

Channel Diagonalization Using A Full-rate Space-Time Block Code

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Abstract— A novel full rate space-time block code that diagonalizes the channel matrix without the channel knowledge at the transmitter is proposed in this research. The main advantage of channel diagonalization is that it allows a simple detection scheme at the receiver end. Our method for channel matrix diagonalization consists of two steps. First, we propose a code structure that can transform the channel matrix to a circulant one. Then, the circulant channel matrix can be diagonalized using DFT and IDFT in the transmitter and the receiver, respectively. The proposed scheme not only diagonalizes any channel matrix with an even number of transmit antennas, but also guarantees that the real and imaginary parts of each received symbol are attenuated equally. Simulation results show that the proposed code structure outperforms the \mathcal{G}_4 code about 2 dB in an environment of four transmit and four receive antennas with the bit error probability equal to 10^{-6} .

Keywords— Space-time block code (STBC), full rate, channel diagonalization and MIMO systems.

I. INTRODUCTION

Spatial diversity can be leveraged to combat severe attenuation in wireless communications. To reduce the receiver complexity while maintaining satisfactory performance with spatial diversity, space-time block codes with simple encoding and decoding structures were proposed and extensively studied for the last several years.

The space-time block code was first proposed by Alamouti [1] for a system with two transmit antennas. This is a complex orthogonal design with a full rate. Later, Tarokh *et al.* [2] showed that Alamouti's scheme cannot be extended to more than two antennas. That is, it is impossible to find a full rate complex orthogonal design if the number of the transmit antennas is more than 2. For the case with more than two transmit antennas, research has been conducted along two directions. One is to allow a lower rate code while maintaining the complex orthogonal design [2]. The other is to keep the full rate but sacrifice the orthogonal property [3], [4]. Although the latter approach leads to a higher rate than the former one, it demands a more complicated detection scheme at the receiver end.

Agrawal *et al.* [5] discussed a series of code structures to diagonalize an arbitrary channel matrix without the channel knowledge at the transmitter. Their design was achieved using appropriate linear processing techniques at both the transmitter and the receiver. This scheme leads to symbol-by-symbol detection and greatly reduces the detection complexity. However, since the real part of the channel matrix is extracted via code design, only real symbols are allowed to be transmitted. Even if k -dimensional complex symbols are transmitted using $2k$ -dimensional

real symbols, it may make the real and imaginary parts of any received complex symbol be attenuated unequally to result in performance degradation. Furthermore, an explicit code construction procedure was not stated in [5]. Here, by following the same direction given by [5], we present a specific procedure to construct a code that diagonalizes the channel matrix without channel knowledge at the transmitter. Furthermore, the proposed code scheme attenuates the real and imaginary parts of the transmit symbols equally.

Our method for channel matrix diagonalization consists of two steps. First, we propose a code structure that can transform the channel matrix to a circulant one. Then, the circulant channel matrix can be diagonalized using DFT and IDFT in the transmitter and the receiver, respectively. The proposed scheme not only diagonalizes any channel matrix with an even number of transmit antennas, but also guarantees that the real and imaginary parts of each received symbol are attenuated equally. Since the channel matrix is diagonalized, symbols under detection are uncorrelated with each other. When additive noise is also uncorrelated, symbol-by-symbol detection gives the optimal detection performance. Consequently, the proposed code can greatly reduce the decoder complexity. It is worthwhile to point out that the proposed full rate code can be easily extended for an even number of transmit antennas. This result is contrary to that of the non-full rate orthogonal design in [2], where it is claimed to be difficult to construct codes with a rate greater than 1/2 when the number of transmit antennas is greater than 4. Moreover, since our code construction is a square matrix, the delay latency of the proposed code scheme will be shorter than that of the non-full rate orthogonal design in [2] if the number of transmit antennas is the same.

The rest of the paper is organized as follows. The proposed space-time block coding system is described in Sec. II. The property of the diagonalized channel is discussed in Sec. III. Experimental results are given in Sec. IV to demonstrate the superior performance of the proposed code structure. In particular, we show that the proposed space-time block code outperforms the \mathcal{G}_4 code [2] by about 2 dB at an bit error rate of 10^{-6} in an environment of four transmit antennas and four receive antennas. Finally, concluding remarks and future research work are given in Sec. V.

