^2$ [4]. For an EGT beamforming vector, more specifically, all the elements in $\mathbf{f}$ have equal magnitude, *i.e.* $f_i = e^{j\theta_i}/\sqrt{N_t}$. After the beamforming processing, the signal is transmitted to a $N_r \times N_t$ MIMO channel, denoted by $\mathbf{H}$. Let h_{ij} be the element of $\mathbf{H}$ in the i-th row and the j-th column; h_{ij} is the channel coefficient from the j-th transmit antenna to the i-th receive antenna, and h_{ij} is assumed to have the i.i.d. complex Gaussian distribution with zero mean.

In the receiver side, the received signal vector is combined by a $N_r \times 1$ combining vector $\mathbf{g}$ and form a scalar z. The mathematical expression is then given by

$$z = \mathbf{g}^\dagger \mathbf{H}\mathbf{f}x + \mathbf{g}^\dagger \mathbf{n}, \tag{1}$$

where $\mathbf{n}$ is a $N_r \times 1$ noise vector. Let the power of x be $\mathcal{E}_x$ and the noise power be $\mathcal{N}_0$. The instantaneous SNR of z can be shown to be

$$\rho = \frac{\mathcal{E}_x}{\mathcal{N}_0}\frac{|\mathbf{g}^\dagger \mathbf{H}\mathbf{f}|^2}{\|\mathbf{g}\|_2^2} = \frac{\mathcal{E}_x}{\mathcal{N}_0}|\mathbf{g}^\dagger \mathbf{H}\mathbf{f}|^2, \tag{2}$$

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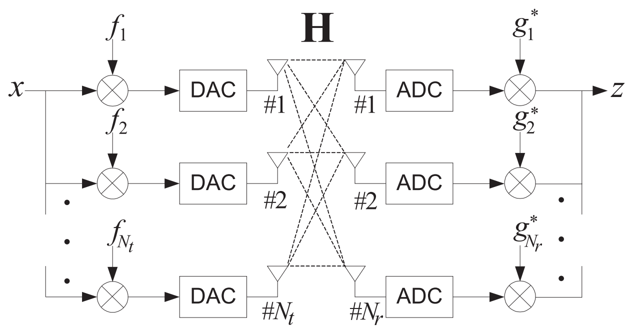

Fig. 1. A MIMO beamforming system.

where $\|\mathbf{g}\|_2^2$ is assumed to be unit without losing the generality [6]. Perform the singular value decomposition (SVD) to $\mathbf{H}$, *i.e.*

$$\mathbf{H} = \mathbf{U}\mathbf{\Sigma}\mathbf{V}^\dagger = [\mathbf{u}_1|\mathbf{U}_{[\bar{1}]}]\mathbf{\Sigma}[\mathbf{v}_1|\mathbf{V}_{[\bar{1}]}]^\dagger, \quad (3)$$

where $\mathbf{u}_1$ and $\mathbf{v}_1$ are the left and the right singular vectors corresponding to the maximum singular value, σ_1, respectively. $\mathbf{U}_{[\bar{1}]}$ and $\mathbf{V}_{[\bar{1}]}$ are the matrices obtained by removing $\mathbf{u}_1$ and $\mathbf{v}_1$ from $\mathbf{U}$ and $\mathbf{V}$, respectively. Then the optimal beamforming system is to let the beamforming vector be the right singular vector and the combining vector be the left singular vector corresponding to the maximum singular value (see [1]), *i.e.*

$$\mathbf{f}_o = \mathbf{v}_1 \quad \text{and} \quad \mathbf{g}_o = \mathbf{u}_1. \quad (4)$$

The optimal instantaneous SNR of z can then be obtained using (3) and (4), and is given by

$$\rho_o = \frac{\mathcal{E}_x}{\mathcal{N}_0}\sigma_1^2. \quad (5)$$

The beamforming and combining vectors in (4) are usually called the MRT and MRC (maximum ratio combiner), respectively.

