Codeword Length Optimization for CPPUWB Systems

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Abstract— **The optimal codeword length that generates the maximum output signal to interference ratio (SIR) for the channel-phase-precoded ultra-wideband (CPPUWB) system proposed in [1] is studied in this work to mitigate the interference in high data rate transmission. As compared with the proposed MMSE receiver in [1], this code length optimization technique demands no additional training symbols, which sacrifices the actual data rate, and maintains a simple receiver structure. This optimization problem is highly nonlinear in nature, and a fast search algorithm is developed to speed up the optimization process. The signal to interference plus noise ratio (SINR) performance of the proposed scheme degrades slightly when the input SNR is low, and it will converge to the maximum SINR as the input SNR becomes large.**

I. Introduction

The ultra wideband (UWB) communication channel has a large number of multipath components due to its exceptional temporal resolution. The multipath diversity provides an effective means to combat channel fading since the probability that all paths experience deep fading simultaneously is low. However, a large number of correlation operations are required to exploit the multipath diversity [2], and the complexity is a main concern in the UWB receiver design.

The time-reversal prefiltering (TRP) technique was proposed by Strohmer *et al.* [3] to address the receiver complexity issue in UWB. With TRP, the transmit signal convolves with the time-reversed version of the channel impulse response (CIR) as a prefiltering operation at the transmitter. Then, the received signal power can be more concentrated at the receiver end to allow a simpler detection scheme, where the number of correlation operations can be greatly reduced. However, the transmitter of TRP demands complete channel information, including both the amplitude and the phase of tap coefficients. They have to be estimated at the receiver and then sent back to the transmitter through a feedback channel. Since the UWB channel has a large number of channel taps, it is expensive to estimate the complete channel information and send it back to the transmitter.

A new UWB communication architecture, called the channel phase precoded UWB (CPPUWB), was proposed in [1] to overcome the drawback of the TRP-based UWB system (TRPUWB). The CPPUWB system encodes the antipodal data symbol with a channelized codeword, which is chosen to be the time-reversed order of the fedback channel phase sequence. Note that the phase of each tap coefficient of the channel model takes only binary values, *i.e.*, 1 or -1. When the estimated phase information is correct, it is apparent that there will be a strong receive signal peak since all responses contributed by the same transmit data symbol are coherently summed. In contrast, the amplitudes of off-peak received signals are much weaker since these responses are combined incoherently. To detect the transmit data, the receiver can sample the peak value of the received signal and perform a simple thresholding operation. Since CPPUWB demands only the phase information of the channel, the complexity in channel estimation can be greatly reduced. Moreover, the transmission cost in channel information feedback can be greatly reduced, where only one bit is needed to represent each tap's phase.

Because CPPUWB can mitigate intersymbol interference (ISI) by concentrating the received signal power, the symbol duration can be reduced to achieve a faster data rate for some fixed noise margin. As the symbol duration becomes smaller at a higher rate, the ISI effect becomes more severe. The peak value used for symbol detection may suffer from ISI and degrade the detection performance. To improve the performance, we may consider the use of more sophisticated receivers to suppress ISI, *e.g.*, the minimum mean square error (MMSE) receiver. Under this case, additional training symbols are demanded to determine the filter coefficients of the MMSE receiver. Moreover, since the MMSE filter contains multiple taps, the decoding complexity is higher than the original CPPUWB receiver, which contains only one single tap [1].

A new ISI mitigation scheme for CPPUWB, called codeword length optimization, is proposed in this work. It does not demand additional training symbols and has a lower decoding complexity as compared with the MMSE receiver. The main idea can be described as follows. Even though it is possible to accumulate more power at a peak in the received signal corresponding to a data symbol with a longer codeword, the long codeword also increases off-peak values to result in interference with neighboring symbols. Thus, the performance of a long codeword may not necessarily better than that of a shorter one. It is interesting to see whether there exists an

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optimal length of the codeword that provides the maximum output signal to interference ratio (SIR) for some fixed data rate.

