Carrier Frequency Offset Estimation Algorithm for OFDM-Based Multi-cell Systems

Ying-Che Hung, Sheng-Yuan Peng, and Shang-Ho (Lawrence) Tsai
Department of Electrical Engineering, National Chiao Tung University, Hsinchu, Taiwan
E-mail: yingjhe.ece95g@g2.nctu.edu.tw, sam955251@hotmail.com, and shanghot@alumni.usc.edu.

***Abstract*—In this paper, we propose an algorithm for estimating the carrier frequency offset (CFO) in OFDM-based multi-cell networks. In 3GPP-LTE single-cell systems, the CFO can be estimated by applying the Schmidl algorithm. However, multi-cell interference (MCI) is induced in multi-cell environments; as a result the MCI degrades estimate accuracy. One solution to mitigate MCI may be via properly designing the training sequences. In this paper, we propose a method for generating training sequences with good orthogonality in both time and frequency domain. Therefore, MCI can be effectively suppressed and CFO estimation algorithms designed for single-user or single-cell environments can be slightly modified, and applied in multi-cell environments. An example is given for showing how to modify the estimation algorithms. Consequently, the computational complexity can be dramatically reduced. Moreover, the training sequences can be applied for detecting cell identity (ID) thanks to its good orthogonality. Simulation results show that the proposed sequences and the CFO estimation algorithms outperform conventional schemes in multi-cell environments.**

I. Introduction

Recently, OFDM based two-tier cell networks, which consist of a conventional macrocell cellular network overlaid with small cells access points (FAPs), attract extensive research attention due to their enhanced capacity and improved coverage, see *e.g.,* [1]-[3]. In many cases, different cells may appear in the same frequency band to increase user capacity and data capacity. However, multi-cell interference (MCI) appears and degrades the accuracy of carrier frequency offset (CFO) estimation.

In this paper, we propose a CFO estimation algorithm for multi-cell environments. We take the single-cell CFO estimation algorithm in [4] as an example, and show how to modify it in multi-cell environments if the training sequences have good orthogonality in time domain multipath channels. As a result, the computational complexity for CFO and channel estimation can be greatly reduced thanks to the mitigation of MCI.

On the other hand, in order to achieve the required sequence property of the proposed CFO estimation algorithm, we proposed a set of training sequences. The proposed sequences are inspired by the Zadoff-Chu (ZC) sequences [5] and the Gold code. We prove that the proposed sequences can achieve mutually orthogonal in multipath channels by appropriately selecting the sequence index. Moreover, since the proposed sequences have good orthogonality, they can also be applied to detect the cell identity (ID). Simulation results show that the proposed sequences as well as the CFO estimation algorithms greatly outperform conventional schemes in multi-cell environments.

The following notations shall run through the contents: Vectors are in lowercase boldface and matrices are in uppercase boldface. $(\cdot)^T$ and $(\cdot)^H$ denote the transpose and conjugate transpose of matrix.

[1]This research was supported by the National Science Council (NSC), Taiwan under Grant NSC 102-2221-E-009-017-MY3.

II. System model and problem formulation

A block diagram describing OFDM-based multi-cell cell search structure is illustrated in Fig. 1. Assume there are I

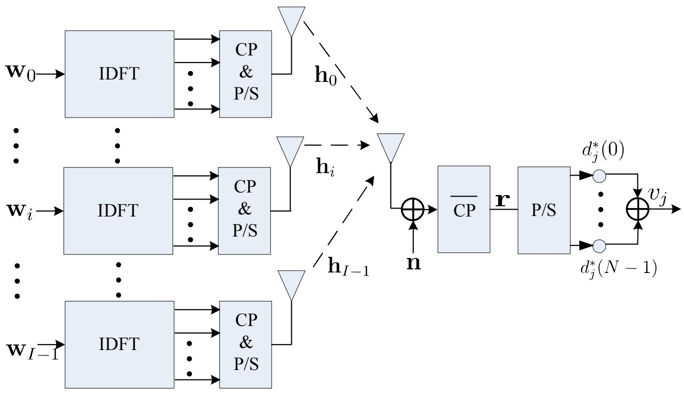

Fig. 1. The block diagram of a OFDM based multi-cell system.