II. PROPOSED SPACE-TIME CODING SYSTEM

The block-diagram of the proposed space-time block coding system is shown in Fig. 1, which has n transmit

antennas and m receive antennas. We assume n is even throughout the article.

A. Transmitter Design

As shown in the figure, the transmitter contains two stages. The input vector \mathbf{v} of dimension $n \times 1$ consists of n modulation symbols. In the first stage, we apply the $n \times n$ discrete Fourier transform (DFT) to \mathbf{v} . The element in the p th row and the q th column defined by the DFT matrix can be written as

$$[\mathbf{F}]_{p,q} = \frac{1}{\sqrt{n}} e^{-j \frac{2\pi}{n} pq}. \quad (1)$$

To describe the encoding procedure in the second stage conveniently, we define a new vector $\bar{\mathbf{x}}$ of dimension $n \times 1$ with its k th element given by

$$\bar{x}[k] = \begin{cases} x^*[k], & |k = 0, 2, \dots, n-2 \\ -x^*[k], & |k = 1, 3, \dots, n-1 \end{cases}. \quad (2)$$

Let the time index be t , where $t = 0, 1, \dots, n-1$ and $x \bmod y$ denote x modulo y . The $n \times 1$ output vector \mathbf{x} of the DFT matrix is encoded as follows.

1. When t is even, we transmit the k th element of \mathbf{x} by the antenna indexed $[(k-t) \bmod n]$.
2. When t is odd, we transmit the k th element of $\bar{\mathbf{x}}$ by the antenna indexed $[(t-k) \bmod n]$.

Example: Let $n = 6$, the 6×6 space-time code construction is given by

$$\mathbf{C}_6 = \begin{pmatrix} x[0] & x[1] & x[2] & x[3] & x[4] & x[5] \\ -x^*[1] & x^*[0] & -x^*[5] & x^*[4] & -x^*[3] & x^*[2] \\ x[2] & x[3] & x[4] & x[5] & x[0] & x[1] \\ -x^*[3] & x^*[2] & -x^*[1] & x^*[0] & -x^*[5] & x^*[4] \\ x[4] & x[5] & x[0] & x[1] & x[2] & x[3] \\ -x^*[5] & x^*[4] & -x^*[3] & x^*[2] & -x^*[1] & x^*[0] \end{pmatrix}.$$

For a general code construction \mathbf{C}_n , the meaning of the element is explained below. The column of \mathbf{C}_n indicates the antenna by which the symbol is transmitted, *i.e.* the space dimension. The row of \mathbf{C}_n indicates the time slot in which the symbol is transmitted, *i.e.* the time dimension. Thus, the element $[\mathbf{C}_n]_{p,q}$ denotes the symbol transmitted by the q th antenna at the p th time slot. Note that, when $n = 2$, the proposed code structure is the same as the Alamouti's scheme [1]. The reason of using DFT and the proposed code structure will become clear later. After space-time encoding, the symbols are transmitted to channel by n antennas.

B. Channel Model

Next, let us examine the channel model. Here, we assume that channels are flat, *i.e.* the channel impulse response is only one-tap. Let $h_{i,j}$ denote the channel impulse response from the i th transmit antenna to the j th receive antenna. After symbols pass through the flat channels, there exists additive noise. We use $e_j[k]$ to denote the k th additive noise received by the j th antenna.

C. Receiver Design

The receiver consists of four stages. In the first stage, the received symbols are buffered up to n symbols and then converted from serial to parallel (S/P). Let the $n \times 1$ S/P converted vector received from the j th antenna denoted by \mathbf{y}_j . In the second stage, \mathbf{y}_j will pass through a selective "symbol conjugate" operation and its output is denoted by \mathbf{z}_j . The relation between the k th elements of \mathbf{z}_j and \mathbf{y}_j is given by

$$z_j[k] = \begin{cases} y_j[k], & |k = 0, 2, \dots, n-2 \\ y_j^*[k], & |k = 1, 3, \dots, n-1 \end{cases}, \quad (3)$$

where y^* is the complex conjugate of y . If the proposed code structure is used, one can verify from (3) that the relation between \mathbf{z}_j and \mathbf{x} is