In addition, using this optimal length can further reduce the feedback overhead. Since this optimization problem is nonlinear in nature, we may not provide a close form solution. However, an exhaustive search algorithm is computationally intensive and not suitable for UWB systems which demand a transceiver of low complexity. In this work, we develop a fast search algorithm to speed up the optimization process and reduce the computational complexity. The proposed fast search scheme can achieve a SINR value that is comparable to that based on exhaustive search. Thus, there is little degradation in the overall system performance.

II. System Model

The proposed system targets at a single user in an indoor data communication environment, where a short range and high data rate communication based on the UWB technology [4] is used. Here, we adopt a simplified UWB channel model proposed in [5], which can be expressed in the following form:

$$h(t) = \sum_{i=0}^{L-1} h_i \delta(t - i\Delta) = \sum_{i=0}^{L-1} p_i \alpha_i \delta(t - i\Delta), \quad (1)$$

where $h_i = p_i \alpha_i$, L is the total number of paths, Δ is the multipath resolution that is assumed to be the same as the time domain pulse width, $p_i \in \{+1, -1\}$ with equal probability is the phase of the ith path, and α_i is the corresponding amplitude which is an independent Rayleigh random variable with the following probability density function (PDF):

$$f_{\alpha_i}(x) = \frac{x}{\sigma_i^2} e^{-x^2/2\sigma_i^2}.$$

The average power of α_i, which is equal to $2\sigma_i^2$, is subject to exponential decay, *i.e.,*

$$E\{\alpha_i^2\} = 2\sigma_i^2 = \Omega \gamma^i, \quad (2)$$

where $E\{\}$ is the expectation operator, Ω is the average power of α_0 and $\gamma \equiv e^{-\Delta/\Gamma}$ where Γ is the decay time constant and $\Gamma > \Delta$.

At the initial stage, the receiver estimates the channel phase information of each tap, which is either $+1$ or -1. The estimated channel phase information $\hat{\mathbf{p}}$ is then fed back to the transmitter as a channelized codeword $\mathbf{c}$, which is of L-chip and unit-power. Let c_k denote the kth component of $\mathbf{c}$. Then, we have

$$\mathbf{c}_L = [c_0, \cdots, c_{L-1}]^T = \frac{1}{\sqrt{L}} [\hat{p}_{L-1}, \cdots, \hat{p}_0]^T.$$

Next, the transmitter encodes each of the bipolar data symbol, $b(i)$, with the channelized codeword. A pulse generator modulates the UWB pulse waveform $w_s(t)$ onto each chip. Therefore, the transmit signal can be expressed as

$$x_s(t) = \sum_{i=-\infty}^{\infty} b(i) \sum_{j=0}^{L-1} c_j w_s(t - j\Delta - iT_s),$$

where $b(i) \in \{+1, -1\}$ is the ith bipolar signal, $w_s(t)$ is the transmit pulse waveform, and T_s is the symbol interval, which is properly chosen to reduce the ISI effect.

Based on the channel model in (1), the discrete received signal for the ith data symbol $b(i)$ after chip-matched filtering and sampling can be written as

$$\mathbf{r}_L(i) = [r_{L,0}(i), \cdots, r_{L,2L-2}(i)]^T = \mathbf{H}_L \mathbf{c}_L b(i) + \mathbf{I}_L(i) + \mathbf{n}_L(i), \quad (3)$$

where $\mathbf{H}_L$ is a $(2L-1) \times L$ Toeplitz matrix whose first column contains $\mathbf{h} = [h_0, h_1, \cdots, h_{L-1}]^T$ from the first to the Lth elements and zeros elsewhere, $\mathbf{I}_L(i) = [I_{L,0}(i), \cdots, I_{L,2L-2}(i)]^T$ is the interference vector that contains ISI, and $\mathbf{n}_L(i) = [n_{L,0}(i), \cdots, n_{L,2L-2}(i)]^T$ is the additive white Gaussian noise (AWGN) vector whose mean is zero and the covariance matrix equals to $\frac{N_0}{2}\mathbf{I}_{2L-1}$, where $\mathbf{I}_{2L-1}$ is a $(2L-1) \times (2L-1)$ identity matrix.