cells in the same frequency band and each cell is assigned a unique synchronization sequence $\mathbf{w}_i = (w_i[0] \cdots w_i[N-1])^T$, $0 \le i \le I-1$. The synchronization sequence $\mathbf{w}_i$ is converted to $\mathbf{d}_i$ through inverse DFT (IDFT) operation, *i.e.,* $\mathbf{d}_i = \mathbf{F}^H \mathbf{w}_i$, where $\mathbf{F}$ is the $N \times N$ DFT matrix with the kth row and the nth column being $[\mathbf{F}]_{kn} = \frac{1}{\sqrt{N}} e^{-j\frac{2\pi}{N}kn}$. Assume there is a user attempting to connect to one of these I cells. For this user, the time-domain received signal after removing cyclic prefix (CP) is represented as

$$\mathbf{r} = \sum_{i=0}^{I-1} \mathbf{\Phi}_i \mathbf{H}_i \mathbf{d}_i + \mathbf{n}, \tag{1}$$

where $\mathbf{\Phi}_i = \text{diag}\left(e^{j\frac{2\pi}{N}N_g\epsilon_i} \ \ldots \ e^{j\frac{2\pi}{N}(N+N_g-1)\epsilon_i}\right)$ is the CFO matrix with N_g being the CP length and ϵ_i being the CFO between this user and the ith cell. $\mathbf{n} \in \mathbb{C}^{N\times 1}$ is the noise vector and $\mathbf{H}_i$ is an $N \times N$ circulant matrix given by

$$\mathbf{H}_i = \left(\begin{array}{cccc} \mathbf{P}_{(0)}\overline{\mathbf{h}}_i & \mathbf{P}_{(1)}\overline{\mathbf{h}}_i & \cdots & \mathbf{P}_{(N-1)}\overline{\mathbf{h}}_i \end{array} \right),$$

where $\overline{\mathbf{h}}_i = [\mathbf{h}_i^T \ \mathbf{0}_{1\times N-L}]^T$ and $\mathbf{h}_i \in \mathbb{C}^{L\times 1}$ are the channel of the ith cell, with distribution $\mathcal{CN}(0,1)$. $\mathbf{P}_{(n)} \in \mathbb{R}^{N\times N}$ denotes a permutation matrix obtained by cyclically down shifting the row of an identity matrix $\mathbf{I}_N$ by n elements. Since the circulant matrix $\mathbf{H}_i$ can be diagonalized using DFT and IDFT matrices, *i.e.,* $\mathbf{H}_i = \mathbf{F}^H\mathbf{\Lambda}_i\mathbf{F}$, where $\mathbf{\Lambda}_i = \text{diag}(\boldsymbol{\lambda}_i)$, and $\boldsymbol{\lambda}_i$ is the frequency response of the ith channel expressed as $\boldsymbol{\lambda}_i = \mathbf{F}\overline{\mathbf{h}}_i$. We can rewrite (1) as

$$\mathbf{r} = \sum_{i=0}^{I-1} \mathbf{\Phi}_i\mathbf{F}^H\mathbf{\Omega}_i\mathbf{F}\overline{\mathbf{h}}_i + \mathbf{n}, \tag{2}$$

where $\mathbf{\Omega}_i = \text{diag}(\mathbf{w}_i)$.

The system would perform cell ID detection before estimating the CFO. Without loss of generality, assume the jth cell is the targeted cell. The matched result can be expressed as

$$\begin{aligned}\mathbf{d}_j^H\mathbf{r} &= \mathbf{d}_j^H\left[\sum_{i=0}^{I-1}\mathbf{\Phi}_i\mathbf{F}^H\mathbf{\Omega}_i\mathbf{F}\overline{\mathbf{h}}_i + \mathbf{n}\right] \\ &= \left\{\underbrace{\mathbf{d}_j^H\mathbf{\Phi}_j\mathbf{F}^H\mathbf{\Omega}_j\mathbf{F}\overline{\mathbf{h}}_j}_{\text{desired signal}} + \sum_{i=0,i\neq j}^{I-1}\underbrace{\mathbf{d}_j^H\mathbf{\Phi}_i\mathbf{F}^H\mathbf{\Omega}_i\mathbf{F}\overline{\mathbf{h}}_i}_{\text{MCI}_{j\leftarrow i}} + \mathbf{d}_j^H\mathbf{n}\right\}.\end{aligned} \tag{3}$$

The MCI would degrade the CFO estimation performance. One solution is to design training sequences to mitigate the MCI effect.