$$\mathbf{z}_j = \mathbf{H}_j \mathbf{x} + \tilde{\mathbf{e}}_j, \quad (4)$$

where

$$\mathbf{H}_j = \begin{pmatrix} h_{0,j} & h_{1,j} & h_{2,j} & h_{3,j} & \dots & h_{n-1,j} \\ h_{1,j}^* & -h_{0,j}^* & h_{n-1,j}^* & -h_{n-2,j}^* & \dots & -h_{2,j}^* \\ h_{n-2,j} & h_{n-1,j} & h_{0,j} & h_{1,j} & \dots & h_{n-3,j} \\ h_{3,j}^* & -h_{2,j}^* & h_{1,j}^* & -h_{0,j}^* & \dots & -h_{4,j}^* \\ \vdots & \vdots & \vdots & \vdots & \ddots & \vdots \\ h_{2,j} & h_{3,j} & h_{4,j} & h_{5,j} & \dots & h_{1,j} \\ h_{n-1,j}^* & -h_{n-2,j}^* & h_{n-3,j}^* & -h_{n-4,j}^* & \dots & -h_{0,j}^* \end{pmatrix}, \quad (5)$$

and

$$\tilde{\mathbf{e}}_j[k] = \begin{cases} e_j[k], & |k = 0, 2, \dots, n-2 \\ e_j^*[k], & |k = 1, 3, \dots, n-1 \end{cases}. \quad (6)$$

We can express the element in the p th row and the q th column of \mathbf{H}_j using a more general expression given by

$$[\mathbf{H}_j]_{p,q} = \begin{cases} h_{(q-p) \bmod n,j} & , & |p = 0, 2, \dots, n-2 \\ (-1)^q h_{(p-q) \bmod n,j}^* & , & |p = 1, 3, \dots, n-1 \end{cases}. \quad (7)$$

In the third stage, \mathbf{z}_j is passed through a matched filter, which can be written as

$$\mathbf{u}_j = \mathbf{H}_j^\dagger \mathbf{z}_j, \quad (8)$$

where \mathbf{H}^\dagger is the transposition plus the complex conjugate operation of \mathbf{H} . In the fourth stage, \mathbf{u}_j is passed through the $n \times n$ IDFT matrix to result in the output vector $\hat{\mathbf{v}}_j$. Based on $\hat{\mathbf{v}}_j$, we can reconstruct \mathbf{v} by combining $\hat{\mathbf{v}}_j$, $j = 0, 1, \dots, m-1$, using techniques such as the equal gain combining (EGC) or the maximum ratio combining (MRC).

III. CHANNEL DIAGONALIZATION

In this section, we show that the space-time coding system described in Sec. II is able to diagonalize the channel matrix $\mathbf{H}_j^\dagger \mathbf{H}_j$, and this property holds for an arbitrary positive even n . The diagonalization of the channel matrix can greatly reduce the detection complexity, thus leading to a simple decoder [5].

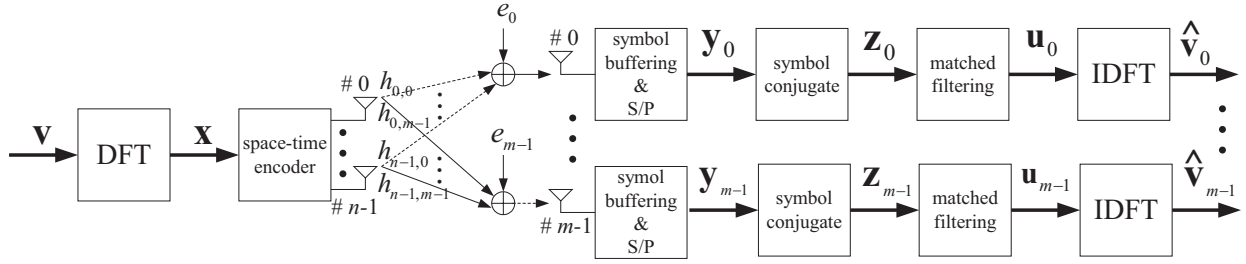


Fig. 1. The block-diagram of the proposed space-time block coding system.