Let $\mathbf{h}_L = [h_{L,0}, \cdots, h_{L,2L-2}]^T = \mathbf{H}_L \mathbf{c}_L$. When the phase estimation is perfect, *i.e.,* $\mathbf{p} = \hat{\mathbf{p}}$, we can expect that

$$\max_j h_{L,j} = \sum_{i=0}^{L-1} \frac{\alpha_i}{\sqrt{L}}$$

occurs at $j = L-1$ since all channel taps are coherently combined. To detect the transmit symbol $b(i)$, we simply apply the decision rule to $r_{L,L-1}(i)$ in (3), *i.e.,*

$$\hat{b}(i) = \text{sign}\{r_{L,L-1}(i)\}.$$

Although the maximum ratio combining (MRC) receiver that combines the peak as well as the off-peak received signals corresponding to the same transmit symbol can enhance the output SNR, it increases the computational cost, too. Please note that MRC demands the amplitude information, and additional training symbols are required to estimate the received amplitude.

III. Codeword Length Optimization

A. Problem Formulation

We consider the case where the peak value of the received signal for symbol detection contains ISI caused by the previous and the following symbols, *i.e.,* $T_s < L\Delta$. We assume that symbol interval is an integer multiple of the time domain pulse width, *i.e.,* $T_s = M\Delta$, where M is a positive integer and $M < L$. Although a longer codeword can combine more channel taps at the peak to achieve a higher peak value, it also leads to the extension of the off-peak signals which may cause more serious ISI. In high data rate transmission, neighboring symbols interfere each other seriously. This serious ISI causes larger impairment than AWGN and will dominate the system performance. Thus, we need to use some interference suppression schemes to combat ISI. In [1], simulation results show that the MMSE receiver can enhance the system performance significantly in high data rate transmission. However, it also increases receiver complexity. Furthermore, more training symbols are demanded to track the MMSE coefficients and the actual data rate is reduced in this case.

An alternative to enhance the system performance while maintaining a simple receiver structure is to optimize the codeword length. When the symbol interval is fixed, there exists an optimal codeword length such that the output signal to interference power ratio (SIR) at the peak value of the received signal is maximized. When the optimal codeword length is less than the channel length, the feedback information can be reduced since we only need to send back the phase tap information partially according to the optimal codeword length. In the following, we assume that the feedback phase information is perfect, *i.e.,* $\hat{\mathbf{p}} = \mathbf{p}$, and the length of the channel impulse response is an integer multiple of the symbol interval, *i.e.,* $L = KM$, where K is a positive integer. Let $L \equiv \Gamma\delta/\Delta$ be an integer and parameter δ can be used to select the effective length of the channel (in the sense that those taps whose power ratio with respect to the first tap is less than $\gamma^L (= e^{-\delta})$ are ignored). Therefore, the second assumption is not as restrictive as it appears, since we can adjust the value of δ so that the channel impulse response meets this requirement. This assumption is made to facilitate the presentation of the proposed fast optimal codeword length selection algorithm to be given in the next subsection.