For the EGT and the corresponding MRC design, the problem to design the beamforming and combining vectors so as to maximize the instantaneous SNR of z is as follows (referring to (2)):

$$(\mathbf{f}_e,\ \mathbf{g}_e) = \arg \max_{\mathbf{g},\ \mathbf{f}} |\mathbf{g}^\dagger \mathbf{H}\mathbf{f}|^2, \quad \text{with} \quad f_i = e^{j\theta_i}/\sqrt{N_t}. \quad (6)$$

In MISO channels, the channel $\mathbf{h}^t$ is a $1 \times N_t$ vector. In such a case, the solution for (6) can be easily obtained; in addition, it is not unique. One solution is as follows (see *e.g.* [6] and [9]):

$$\mathbf{f}_{e-miso} = \frac{e^{j\angle h_1}}{\sqrt{N_t}}\left(e^{-j\angle h_1}\ e^{-j\angle h_2}\ e^{-j\angle h_3} \cdots e^{-j\angle h_{N_t}}\right)^t,$$

and g_e is a scalar given by

$$g_{e-miso} = \frac{\mathbf{h}^t\mathbf{f}_{e-miso}}{|\mathbf{h}^t\mathbf{f}_{e-miso}|}.$$

The instantaneous SNR can be obtained using the above solution and (2) as follows:

$$\rho_{e-miso} = \frac{\mathcal{E}_x}{\mathcal{N}_0}\frac{\|\mathbf{h}\|_1^2}{N_t}, \quad (7)$$

where $\|\mathbf{x}\|_1^2$ is the square of the L_1 norm for vector $\mathbf{x}$ defined by $\|\mathbf{x}\|_1^2 = (|x_1| + |x_2| + \cdots |x_n|)^2$ [4]. In MISO channels, $\sigma_1^2 = \|\mathbf{h}\|_2^2$. From (5) and (7), the average SNR loss between the MRT and the EGT in MISO channels is thus given by [8]

$$\frac{\mathbb{E}\{\rho_o\}}{\mathbb{E}\{\rho_{e-miso}\}} = \frac{N_t\mathbb{E}\{\|\mathbf{h}\|_2^2\}}{\mathbb{E}\{\|\mathbf{h}\|_1^2\}} = \frac{N_t}{1+(N_t-1)\frac{\pi}{4}}. \quad (8)$$

The average SNR loss in (8) is at most 1.05 dB, no matter how N_t increases [8]. It is worth pointing out that Brennan showed the SNR loss between the receive MRC and the receive EGC is also 1.05 dB (see [3]).

In MIMO channels, unfortunately, there is no simple closed-form solution for (6) (see [6] and [9]); hence, little knowledge is available about the performance of EGT in MIMO channels. Is the SNR loss still at most 1.05 dB for EGT in MIMO channels? Interestingly, the answer is positive and this is introduced in the following section.

III. Proposed MIMO EGT and its Performance

In MISO channels, the best EGT design can be $e^{-j\angle\mathbf{h}}/\sqrt{N_t}$, which uses the phase of $\mathbf{h}^*$. $\mathbf{h}^*/\|\mathbf{h}\|_2$ is actually the right singular vector of the MISO channel $\mathbf{h}^t$. Similarly, in MIMO channels, the phase of the right singular vector $\mathbf{v}_1$ (corresponding to the maximum singular value) of $\mathbf{H}$ may be used for MIMO EGT design. For notation convenience, define

$$\angle\mathbf{v}_1 = (\theta_1\ \theta_2\ \cdots\theta_{N_t}). \quad (9)$$

The proposed EGT and the corresponding MRC for MIMO channels are then as follows:

$$\mathbf{f}_{e-mimo} = \left(1\ e^{j(\theta_2-\theta_1)}\ e^{j(\theta_3-\theta_1)} \cdots e^{j(\theta_{N_t}-\theta_1)}\right)^t/\sqrt{N_t} \quad (10)$$

and

$$\mathbf{g}_{e-mimo} = \frac{\mathbf{H}\mathbf{f}_{e-mimo}}{\|\mathbf{H}\mathbf{f}_{e-mimo}\|_2}. \quad (11)$$

From (2), the instantaneous SNR using the solution in (10) can be shown to be

$$\rho_{e-mimo} = \frac{\mathcal{E}_x}{\mathcal{N}_0}\frac{\|\mathbf{H}\mathbf{f}_{e-mimo}\|_2^2}{N_t}. \quad (12)$$

The following lemma gives a lower bound for the instantaneous SNR in (12).