III. Proposed Multi-cell CFO estimation algorithm

In multi-cell environments, the MCI would affect the accuracy of CFO estimation. In this case, the Schmidl scheme may not be directly applied to estimate CFO [4]. To overcome this issue, we propose an CFO estimation algorithm for multi-cell environments. It is noted that this algorithm is modified from the Schmidl scheme. From (1), two consecutively received signals are expressed as

$$\begin{aligned}\mathbf{r}^{(1)} &= \sum_{i=0}^{I-1}\underbrace{\mathbf{\Phi}_i^{(1)}\mathbf{F}^H\mathbf{\Lambda}_i\mathbf{w}_i}_{\mathbf{s}_i^{(1)}} + \mathbf{n}^{(1)} \\ \mathbf{r}^{(2)} &= \sum_{i=0}^{I-1}\underbrace{\mathbf{\Phi}_i^{(2)}\mathbf{F}^H\mathbf{\Lambda}_i\mathbf{w}_i}_{\mathbf{s}_i^{(2)}} + \mathbf{n}^{(2)},\end{aligned}$$

where $\mathbf{\Phi}_i^{(1)}$ and $\mathbf{\Phi}_i^{(2)}$ are diagonal CFO matrices whose elements are respectively represented as $\left[\mathbf{\Phi}_i^{(1)}\right]_{nn} = e^{j\frac{2\pi}{N}(n+N_g)\epsilon_i}$ and $\left[\mathbf{\Phi}_i^{(2)}\right]_{nn} = e^{j\frac{2\pi}{N}(n+N+2N_g)\epsilon_i}$, $0 \leq n \leq N-1$. $\mathbf{n}^{(1)}$ and $\mathbf{n}^{(2)}$ are thermal noises and can be ignored at high SNR.

To explicitly explain why the Schmidl scheme may not work in multi-cell environments, we give an example which consists of three cells in the same frequency band. Moreover, we assume the user already determines which cell to link before the CFO estimation; this assumption is reasonable because cell ID is in general determined before CFO estimation. Also, we consider the system is at high SNR. Therefore, the received signals can be expressed as

$$\begin{aligned}\mathbf{r}^{(1)} &= \mathbf{s}_1^{(1)} + \mathbf{s}_2^{(1)} + \mathbf{s}_3^{(1)} \\ \mathbf{r}^{(2)} &= \mathbf{s}_1^{(2)} + \mathbf{s}_2^{(2)} + \mathbf{s}_3^{(2)}\end{aligned} \tag{4}$$

The Schmidl algorithm is applied to estimate the CFO of cell 1 and the value q is given by

$$\begin{aligned}q &= \left[\mathbf{r}^{(1)}\right]^H\mathbf{r}^{(2)} = \left[\mathbf{s}_1^{(1)} + \mathbf{s}_2^{(1)} + \mathbf{s}_3^{(1)}\right]^H\left[\mathbf{s}_1^{(2)} + \mathbf{s}_2^{(2)} + \mathbf{s}_3^{(2)}\right] \\ &= \left[\mathbf{s}_1^{(1)}\right]^H\mathbf{s}_1^{(2)} + \left[\mathbf{s}_2^{(1)}\right]^H\mathbf{s}_2^{(2)} + \left[\mathbf{s}_3^{(1)}\right]^H\mathbf{s}_3^{(2)} \\ &+ \sum_{j=1}^{3}\sum_{i=1,i\neq j}^{3}\left[\mathbf{s}_j^{(1)}\right]^H\mathbf{s}_i^{(2)},\end{aligned} \tag{5}$$