Let the length of the codeword $\mathbf{c}_l$ be l, *i.e.,* $\mathbf{c}_l = [c_0, \cdots, c_{l-1}]^T = \frac{1}{\sqrt{l}}[p_{l-1}, \cdots, p_0]^T$. The input-output relationship in (3) can be remodelled as

$$\begin{aligned}\mathbf{r}_l(i) &= [r_{l,0}(i), \cdots, r_{l,L+l-2}(i)]^T \\ &= \mathbf{H}_l \mathbf{c}_l b(i) + \mathbf{I}_l(i) + \mathbf{n}_l(i) \\ &= \mathbf{h}_l b(i) + \mathbf{I}_l(i) + \mathbf{n}_l(i),\end{aligned}$$

where $\mathbf{h}_l = \mathbf{H}_l \mathbf{c}_l = [h_{l,0}, \cdots, h_{l,L+l-1}]^T$, $\mathbf{H}_l$ is the $(L+l-1) \times l$ Toeplitz matrix, $\mathbf{I}_l(i)$ and $\mathbf{n}_l(i)$ are the interference and AWGN, respectively. The maximum signal power occurs at $h_{l,l-1}(i)$ and the average output SIR at $r_{l,l-1}(i)$ in this case is

$$\bar{\nu}(l) = \frac{E\{h_{l,l-1}^2\}}{\sum_{j=1}^{L_1} E\{h_{l,l+jM-1}^2\} + \sum_{j=1}^{L_2} E\{h_{l,l-jM-1}^2\}}, \quad (4)$$

where $E\{\}$ is the expectation operator, $L_1 = \lfloor (L-1)/M \rfloor$, $L_2 = \lfloor (l-1)/M \rfloor$, and $\lfloor x \rfloor$ is the floor function of x. The optimized codeword length $\bar{L}$, can be determined via

$$\bar{L} = \arg \max_{0<l\leq L} \bar{\nu}(l). \quad (5)$$

Note that we may not be able to get a closed-form solution to the above optimization problem due to its nonlinear nature. One way to solve this problem is to perform an exhaustive search for all possible values of $l \in \{1, \cdots, L\}$ and pick up the one with the largest SIR value. Since the UWB channel usually has a huge number of taps, this method is computationally expensive. As a result, this scheme is not suitable for a simple receiver, whose computational power is usually limited. In what follows, we develop a fast search algorithm for the optimal value of $\bar{L}$ such that the output SIR at the peak received signal is maximized.

When AWGN and ISI are considered jointly, the codeword length optimization problem, which selects the best codeword length to maximize the output SINR, can be stated as

$$\bar{L} = \arg \max_{0<l\leq L} \frac{E\{h_{l,l-1}^2\}}{\frac{N_0}{2} + \sum_{j=1}^{L_1} E\{h_{l,l+jM-1}^2\} + \sum_{j=1}^{L_2} E\{h_{l,l-jM-1}^2\}}. \quad (6)$$

In principle, an exhaustive search algorithm can be used to find out the solution.

B. Fast Algorithm for Optimal Code Length Selection

In this section, we propose a fast algorithm for optimal code length selection. Multiplying both the nominator and dominator in (4) by l and perform some manipulations, we can rewrite (4) as

$$\bar{\nu}(l) = \frac{E\left\{\left(\sum_{j=0}^{l-1} \alpha_j\right)^2\right\}}{\sum_{j=0}^{l-1} I^{(j)}},$$

where $I^{(j)}$ is the normalized interference power. It is generated by combining the jth channel tap and given by

$$\begin{aligned}I^{(j)} &\equiv \sum_{m=1}^{\lfloor j/M \rfloor} E\left\{(\alpha_{j-mM} p_j p_{j-mM} b(i+m))^2\right\} \\ &+ \sum_{n=1}^{\lfloor (L-j)/M \rfloor} E\left\{(\alpha_{j+nM} p_j p_{j+nM} b(i-n))^2\right\} \\ &= \sum_{m=1}^{\lfloor j/M \rfloor} \Omega\gamma^{j-mM} + \sum_{n=1}^{\lfloor (L-j)/M \rfloor} \Omega\gamma^{j+nM}, \quad (7)\end{aligned}$$

where $\sum_{m=1}^{\lfloor j/M \rfloor} \Omega\gamma^{j-mM}$ and $\sum_{n=1}^{\lfloor (L-j)/M \rfloor} \Omega\gamma^{j+nM}$ are the post-cursor and pre-cursor ISI corresponding to the ith transmitted symbol $b(i)$. For convenience, we define β_j as the ratio of the average power of the jth tap and $I^{(j)}$, *i.e.,*