Lemma 1: The instantaneous SNR of the proposed MIMO EGT in (10) is lower bounded by

$$\rho_{e-mimo} \geq \frac{\mathcal{E}_x}{\mathcal{N}_0}\frac{\sigma_1^2\|\mathbf{v}_1\|_1^2}{N_t}. \quad (13)$$

Proof. Let $\mathbf{p} = (e^{j\theta_1}\ e^{j\theta_2} \cdots e^{j\theta_{N_t}})^t/\sqrt{N_t}$, where θ_i is defined in (9). Using Cauchy-Schwarz inequality that $|\mathbf{x}^\dagger\mathbf{y}|^2 \leq \|\mathbf{x}\|_2^2\|\mathbf{y}\|_2^2$ and the fact that $\|\mathbf{u}_1\|_2^2$=1, it leads to

$$\left|\mathbf{u}_1^\dagger\mathbf{H}\mathbf{p}\right|^2 \leq \|\mathbf{u}_1\|_2^2\|\mathbf{H}\mathbf{p}\|_2^2 = \|\mathbf{H}\mathbf{p}\|_2^2$$

Since $\|\mathbf{H}\mathbf{p}\|_2^2 = \|\mathbf{H}\mathbf{p}e^{-j\theta_1}\|_2^2 = \|\mathbf{H}\mathbf{f}_{e-mimo}\|_2^2$, it results in

$$\left|\mathbf{u}_1^\dagger\mathbf{H}\mathbf{p}\right|^2 \leq \|\mathbf{H}\mathbf{f}_{e-mimo}\|_2^2. \quad (14)$$

The following equality can be obtained from (12) and (14):

$$\rho_{e-mimo} \geq \frac{\mathcal{E}_x}{\mathcal{N}_0} \frac{\left|\mathbf{u}_1^\dagger \mathbf{H}\mathbf{p}\right|^2}{N_t}. \quad (15)$$

The SVD of $\mathbf{H}$ can also be expressed as follows:

$$\mathbf{H} = \sum_{i=1}^{r} \sigma_i \mathbf{u}_i \mathbf{v}_i^\dagger, \quad (16)$$

where r is the rank of $\mathbf{H}$. Using (15) and (16) leads to the result in (13).

■

Before analyzing the lower bounded value in (13) theoretically, the following lemmas are introduced for aiding the derivation.

***Lemma** 2:* Let σ_i and $\mathbf{v}_i$ be the singular values and the right singular vectors of $\mathbf{H}$, where $\mathbf{H}^\dagger\mathbf{H}$ is with the Wishart distribution. Then, σ_i is distributed independently of $\mathbf{v}_i$. Moreover, $\mathbf{v}_i$ has the conditional Haar invariant distribution (see pp. 536-538 in [2]).

■

***Lemma** 3:* Let $\mathbf{H}$ be a $N_r \times N_t$ matrix and $\mathbf{h}^t$ be a $1 \times N_t$ vector whose elements all have the i.i.d. complex Gaussian distribution with zero mean and variance σ_h^2. Let $\mathbf{v}_i$ be the right singular vectors of $\mathbf{H}$. Then, the distribution of $\mathbf{v}_i$ is equivalent to the distribution of $\mathbf{h}^*/\|\mathbf{h}\|_2$.