A. Channel Diagonalization Proof

Let us examine the reconstructed symbols $\hat{\mathbf{v}}_j$. Based on (4) and (8), the relation between $\hat{\mathbf{v}}_j$ and \mathbf{v} can be written as

$$\hat{\mathbf{v}}_j = \mathbf{F}^\dagger \mathbf{H}_j^\dagger \mathbf{H}_j \mathbf{F} \mathbf{v} + \mathbf{F}^\dagger \mathbf{H}_j^\dagger \tilde{\mathbf{e}}_j. \quad (9)$$

It is well known [6] that, if $\mathbf{H}_j^\dagger \mathbf{H}_j$ in (9) is a circulant matrix, $\mathbf{F}^\dagger \mathbf{H}_j^\dagger \mathbf{H}_j \mathbf{F}$ is a diagonal matrix. A direct consequence is that $\hat{\mathbf{v}}_j[k]$ and $\hat{\mathbf{v}}_j[k']$ are uncorrelated for $k \neq k'$ and the symbol-by-symbol detection scheme is the optimal solution under such a situation.

In the following, we will prove that, if the proposed code structure is used for any positive even n , the matched channel matrix $\mathbf{H}_j^\dagger \mathbf{H}_j$ is circulant. For convenience, let us temporarily ignore subscript j in \mathbf{H}_j and $h_{i,j}$. From (7), the element in the r th row and the s th column of $\mathbf{H}^\dagger \mathbf{H}$ can be written as

$$\begin{aligned} [\mathbf{H}^\dagger \mathbf{H}]_{r,s} &= \sum_{l=0,2,\dots}^{n-2} [\mathbf{H}]_{l,r}^* [\mathbf{H}]_{l,s} + \sum_{l=1,3,\dots}^{n-1} [\mathbf{H}]_{l,r}^* [\mathbf{H}]_{l,s} \\ &= \underbrace{\sum_{l=0,2,\dots}^{n-2} h_{(r-l) \bmod n}^* h_{(s-l) \bmod n}}_{\mathcal{E}} \\ &\quad + \underbrace{\sum_{l=1,3,\dots}^{n-1} (-1)^{r+s} h_{(l-r) \bmod n} h_{(l-s) \bmod n}^*}_{\mathcal{O}}. \end{aligned} \quad (10)$$

Let $c = r - s$. The summation of the even part in (10) is

$$\mathcal{E} = \sum_{l=0,2,\dots}^{n-2} h_{(r-l) \bmod n}^* h_{(r-l-c) \bmod n}. \quad (11)$$

Let $f = r - l$, we can rewrite (11) as

$$\mathcal{E} = \begin{cases} \sum_{f=0,2,\dots}^{n-2} h_f^* h_{(f-c) \bmod n} & | r \text{ even} \\ \sum_{f=1,3,\dots}^{n-1} h_f^* h_{(f-c) \bmod n} & | r \text{ odd} \end{cases}. \quad (12)$$

Next, let us examine the summation of the odd part in (10). Let $c = r - s$. We have

$$\mathcal{O} = (-1)^{r+s} \sum_{l=1,3,\dots}^{n-1} h_{(l-s) \bmod n}^* h_{(l-s-c) \bmod n}. \quad (13)$$

Let $g = l - s$, we can rewrite (13) as

$$\mathcal{O} = \begin{cases} -\sum_{g=0,2,\dots}^{n-2} h_g^* h_{(g-c) \bmod n} & | r \text{ even, } s \text{ odd} \\ \sum_{g=1,3,\dots}^{n-1} h_g^* h_{(g-c) \bmod n} & | r \text{ even, } s \text{ even} \\ \sum_{g=0,2,\dots}^{n-2} h_g^* h_{(g-c) \bmod n} & | r \text{ odd, } s \text{ odd} \\ -\sum_{g=1,3,\dots}^{n-1} h_g^* h_{(g-c) \bmod n} & | r \text{ odd, } s \text{ even} \end{cases} \quad (14)$$

Based on (10), (12) and (14), it is straightforward to show that the element in the p th row and the q th column of $\mathbf{H}_j^\dagger \mathbf{H}_j$ is

$$[\mathbf{H}_j^\dagger \mathbf{H}_j]_{r,s} = \begin{cases} \sum_{i=0}^{n-1} |h_i|^2, & | r - s = 0 \\ \sum_{i=0}^{n-1} h_{i,j}^* h_{(i-r+s) \bmod n,j} & | r - s \text{ even} \\ 0, & | r - s \text{ odd} \end{cases}. \quad (15)$$

As shown in (15), since the element of $\mathbf{H}_j^\dagger \mathbf{H}_j$ only depends on the difference between the row and column indexes, *i.e.* $r - s$, $\mathbf{H}_j^\dagger \mathbf{H}_j$ is a circulant matrix for any positive even n . Note that we do not impose any constraint on $h_{i,j}$ in the derivation. Thus, Eqn. (15) holds for arbitrary flat fading channels $h_{i,j}$.