$$\beta_j \equiv \frac{E\left\{\alpha_j^2\right\}}{I^{(j)}}, \quad (8)$$

which indicates the output SIR when only the jth channel tap is combined using a sparse codeword whose $(L-j)$th element is one and zero elsewhere. By applying (2) and (7) in the above definition, we can simplify β_j as

$$\beta_j \equiv \frac{E\left\{\alpha_j^2\right\}}{I^{(j)}} = \frac{1}{\sum_{m=1}^{\lfloor j/M \rfloor} \gamma^{-mM} + \sum_{n=1}^{\lfloor (L-j)/M \rfloor} \gamma^{nM}}. \quad (9)$$

From (9), we can get

$$\begin{aligned}&\beta_0 = \cdots = \beta_{M-1} > \beta_M = \cdots = \beta_{2M-1} \\ &> \cdots > \beta_{(K-1)M} = \cdots = \beta_{KM-1}.\end{aligned}$$

In other words, we partition all L taps into K disjoint groups such that elements of the same group have the same β value. The first group contains the 0th to the $(M-1)$th elements, the second group contains the Mth to the $(2M-1)$th elements, and so on.

The following lemma will be repeatedly used in developing the proposed fast search algorithm.

Lemma 1: If S_1, S_2, I_1 and I_2 are all positive numbers, then

$$\frac{S_2}{I_2} < \frac{S_1}{I_1} \Longleftrightarrow \frac{S_2}{I_2} < \frac{S_1+S_2}{I_1+I_2} < \frac{S_1}{I_1}$$

Proof: This can be easily verified by direct multiplication. ■

When the codeword length is limited by M, we have the following proposition.

Proposition 1: If $0 < l \leq M$, then

$$M = \arg \max_{0<l\leq M} \bar{\nu}(l).$$

Proof: Let $l = k < M$, we can simplify $\bar{\nu}(k)$ as

$$\begin{aligned}\bar{\nu}(k) &= \frac{E\left\{\left(\sum_{i=0}^{k-1} \alpha_i\right)^2\right\}}{\sum_{i=0}^{k-1} I^{(i)}} = \frac{E\left\{\sum_{i=0}^{k-1} \alpha_i^2 + \sum_{i\neq j} \alpha_i \alpha_j\right\}}{I^{(0)} \sum_{i=0}^{k-1} d^{2i}} \\ &= \frac{\Omega\left(\sum_{i=0}^{k-1} d^{2i} + \frac{\pi}{2} \sum_{n=1}^{k-1} \sum_{m=1}^{n} d^{n+m-1}\right)}{I^{(0)} \sum_{i=0}^{k-1} d^{2i}} \\ &= \beta_0 + \frac{\Omega\pi}{2I^{(0)}} \frac{\sum_{n=1}^{k-1} \sum_{m=1}^{n} d^{n+m-1}}{\sum_{i=0}^{k-1} d^{2i}} \\ &= \beta_0 + \frac{\Omega\pi}{2I^{(0)}} g(k), \quad (10)\end{aligned}$$

where $0 < d = \gamma^{1/2} < 1$ and

$$g(k) = \frac{\sum_{n=1}^{k-1}\sum_{m=1}^{n} d^{n+m-1}}{\sum_{i=0}^{k-1} d^{2i}} > 0.$$

Next, we show that $g(k+1) > g(k)$ for $0 < k \leq M$. Note that

$$\begin{aligned} g(k+1) &= \frac{\sum_{n=1}^{k}\sum_{m=1}^{n} d^{n+m-1}}{\sum_{i=0}^{k} d^{2i}} \\ &= \frac{\sum_{n=1}^{k-1}\sum_{m=1}^{n} d^{n+m-1} + \sum_{m=1}^{k} d^{k+m-1}}{\sum_{i=0}^{k-1} d^{2i} + d^{2k}}. \end{aligned}$$