where the term $\left[\mathbf{s}_j^{(1)}\right]^H\mathbf{s}_i^{(2)}$ represents the MCI. If the synchronization sequences have good orthogonality, $\left[\mathbf{s}_j^{(1)}\right]^H\mathbf{s}_i^{(2)} = 0$ if we appropriately grouping the sequences. However, even the sequences have good orthogonality, there still exists the interference terms $\left[\mathbf{s}_2^{(1)}\right]^H\mathbf{s}_2^{(2)}$ and $\left[\mathbf{s}_3^{(1)}\right]^H\mathbf{s}_3^{(2)}$. Therefore, we propose a method to mitigate the interferences to improve the CFO estimation accuracy. More specifically, $\mathbf{r}^{(1)}$ and $\mathbf{r}^{(2)}$ are both multiplied by the Hermitian of the time domain synchronization sequence for the targeted cell before performing inner product. Below we explain how to mitigate the MCI by applying the proposed method for CFO estimation:

The value q using the proposed method is expressed as

$$q = \left[\mathbf{d}_j^H\mathbf{r}^{(1)}\right]^H\left[\mathbf{d}_j^H\mathbf{r}^{(2)}\right]. \tag{6}$$

From (1), (6) can be expressed to (7), shown at the bottom of the next page, and it can be manipulated as

$$\begin{aligned}&\left[\mathbf{d}_j^H\mathbf{\Phi}_j^{(1)}\mathbf{F}^H\mathbf{\Lambda}_j\mathbf{w}_j + \sum_{i=0,i\neq j}^{I-1}\text{MCI}_{j\leftarrow i,\text{ domi}}^{(1)} + \text{MCI}_{j\leftarrow i,\text{ resi}}^{(1)}\right]^H \\ &\cdot\left[\mathbf{d}_j^H\mathbf{\Phi}_j^{(2)}\mathbf{F}^H\mathbf{\Lambda}_j\mathbf{w}_j + \sum_{i=0,i\neq j}^{I-1}\text{MCI}_{j\leftarrow i,\text{ domi}}^{(2)} + \text{MCI}_{j\leftarrow i,\text{ resi}}^{(2)}\right].\end{aligned} \tag{8}$$

From [6], if the sequences have good orthogonality in multipath channels, $\text{MCI}_{j\leftarrow i,\text{ resi}}$ is sufficiently small to be ignored when CFO is small. Moreover, $\text{MCI}_{j\leftarrow i,\text{ domi}}$ is zero due to the sequence property. Therefore, (8) can be approximated to

$$\begin{aligned}q &\approx \left[\mathbf{d}_j^H\mathbf{\Phi}_j^{(1)}\mathbf{F}^H\mathbf{\Lambda}_j\mathbf{w}_j\right]^H\left[\mathbf{d}_j^H\mathbf{\Phi}_j^{(2)}\mathbf{F}^H\mathbf{\Lambda}_j\mathbf{w}_j\right] \\ &= e^{j2\pi\epsilon_j\left(1+\frac{N_g}{N}\right)}\|\mathbf{d}_j^H\mathbf{\Phi}_j^{(1)}\mathbf{F}^H\mathbf{\Lambda}_j\mathbf{w}_j\|^2.\end{aligned}$$

Therefore, the estimated CFO $\widehat{\epsilon}_j$ of the targeted cell can be obtained by

$$\widehat{\epsilon}_j = \frac{1}{2\pi(1+\frac{N_g}{N})}\text{angle}(q).$$

It is worth to emphasize that the proposed CFO estimation is modified from the Schmidl algorithm, which is designed for single cell environment. The proposed CFO estimation by using the designed sequences for multi-cell environments in concluded in Algorithm 1.

Algorithm 1: Proposed CFO estimation algorithm for multi-cell environments.

1: Multiply the Hermitian of the time-domain synchronization sequence of the targeted cell $\mathbf{d}_j$ to the two consecutively received signals $\mathbf{r}^{(1)}$ and $\mathbf{r}^{(2)}$.
2: Calculate $q = \left[\mathbf{d}_j^H \mathbf{r}^{(1)}\right]^H \left[\mathbf{d}_j^H \mathbf{r}^{(2)}\right]$.
3: The CFO of the targeted cell ϵ_j is obtained by $\widehat{\epsilon}_j = \frac{1}{2\pi(1+\frac{N_g}{N})}\text{angle}(q)$.