Proof. $\mathbf{H}^\dagger\mathbf{H}$ and $(\mathbf{h}^t)^\dagger\mathbf{h}^t$ are both with the Wishart distribution, since the elements of $\mathbf{H}$ and $\mathbf{h}^t$ are with the complex Gaussian distribution [2]. The right singular vectors of $\mathbf{H}$ and $\mathbf{h}^t$ thus belong to the conditional Haar invariant distribution, as indicated in Lemma 2. Consequently, $\mathbf{h}^*/\|\mathbf{h}\|_2$ and $\mathbf{v}_i$ both have the same distribution, *i.e.* the conditional Haar invariant distribution, since $\mathbf{h}^*/\|\mathbf{h}\|_2$ is the right singular vector of $\mathbf{h}^t$.

■

***Lemma** 4:* Let $\mathbf{h}^t$ be a $1 \times N_t$ vector with the i.i.d. complex Gaussian distributed elements. Then, the following approximation can be made (Please notice that $\mathbb{E}\{x/y\} \neq \mathbb{E}\{x\}/\mathbb{E}\{y\}$ generally):

$$\mathbb{E}\left\{\frac{\|\mathbf{h}\|_1^2}{\|\mathbf{h}\|_2^2}\right\} \approx \frac{\mathbb{E}\left\{\|\mathbf{h}\|_1^2\right\}}{\mathbb{E}\left\{\|\mathbf{h}\|_2^2\right\}} = 1 + \frac{\pi}{4}(N_t - 1). \quad (17)$$

Proof. Using the following equality:

$$\frac{\|\mathbf{h}\|_1^2}{\|\mathbf{h}\|_2^2} = 1 + \sum_{i=1}^{N_t} \sum_{j=1, j\neq i}^{N_t} \frac{|h_i||h_j|}{\|\mathbf{h}\|_2^2}, \quad h_i, h_j \in \mathbf{h},$$

$\mathbb{E}\left\{\frac{\|\mathbf{h}\|_1^2}{\|\mathbf{h}\|_2^2}\right\}$ can be shown to be

$$\begin{aligned} & 1 + \mathbb{E}\left\{ \left. \frac{\sum_{i=1}^{N_t}\sum_{j=1,j\neq i}^{N_t} |h_i||h_j|}{\|\mathbf{h}\|_2^2} \right| h_i, h_j \in \mathbf{h} \right\} \\ = \; & 1 + N_t(N_t - 1)\mathbb{E}\left\{ \left. \frac{|h_i||h_j|}{\|\mathbf{h}\|_2^2} \right| h_i, h_j \in \mathbf{h} \right\}. \end{aligned} \quad (18)$$

Let the mean values of x and y be μ_x and μ_y, respectively. The mean value of a function $g(x, y)$ of two random variables can be approximated by (see p. 215 in [5]):

$$\begin{aligned} \mathbb{E}\{g(x,y)\} \approx \; & g(\mu_x, \mu_y) + \frac{1}{2}\left(\frac{\partial^2 g(\mu_x,\mu_y)}{\partial x^2}\sigma_x^2 \right. \\ + \; & \left. 2\frac{\partial^2 g(\mu_x,\mu_y)}{\partial x \partial y} C_{xy} + \frac{\partial^2 g(\mu_x,\mu_y)}{\partial y^2}\sigma_y^2\right), \end{aligned}$$

where C_{xy} is the covariance defined by

$$C_{xy} = \mathbb{E}\{xy\} - \mu_x\mu_y.$$

Let $x = |h_i||h_j|$, $y = \|\mathbf{h}\|_2^2$ and $g(x, y) = x/y$; the following approximation holds:

$$\mathbb{E}\left\{\frac{x}{y}\right\} \approx \frac{\mu_x}{\mu_y} + \frac{1}{2}\left(\frac{-2}{\mu_y^2}C_{xy} + \frac{2\mu_x}{\mu_y^3}\sigma_y^2\right), \quad (19)$$