It can be shown that

$$\frac{\sum_{m=1}^{n} d^{n+m-1}}{\sum_{i=0}^{k-1} d^{2i}} < 1, \quad \forall 1 \leq n \leq (k-1).$$

Thus, we can get the following upper bound for $g(k)$, *i.e.*,

$$g(k) = \sum_{n=1}^{k-1} \frac{\sum_{m=1}^{n} d^{n+m-1}}{\sum_{i=0}^{k-1} d^{2i}} < \sum_{n=1}^{k-1} 1 = k-1.$$

In addition,

$$\frac{\sum_{m=1}^{k} d^{k+m-1}}{d^{2k}} = \sum_{m=1}^{k} d^{m-k-1} > \sum_{m=1}^{k} 1 = k.$$

The above inequality is due to the fact that $d = \gamma^{1/2} < 1$ and $m - k - 1 < 0$ for $1 \leq m \leq k$. Thus,

$$g(k) = \frac{\sum_{n=1}^{k-1}\sum_{m=1}^{n} d^{n+m-1}}{\sum_{i=0}^{k-1} d^{2i}} < \frac{\sum_{m=1}^{k} d^{k+m-1}}{d^{2k}}.$$

By Lemma 1, we conclude that $g(k+1) > g(k)$. Thus, $\bar{\nu}(k)$ is a monotonic increasing function of k, when $0 < k \leq M$, and the maximum value of $\bar{\nu}(k)$ must occur at $k = M$. ■

Proposition 1 suggests that a higher SIR value is expected if we combined more channel taps within the first group. Once the codeword length is more than M, both the signal power and interference power will increase. However, the output SIR may go up or down depending on the ratio between the increased amount of the signal power and that of the interference power.

Consider the case when $M < l \leq 2M$. Let $l = M + \bar{l}$ and $1 \leq \bar{l} \leq M$. We can rewrite $\bar{\nu}$ as

$$\bar{\nu}(l) = \frac{S(l)}{I(l)} = \frac{S(M) + \Delta S(\bar{l})}{I(M) + \Delta I(\bar{l})},$$

where $S(l)$ and $I(l)$ are the signal power and the interference power by combining the first l taps, and $\Delta S(\bar{l}) \equiv S(l) - S(M)$ and $\Delta I(\bar{l}) \equiv I(l) - I(M)$ are the increased power of the signal and that of the interference, respectively, owing to additional $\bar{l}$ taps combined. Note that $\Delta I(\bar{l})$ contains the interference power by combining of the $(M+1)$th to the lth taps, *i.e.*, $\Delta I(\bar{l}) = \sum_{j=M+1}^{l} I^{(j)}$, but $\Delta S(\bar{l})$ includes the square term of the first $\bar{l}$ elements in the 2nd group as well as the cross product between elements in the 2nd group and elements in different groups. It is straightforward to show the following lemma.

Lemma 2: $\frac{\Delta S(\bar{l})}{\Delta I(\bar{l})}$ is a monotonic increasing function of l when $1 \leq \bar{l} \leq M$.

Then, we are led to the following proposition.

Proposition 2: If $0 < l \leq 2M$, we have

$$\arg \max_{0<l\leq 2M} \bar{\nu}(l) = M \text{ or } 2M.$$

Proof: See Appendix. ■

By Proposition 2, the maximum value of $\bar{\nu}(l)$ occurs at $l = M$ or $l = 2M$ when the length of codeword is limited by $2M$. Actually, we can generalize the result in Proposition 2 and get the following proposition.

Proposition 3: The optimal codeword length that maximizes the output SIR must be an integer multiple of M. *i.e.,*

$$\bar{L} = \arg \max_{0<l\leq L} \bar{\nu}(l) = p \cdot M,$$

where $p \in \{1, 2, \cdots, K\}$.