IV. The Proposed sequences

To mitigate the MCI, we refer to the concept for generating the Gold sequence. Since ZC sequence has constant amplitude and zero autocorrelation (CAZAC) property; while Gold sequences have good orthogonality, one intuition to generate a set of sequences that can preserve both properties is to combine the generation of these two kinds of sequences. More specifically, the proposed sequences are generated by combining two ZC sequences with different roots u_1 and u_2. The proposed sequences can be collected to form a matrix $\mathbf{W} = [\mathbf{w}_0, \mathbf{w}_1, \ldots, \mathbf{w}_{N-1}]$, where $\mathbf{w}_i \in \mathbb{C}^{N\times 1}$ is the synchronization sequence and is the combination of two ZC sequences $\mathbf{x}$ and $\mathbf{y}$, $0 \le i \le N-1$. Each column in $\mathbf{W}$ is generated by the following formula

$$\mathbf{w}_i = \mathbf{Y}\mathbf{P}_{(i)}\mathbf{X}, \tag{9}$$

where $\mathbf{Y} = \text{diag}(\mathbf{y})$ and $\mathbf{X} = \text{diag}(\mathbf{x})$. For example, the proposed sequences for length $N = 4$ are expressed as

$$\mathbf{W} = \begin{pmatrix} y[0]x[0] & y[0]x[3] & y[0]x[2] & y[0]x[1] \\ y[1]x[1] & y[1]x[0] & y[1]x[3] & y[1]x[2] \\ y[2]x[2] & y[2]x[1] & y[2]x[0] & y[2]x[3] \\ y[3]x[3] & y[3]x[2] & y[3]x[1] & y[3]x[0] \end{pmatrix}.$$

It is noted that the entry of the proposed sequences can be summarized as

$$w_i[k] = e^{-j\frac{\pi}{N}\alpha_i[k]}, \tag{10}$$

where

$$\alpha_i[k] = \begin{cases} u_1(k-i)^2 + u_2 k^2, & \text{if } N \text{ is even.} \\ u_1(k-i)(k-i+1) + u_2 k(k+1), & \text{if } N \text{ is odd.} \end{cases} \tag{11}$$

We would show that the proposed sequences have good orthogonality in multipath fading channels if we appropriately select the sequence indices.

Theorem 1 *Let the root u_1 for generating $\mathbf{W}$ be odd and the sequence length $N = 2^s$, $s \ge 2$. Let the parameter G be $G = 2^{s'} \ge L$, $s' \in \mathbb{N}$. The number of mutually orthogonal cells is N/G. More specifically, if cell j is selected, the indices of the other mutually orthogonal cells are*

$$i = ((\ j + oG\ ))_N,\ o \in \mathbb{Z}.$$

Proof: From [6], there are $L-1$ interfering cells to the cell j and the corresponding indices can be obtained through

$$((\ (j-i)\ ))_N = n, \quad 1 \le n \le L-1.$$

For $u_1 = 1$, the cell j is interfered by the cells with indices $i = ((\ j+\tau\ ))_N$, $1 \le \tau \le L-1$, *i.e.,* the neighboring $L-1$ cell indices. Therefore, if we select the cell indices i' that are uniformly separated by a period $G \ge L$, *i.e.,* $i' = j + oG$, $o \in \mathbb{Z}$, these cells would not interfere with each other. In other words, they are mutually orthogonal.