μ_x and μ_y were shown in [3] and [8] to be

$$\mu_x = \frac{\pi}{4}\sigma_h^2 \text{ and } \mu_y = N_t\sigma_h^2. \quad (20)$$

y is with the chi-square distribution; hence σ_y^2 can be shown to be

$$\sigma_y^2 = N_t\sigma_h^4. \quad (21)$$

To obtain C_{xy}, we need $\mathbb{E}\{xy\}$. From the definition of x and y, $\mathbb{E}\{xy\} = \mathbb{E}\left\{ \|\mathbf{h}\|_2^2|h_i||h_j| \;\middle|\; h_i, h_j \in \mathbf{h}\right\}$ is shown to be

$$\begin{aligned} & \mathbb{E}\left\{|h_i|^3|h_j|\right\} + \mathbb{E}\left\{|h_i||h_j|^3\right\} \\ + \; & \sum_{k=1,k\neq i,j}^{N_t} \mathbb{E}\left\{|h_k|^2|h_i||h_j|\right\} = 2\mathbb{E}\left\{|h_i|^3\right\}\mathbb{E}\left\{|h_j|\right\} \\ + \; & (N_t - 2)\mathbb{E}\left\{|h_k|^2\right\}(\mathbb{E}\{|h_i|\})^2. \end{aligned} \quad (22)$$

From [5], $\mathbb{E}\left\{|h_i|^3\right\} = \frac{3}{4}\sqrt{\pi}\sigma_h^3$. Hence,

$$\mathbb{E}\{xy\} = \frac{1}{4}N_t\pi\sigma_h^4 + \frac{1}{4}\pi\sigma_h^4. \quad (23)$$

As a result,

$$C_{xy} = \frac{\pi\sigma_h^4}{4}. \quad (24)$$

From (20)-(24), $\left(\frac{-2}{\mu_y^2}C_{xy} + \frac{2\mu_x}{\mu_y^3}\sigma_y^2\right)$ in (19) is zero. Thus,

$$\mathbb{E}\left\{ \left. \frac{|h_i||h_j|}{\|\mathbf{h}\|_2^2} \right| h_i, h_j \in \mathbf{h} \right\} = \frac{\pi}{4N_t}. \quad (25)$$

The result in (17) can hence be obtained using (18) and (25).

■

The approximation in (17) is very accurate. The approximation error is less than 0.001 dB, from the Monte Carlo simulation for $N_t \leq 64$ (see Fig. 2). Using Lemmas 1-4 leads to the following theorem:

***Theorem** 1:* The average SNR loss between the MRT and the proposed EGT in MIMO channels is at most 1.05 dB, which is the same maximum SNR loss as that in MISO channels.

Proof. From (5) and (13), the average SNR loss between the MRT and the proposed EGT in MIMO channels is upper bounded by

$$\frac{\mathbb{E}\{\rho_o\}}{\mathbb{E}\{\rho_{e-mimo}\}} \leq \frac{N_t\mathbb{E}\left\{\sigma_1^2\right\}}{\mathbb{E}\left\{\sigma_1^2\|\mathbf{v}_1\|_1^2\right\}}. \quad (26)$$

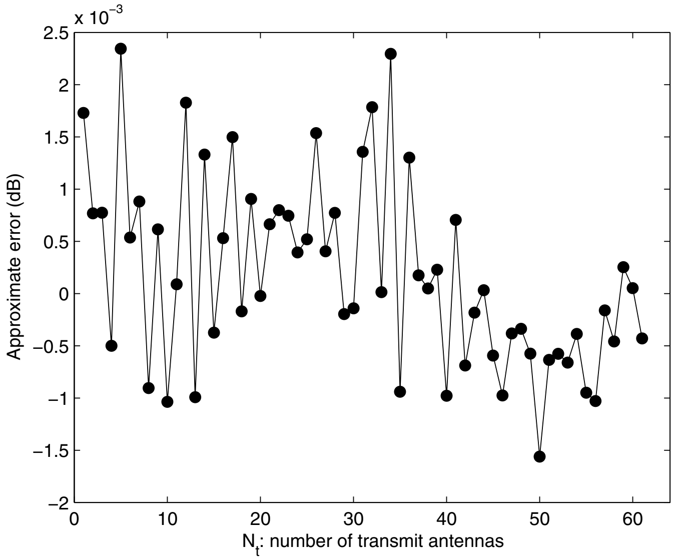

Fig. 2. Approximation error of Lemma 24.