Proof: The codeword length l subject to $0 < l \leq L$ can be represented as $l = mM + \bar{l}$, where $0 < m < K$ and $0 < \bar{l} \leq M$ are positive integers. By utilizing the same techniques in proving Lemma 2 and Proposition 2, we can get the upper bound for $\bar{\nu}(l)$ as

$$\bar{\nu}(l) \leq \bar{\nu}(mM) \text{ or } \bar{\nu}((m+1)M), \tag{11}$$

which suggests that the maximum output SIR must occur when l is an integer multiple of M. ■

As a direct consequence of Proposition 3, we conclude

$$\bar{L} = \arg \max_{0<l\leq L} \bar{\nu}(l) = M \cdot \arg \max_{0<p\leq K} \bar{\nu}(pM). \tag{12}$$

By using Proposition 3, it is easy to develop a fast codeword length search algorithm. That is, we first search over all K possible values of p and then select the one with the largest SIR. Compared with the exhaustive search algorithm given by (5), the proposed algorithm given by (12) reduces the search number by a factor of M.

Our fast search algorithm may not guarantee the greatest output SINR, especially when the input SNR is low. However, it will be shown later that, even when the input SNR is low (*i.e.*, AWGN is stronger than ISI), the performance difference between two systems with different optimization criteria is small. Hence, we can use the fast search algorithm to save the computational complexity.

IV. Simulation Results

Example 1. The BER Performance

In this example, we evaluate the bit error rate (BER) performance for the CPPUWB system with the optimized codeword length for various input SNR at different data rates. The system parameters are chosen as follows. Δ = 1ns, L = 120, Γ = 20.5ns (CM3). M = 40 and M = 20 are selected to achieve a data rate of 25 Mbps and 50 Mbps, respectively. In addition, the simulation result is obtained by averaging 1000 channel realizations.

To compare the system performance, we plot the BER curves in Fig. 1 for the conventional CPPUWB system without ISI mitigation and the CPPUWB system with different ISI mitigation schemes: the MMSE receiver and two codeword length optimization schemes for output SIR and SINR maximization, respectively. It is observed that, by properly adjusting the

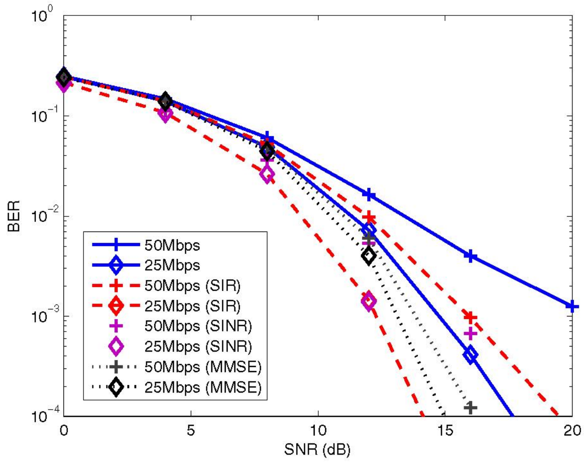

Fig. 1. The BER as a function of input SNR for different code lengths and data rates.

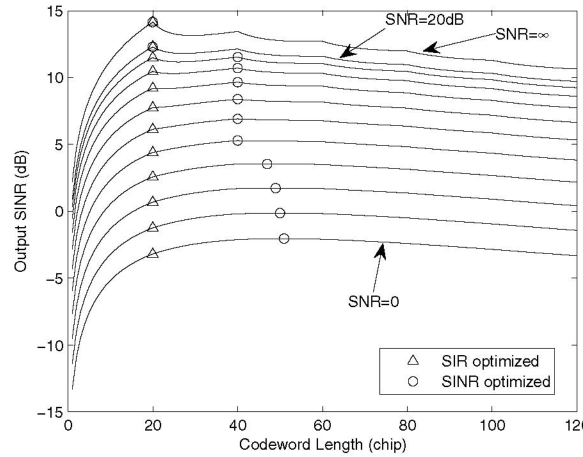

Fig. 2. The output SINR as a function of the codeword length parameterized by different input SNR values.