Since $\mathbf{H}_j^\dagger \mathbf{H}_j$ is circulant, it can be diagonalized by DFT and IDFT matrices. Thus Eqn. (9) becomes

$$\hat{\mathbf{v}}_j = \mathbf{\Lambda}_j \mathbf{v} + \hat{\mathbf{e}}_j, \quad (16)$$

where $\hat{\mathbf{e}}_j = \mathbf{F}^\dagger \mathbf{H}_j^\dagger \tilde{\mathbf{e}}_j$ and $\mathbf{\Lambda}_j$ is a diagonal matrix whose diagonal elements are the DFT of the first column of $\mathbf{H}_j^\dagger \mathbf{H}_j$ [6]. Let $\lambda_j[k]$ be the k th element of $\mathbf{\Lambda}_j$. We have

$$\mathbf{\Lambda}_j = \text{diag}(\lambda_j[0], \lambda_j[1], \dots, \lambda_j[n-1]),$$

where the last $n/2$ elements of $\lambda_j[k]$ is the same as its first $n/2$ elements, *i.e.* $\lambda_j[n/2+l] = \lambda_j[l]$, $l = 0, 1, \dots, n/2-1$. This is because that, the odd-indexed elements of the first column of $\mathbf{H}_j^\dagger \mathbf{H}_j$ is zero according to (15). Thus, it can be viewed as applying the 1:2 upsampling to a sequence consisting of even-indexed elements. The DFT of such an input yields a repeated output.

B. Symbol Detection and Pre-processing

As shown in (16), the received data $\hat{v}_j[k]$ consists of two parts: the attenuated signal and the additive noise. Since $\mathbf{\Lambda}_j$ is diagonal, the attenuated symbols are uncorrelated.

Moreover, if $e_j[k]$ is the white Gaussian noise with variance σ^2 , $\hat{e}_j[k]$ is also white Gaussian noise with variance $\lambda_j[k]\sigma^2$. That is, the received noise is also uncorrelated. Under such a scenario, the symbol-by-symbol detection provides the optimal solution, and the decoding complexity can be greatly reduced. If there is no additive noise, $v[k]$ can be perfectly reconstructed by multiplying $\hat{v}_j[k]$ with $\lambda_j^{-1}[k]$.

Let us examine the SNR of each reconstructed symbol. From (16), we have the SNR of $\hat{v}_j[k]$ as

$$SNR_j[k] = \frac{\lambda_j[k]\sigma_v^2[k]}{\sigma^2}, \quad (17)$$

where $\sigma_v^2[k]$ is the variance of $v[k]$. Since the transmitter does not have the channel information, flat input power is used. Thus, $\sigma_v^2[k] = \sigma_v^2$ for $0 \leq k \leq n-1$. It is possible that some $\lambda_j[k]$ in (17) are much smaller than others. This may lead to a serious symbol attenuation effect. This occurs when $h_{i,j}$ are highly correlated for a fixed j and $0 \leq i \leq n-1$. An extreme case is that $h_{i,j}$, $0 \leq i \leq n-1$, are all real and equal for given j . Thus, $\lambda_j[k] \neq 0$ for $k = 0, n/2 - 1$, but $\lambda_j[k] = 0$ for other values of k . The zero eigenvalue will make the corresponding attenuated symbols undetectable.

The situation can be avoided by randomly multiplying the input symbol with a phase term before transmission. This can reduce the channel correlation effectively. Another method to reduce the probability of serious attenuation of received symbols is the use of the receive antenna diversity. When the MRC rule is applied to multiple receive antennas, the overall SNR contributed from all m receive antennas is given by

$$\widetilde{SNR}[k] = \frac{\sigma_v^2[k]}{\sigma^2} \sum_{j=0}^{m-1} \lambda_j[k]. \quad (18)$$

As given above, the eigenvalues from different receive antennas are summed up so that the probability for one symbol to attenuate seriously become smaller than the case of using data received by only one antenna.