From Lemma 2, the distribution of σ_1 and $\mathbf{v}_1$ are independent;

$$\mathbb{E}\left\{\sigma_1^2\|\mathbf{v}_1\|_1^2\right\} = \mathbb{E}\left\{\sigma_1^2\right\}\mathbb{E}\left\{\|\mathbf{v}_1\|_1^2\right\}.$$

(26) can therefore be rewritten as

$$\frac{\mathbb{E}\left\{\rho_o\right\}}{\mathbb{E}\left\{\rho_{e-mimo}\right\}} \leq \frac{N_t}{\mathbb{E}\left\{\|\mathbf{v}_1\|_1^2\right\}}. \quad (27)$$

From Lemma 3, $\mathbf{v}_1$ and $\mathbf{h}^*/\|\mathbf{h}\|_2$ both have the conditional Haar invariant distribution, where $\mathbf{h}^*/\|\mathbf{h}\|_2$ may be regarded as a normalized MISO channel. Hence, (27) is rewritten as

$$\frac{\mathbb{E}\left\{\rho_o\right\}}{\mathbb{E}\left\{\rho_{e-mimo}\right\}} \leq \frac{N_t}{\mathbb{E}\left\{\|\mathbf{h}\|_1^2/\|\mathbf{h}\|_2^2\right\}}. \quad (28)$$

From Lemma 4, (28) is lower bounded approximately by

$$\frac{\mathbb{E}\left\{\rho_o\right\}}{\mathbb{E}\left\{\rho_{e-mimo}\right\}} \leq \frac{N_t}{1+(N_t-1)\frac{\pi}{4}}. \quad (29)$$

$N_t/(1+(N_t-1)\frac{\pi}{4})$ is the same with that in (8), and it is a monotonically increasing function of N_t. The maximum value is 1.05 dB as N_t grows to ∞.

■

Even if in the MIMO channels, the SNR loss between the MRT and the proposed EGT is again at most 1.05 dB no matter how the number of transmit antennas increases, as the result indicated in Theorem 1. The SNR loss in MIMO channels may even be smaller than that in MISO channels, since the loss in (29) is an upper bound while that in (8) is an equality. Moreover, the SNR loss for the proposed EGT in MIMO channels is irrelevant to N_r, as observed from (29).

IV. Simulation Result

In this section, computer simulation was demonstrated to corroborate the theoretical results derived in Sec. III. Two kinds of performance curves were shown, *i.e.* the SNR loss and the BER (bit error rate). The simulation was conducted using the following settings: The channel coefficients were assumed to have the i.i.d. complex Gaussian distribution with zero mean. More than 60,000 different channel realizations were used to evaluate the performance. The 16-QAM modulation was used for the BER evaluation. Additionally, the notation mTnR was used to denote $N_t = m$ and $N_r = n$.

Example 1: **SNR Loss as a function of N_t.**

Let the instantaneous SNR be $\rho = \frac{\mathcal{E}_x}{\mathcal{N}_0}|\gamma|^2$. The beamforming gain $|\gamma|^2$ for EGT and MRT is shown in Figs. 3-5, for N_r =1 (MISO channel), and N_r =2 and 3 (MIMO channels), respectively. Some observations are summarized as follows: *1.)* As N_t grows, the SNR loss of EGT increases (toward the upper bound of 1.05 dB). When $N_t = 16$, the loss is around 0.98 dB for MISO channel and that is around 0.93 dB for MIMO channels. This is reasonable since the SNR loss of the proposed EGT in MIMO channels is smaller than that in MISO channels (see Sec. III). *2.)* The SNR loss of the EGT for $N_r = 2$ and $N_r = 3$ is nearly the same, by comparing Fig. 4 and Fig. 5. This corroborates the result that the SNR loss of the proposed EGT is irrelevant to N_r in MIMO channels (see Sec. III).