codeword length, the proposed scheme can even outperform the MMSE receiver at the data rate of 25 Mbps. However, when the data rate increases, it causes more serious ISI. In this case, the MMSE receiver can suppress more ISI than the proposed scheme. Furthermore, it is observed that the performance difference between two different codeword length optimization schemes is small and the gap approaches zero asymptotically as SNR goes up. This is because that the SINR converges to SIR when the noise power decreases.

Example 2. The noise effect

In Example 1, we have shown that the performance difference between different codeword length optimization schemes is small even in the low SNR region. To better illustrate this point, we plot the output SINR as a function of the codeword length at different input SNR when the data rate is fixed at 50 Mbps. The eleven lower curves in Fig. 2 correspond to cases with the input SNR ranging from 0 to 20 dB with a step size of 2 dB while the top curve represents the case where the noise power is zero. That is, it gives the output SIR value. The circle and triangle marks on each curve denote the output SINR one can get using different optimization criteria. It is observed that, when the input SNR is low, the codeword length which optimizes the output SINR is different from that obtained by the proposed method. However, the gap becomes smaller as the SNR value increases. Even though the proposed algorithm cannot guarantee the highest output SINR when input SNR is less than 20 dB, the SINR gap between our fast algorithm and the maximum SINR using (6) is small (less than 1 dB). This is because when the noise power is stronger than the interference power, *e.g.,* in low SNR environment, the output SINR is less sensitive to the interference variation. Thus, the performance variation due to the codeword length change is small.

V. Appendix: Proof of Proposition 2

With Proposition 1, it is easy to see that

$$\arg \max_{0<l\leq 2M} \bar{\nu}(l) = \arg \max_{M\leq l\leq 2M} \bar{\nu}(l). \tag{13}$$

Thus, to prove Proposition 2, we would like to show that either one of the following statements is true.

1) If $\bar{\nu}(2M) \geq \bar{\nu}(M)$ is true, then $\max_{M\leq l\leq 2M} \bar{\nu}(l) = \bar{\nu}(2M)$.
2) If $\bar{\nu}(M) \geq \bar{\nu}(2M)$ is true, then $\max_{M\leq l\leq 2M} \bar{\nu}(l) = \bar{\nu}(M)$.

Let $l = M + \bar{l}$, where $1 \leq \bar{l} \leq M$. Consider the first case where $\bar{\nu}(2M) \geq \bar{\nu}(M)$. If $\frac{\Delta S(\bar{l})}{\Delta I(\bar{l})} \leq \frac{S(M)}{I(M)}$, Lemma 1 implies that

$$\begin{aligned}\bar{\nu}(l) &= \frac{S(M)+\Delta S(\bar{l})}{I(M)+\Delta I(\bar{l})} \leq \frac{S(M)}{I(M)} \\ &= \bar{\nu}(M) \leq \bar{\nu}(2M).\end{aligned} \tag{14}$$

Otherwise, if $\frac{\Delta S(\bar{l})}{\Delta I(\bar{l})} \geq \frac{S(M)}{I(M)}$, we have $\frac{\Delta S(\bar{l})}{\Delta I(\bar{l})} \leq \frac{\Delta S(M)}{\Delta I(M)}$ from Lemma 2. Then, we can use Lemma 1 to show that

$$\bar{\nu}(l) = \frac{S(M)+\Delta S(\bar{l})}{I(M)+\Delta I(\bar{l})} \leq \frac{S(M)+\Delta S(M)}{I(M)+\Delta I(M)} = \bar{\nu}(2M). \tag{15}$$

Based on (14) and (15), we claim that statement 1 is true. Using similar arguments, we can prove statement 2 as well. ∎

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