IV. SIMULATION RESULTS

In this section, we evaluate the performance of the proposed space-time block code and the benchmark orthogonal space-time block codes for more than 2 transmit antennas given in [2]. The simulation parameters are chosen as follows. The QAM constellation is used. The transmission power of each antenna is normalized to the same value. Thus, each transmit antenna shares $1/n$ th of the overall transmission power. Furthermore, flat channels are assumed, where the amplitudes are i.i.d. Rayleigh distributed and the phases are uniformly distributed. The Monte Carlo method is adopted to run more than 10^5 trials of different channel matrices. For the multiple receive antenna case, MRC is used to optimize the performance. The SNR is defined as $SNR = \sigma_v^2 / \sigma^2$.

Let us first fix the bit rate to 2 bits/sec/Hz. For the proposed code structure, since it is a rate 1 code, 4-QAM is used to achieve the given bit rate. For the rate $1/2$ \mathcal{G}_3 and \mathcal{G}_4 codes [2], 16-QAM is used to achieve the given bit rate. The performance measure in terms of the bit error rate for 1, 2 and 4 receive antennas is shown in Figs. 2, 3 and 4, respectively. Note that experimental results for cases of uncoded signals transmitted with one antenna, the \mathcal{G}_3 coded signal transmitted with three antennas and the \mathcal{G}_4 coded signal transmitted with four antennas are shown in all these figures for performance comparison. The results of uncoded, \mathcal{G}_3 coded and \mathcal{G}_4 coded signals given in Fig. 2 are consistent with those data given in [7].

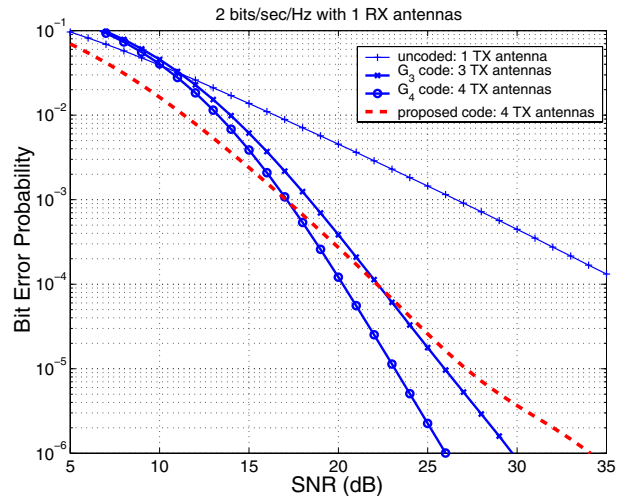


Fig. 2. The bit error rate versus SNR for space-time block codes at 2 bit/sec/Hz with one receive antenna.

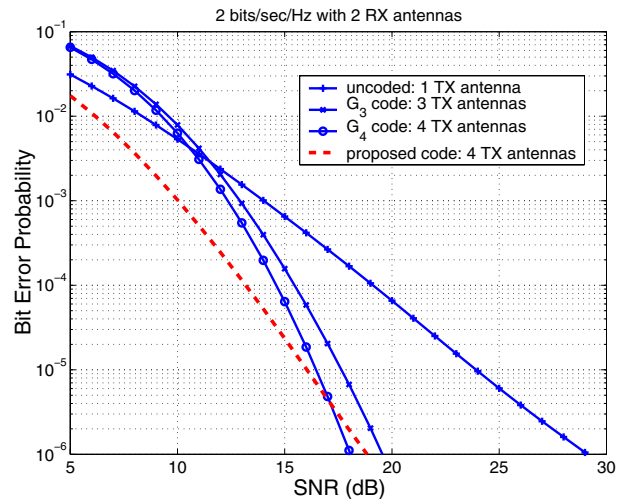


Fig. 3. The bit error rate versus SNR for space-time block codes at 2 bit/sec/Hz with two receive antennas.

For the 1 receive antenna case, as compared with the benchmark \mathcal{G}_4 code [2], we see that the proposed space-

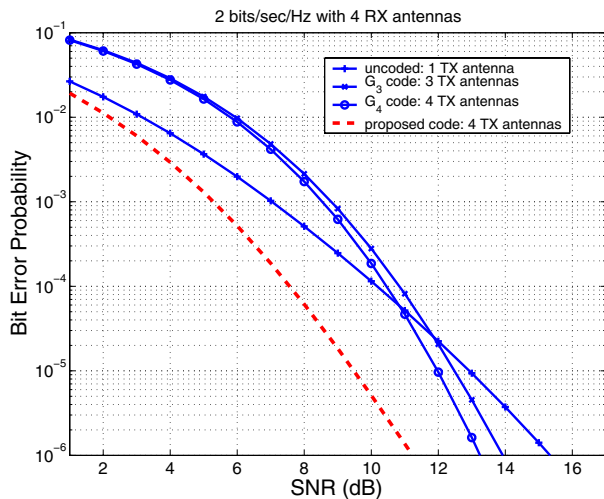


Fig. 4. The bit error rate versus SNR for space-time block codes at 2 bit/sec/Hz with four receive antennas.