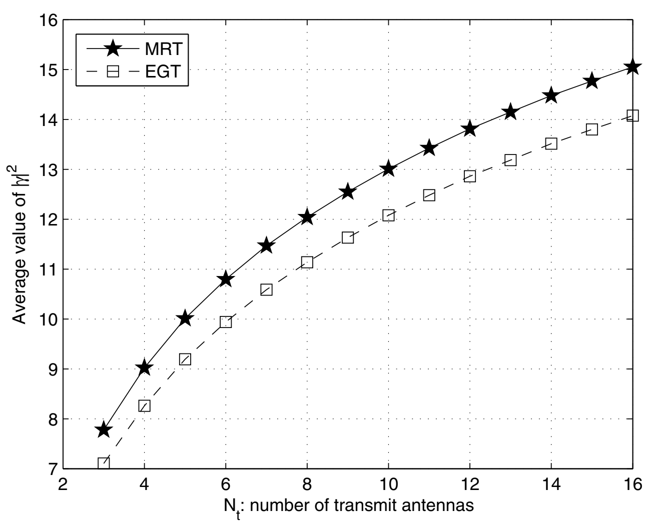

Fig. 3. $|\gamma|^2$ as a function of the number of transmit antennas for $N_r = 1$.

Example 2: **BER comparison of MRT and EGT.**

The BER performance of the MRT and the proposed EGT, as a function of SNR for $N_r = 1$ is shown in Fig. 6. For $N_t = 8$, the performance gap between the MRT and the EGT is about 1.03 dB ; the above performance gaps decreases to 0.9 dB, as N_t decreases from 8 to 4.

Let N_r be replaced from 1 by 2, the BER performance is shown in Fig. 7. The performance gaps for $N_r = 2$ is slightly smaller than that for $N_r = 1$. This is reasonable because the improvement of the proposed EGT schemes in fact is more pronounced in MIMO channels than in MISO channels. The above results generally match the theoretical results introduced in Secs. III and the results in Example. 1

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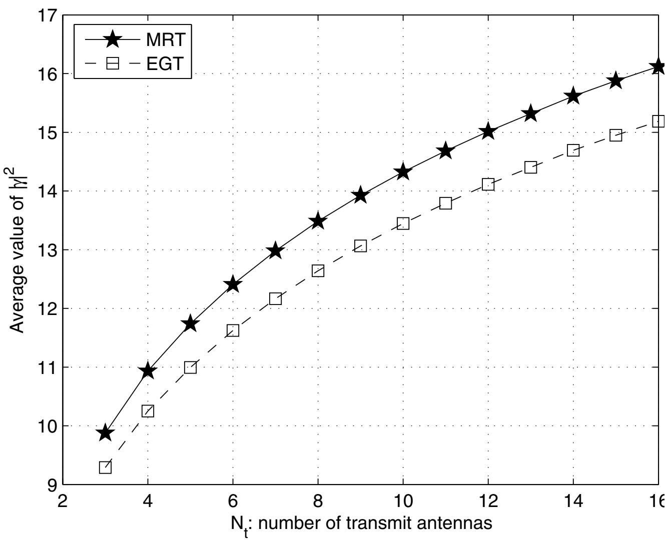

Fig. 4. $|\gamma|^2$ as a function of the number of transmit antennas for $N_r = 2$.