For odd number $u_1 \ne 1$, the interfering cell indices to the cell j can be obtained through $i = j + (n+gN)/u_1$, $1 \le n \le L-1$, $g \in \mathbb{Z}$. Since

$$n+gN = n+g2^s = n+g(2^{s-s'})G \ne u_1 oG, \text{for } 1 \le n \le L-1,$$

the selected indices i' would not become an interferer. More specifically, $i' = j+oG \ne j+(n+gN) = i$, for $1 \le n \le L-1$. Therefore, they are still mutually orthogonal. ■

V. Simulation Results

Example. CFO estimation in multi-cell environments: This example shows the accuracy of CFO estimation for the proposed sequences in multi-cell environments. In this simulation setting, three cells are deployed in the same frequency band and its layout is shown in Fig. 2. User is located at different locations, *i.e.,* Location 1 $= (18, 0)$ and Location 3 $= (0, 20/\sqrt{3})$, for comparing the estimation accuracy between

$$\begin{aligned} q &= \left[\mathbf{d}_j^H \sum_{i=0}^{I-1} \mathbf{\Phi}_i^{(1)} \mathbf{F}^H \mathbf{\Lambda}_i \mathbf{w}_i\right]^H \left[\mathbf{d}_j^H \sum_{i=0}^{I-1} \mathbf{\Phi}_i^{(2)} \mathbf{F}^H \mathbf{\Lambda}_q \mathbf{w}_q\right] \\ &= \left[\mathbf{d}_j^H \mathbf{\Phi}_j^{(1)} \mathbf{F}^H \mathbf{\Lambda}_j \mathbf{w}_j + \sum_{i=0, i\ne j}^{I-1} \underbrace{\mathbf{d}_j^H \mathbf{\Phi}_i^{(1)} \mathbf{F}^H \mathbf{\Lambda}_i \mathbf{w}_i}_{\text{MCI}_{j\leftarrow i}}\right]^H \left[\mathbf{d}_j^H \mathbf{\Phi}_j^{(2)} \mathbf{F}^H \mathbf{\Lambda}_j \mathbf{w}_j + \sum_{i=0, i\ne j}^{I-1} \underbrace{\mathbf{d}_j^H \mathbf{\Phi}_i^{(2)} \mathbf{F}^H \mathbf{\Lambda}_i \mathbf{w}_i}_{\text{MCI}_{j\leftarrow i}}\right]. \end{aligned} \tag{7}$$

the primary synchronization sequence (PSS) applied in 3GPP-LTE specification [1] and the proposed sequences. The path loss model in this example is $38.5 + 20\log(r)$ dB [7]. Also, we let the CFO be uniformly distributed between -0.1 and 0.1 (subcarrier spacing), or normally distributed with outage probability $\Pr\{|\text{CFO}| > 0.1\} = 10^{-5}$. The column indices of the proposed sequences are $\{0,\ 4\ ,8\}$ chosen from $\mathbf{W}$. The channel length L is 4.

The estimation accuracy between the proposed sequence and the PSS is shown in Fig. 3 for the user at Location 1, and in Fig. 4 at Location 3. Note that Location 3 is the boundary region of the three cells. Therefore the receiver needs to perform cell detection before CFO estimation.

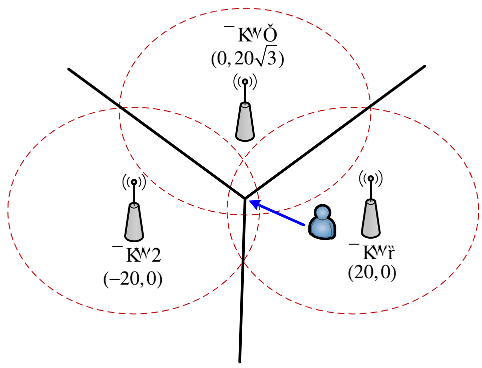

Fig. 2. A multi-cell scenario.

From the simulation results we see that the estimation accuracy of the proposed sequence is much better than that of the PSS at both locations. The improved accuracy is due to that the proposed sequence can completely eliminate the dominant MCI arisen from the CFO, and only leave certain ignorable residual MCI. Moreover, we see from the figure that the performance improvement is more pronounced when the user is at the boundary region of different cells. In this case, the induced MCI is much stronger. Thus, using the proposed sequences can better overcome the MCI issue.

VI. Conclusion

In this paper we proposed a CFO estimation algorithm for multi-cell environments. We can slightly modify the CFO estimation algorithms designed for single-user or single-cell environments and applied them in multi-cell environments. We took a popular CFO estimation for instance, modified and applied it in multi-cell environments. Finally, simulation results showed the analytical results are accurate, and the proposed sequences and CFO estimation algorithms outperform conventional schemes in multi-cell environments. The performance improvement of the proposed sequences is more pronounced when the users is at the boundary regions of several cells, because serious MCI appears in this case. We conclude that with low computational complexity, the proposed sequences and algorithms can efficiently overcome the MCI issue in multi-cell networks.