time coding system has better performance when the SNR is low (*i.e.* lower than 17dB), but worse performance where the SNR is high. This can be explained as follows. When the SNR is low, the full transmission rate is more important [4] so that the proposed code outperforms the half-rate \mathcal{G}_4 code. However, as discussed in Sec. III, the performance of the proposed code is dominated by the minimum eigenvalue of $\mathbf{H}^\dagger \mathbf{H}$. When SNR is high, the symbol error rate corresponding to the minimum eigenvalues will dominate the performance. Thus, our system has worse performance than the \mathcal{G}_3 and \mathcal{G}_4 codes.

When the number of receive antennas becomes 2 and 4 (whose performance is shown in Figs. 3 and 4, respectively), we see that the proposed code has a lower bit error rate than that of the \mathcal{G}_4 code. The performance gap becomes more significant as the number of receive antennas increases. When the receive antenna number is 4, the proposed code outperforms the \mathcal{G}_4 code by about 2 dB at a bit error probability of 10^{-6} . This result confirms our discussion in Sec. III that the receive antenna diversity will mitigate the serious performance degradation caused by the minimum eigenvalue of $\mathbf{H}^\dagger \mathbf{H}$. That is, as the number of receive antennas increases, the ratio between the maximum and the minimum eigenvalues $\mathbf{H}^\dagger \mathbf{H}$ decreases. Thus, it implies that symbols are attenuated by eigenvalues of a more similar value.

Finally, let us examine another example for the higher bit rate case. For a higher level modulation scheme with 4 bits/sec/Hz, we plot the bit error rate as a function of the SNR for several transmitting systems with 4 receive antennas in Fig. 5. Again, the proposed space-time block coding system outperforms the \mathcal{G}_4 code by about 2 dB at a bit error probability of 10^{-6} .

By comparing Figs. 4 and 5, the performance degrades about 6.5dB (from 11dB to 17.5dB) for the proposed code. This is reasonable since it demands about 3 dB increase

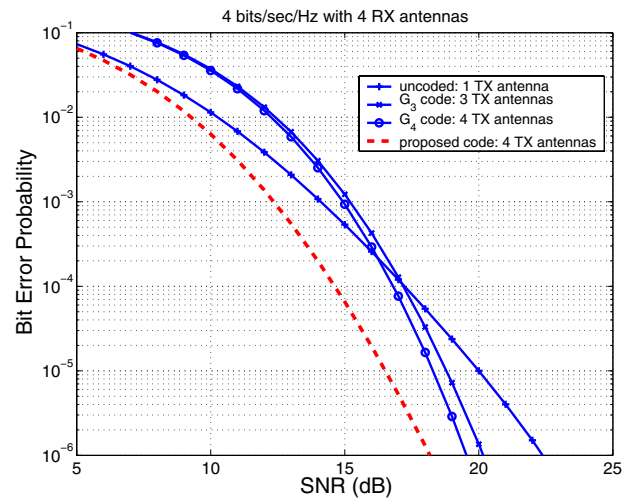


Fig. 5. The bit error rate versus SNR for space-time block codes at 4 bit/sec/Hz with four receive antennas.

per bit for the QAM modulation. For higher bit rate transmission, the higher SNR is needed to maintain the same bit error rate. However, the higher SNR means higher transmission power so that the complexity of the power amplifier increases as well. In this situation, the combined transmitter and receiver diversity can effectively reduce the overall transmit power while maintaining a low bit error rate. This offers an additional advantage for the proposed full rate code since it can be easily extended for any positive even number n of transmit antennas. In contrast, the complex orthogonal design with a rate greater than $1/2$ is difficult to construct for $n > 4$ [2].

V. CONCLUSION AND FUTURE WORK

A specific space-time block code that can diagonalize the channel matrix for any even number of transmit antennas was constructed in this work. This system enables a simple decoding procedure. We will continue to examine the diversity order and the theoretical performance of the proposed code scheme in the future.

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