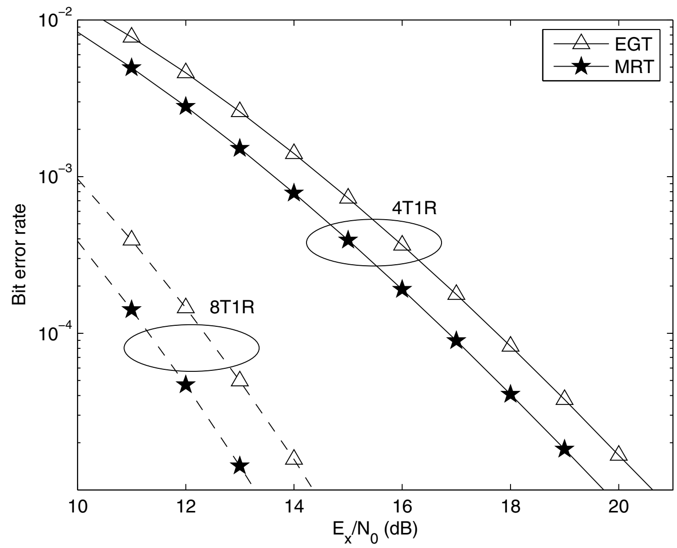

Fig. 6. BER comparison of various beamforming schemes for $N_r = 1$.

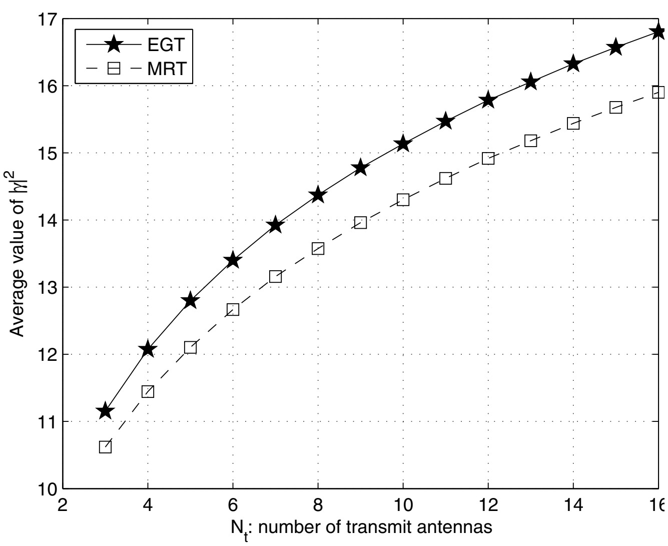

Fig. 5. $|\gamma|^2$ as a function of the number of transmit antennas for $N_r = 3$.

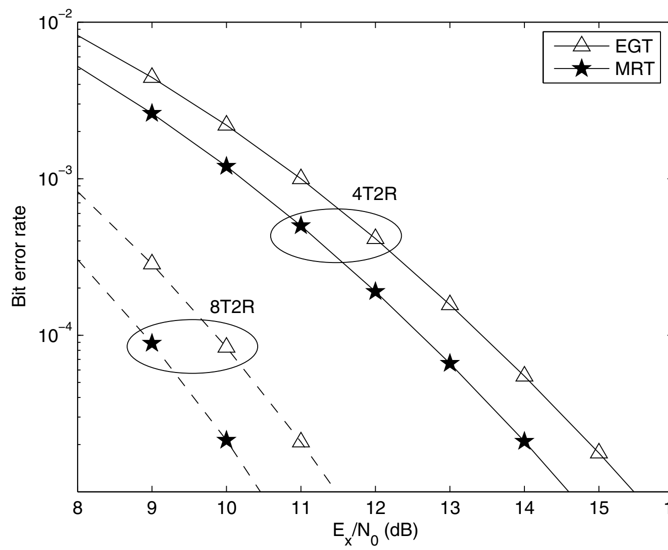

Fig. 7. BER comparison of various beamforming schemes for $N_r = 2$.

V. Conclusion

An EGT design for MIMO channels was proposed. The theoretical results showed that the maximum SNR loss of the proposed EGT and the optimal MRT is 1.05 dB. Conclusion can hence be made that the SNR loss of a well designed EGT is at most 1.05 dB both in MISO and in MIMO channels, no matter how the number of transmit antenna increases.

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