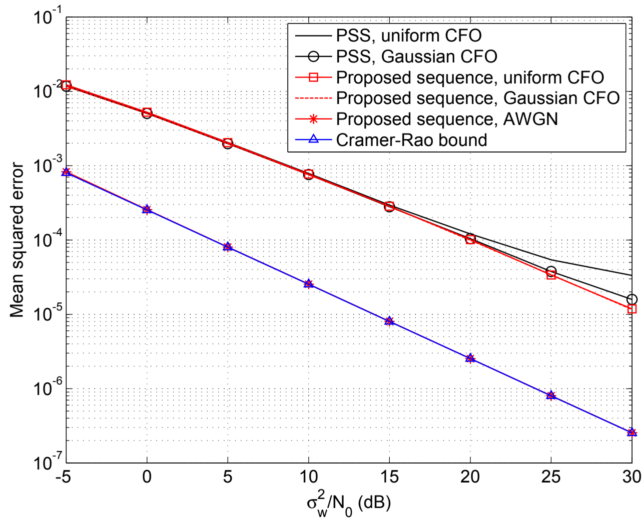

Fig. 3. Multi-cell CFO estimation at Location 1.

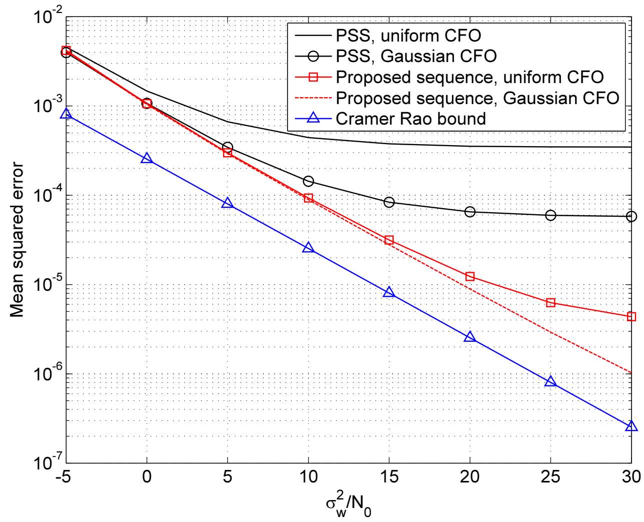

Fig. 4. Multi-cell CFO estimation at Location 3.

References

[1] 3GPP TS36.211 v8.9.0, Physical Channels and Modulation (Release 8), Dec. 2009.

[2] V. Chandrasekhar, J. G. Andrews, and A. Gatherer, "Femtocells networks: a survey," *IEEE Commun. Mag.,* vol. 46, no. 9, pp. 59 - 67, Sep. 2008.

[3] A. Attar, V. Krishnamurthy, and O. N. Gharehshiran, "Interference management using cognitive base station for UMTS LTE," *IEEE Commun. Mag.,* vol. 49, no. 8, pp. 152 - 159, Aug. 2011.

[4] T. M. Schmidl and D. C. Cox, "Robust frequency and timing synchronization for OFDM," *IEEE Trans. Commun.,* vol. 45, no. 12, pp. 1613 - 1621, Dec. 1997.

[5] B. M. Popovic, "Generalized chirp-like polyphase sequence with optimum correlation properties," *IEEE Trans. Inf. Theory,* vol. 38, no. 4, pp. 1406 - 1409, July 1992.

[6] Y. C. Hung, S. Y. Peng and S. H. Tsai, "Sequence designs for interference mitigation in multi-cell networks," *IEEE Trans. Wireless Commun.,* vol. 13, no. 1, pp. 394 - 406, Jan, 2014.

[7] H. Claussen, L. T. W. Ho, and L. G. Samuel, "An overview of the femtocell concept," *Bell Labs Technical Journal,* vol. 13, no. 1, pp. 221 - 245, March 2008.