COMBINED BIT SWAP AND POWER GAIN ADAPTATION FOR ERROR RATE EQUALIZATION IN DMT SYSTEMS

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ABSTRACT

A new adaptation algorithm to equalize the symbol error rates over all subchannels is proposed to overcome the slow time-varying noise environment existing in xDSL channels. The proposed algorithm combines the conventional bit swap algorithm with power gain adaptation. The overall transmission power before and after adjustment is kept the same. Simulation results demonstrate that the proposed algorithm has a faster convergence speed and a smaller dynamic range of the equalized noise variance than the bit swap algorithm [1].

1. INTRODUCTION

The discrete multi-tone (DMT) technique has been adopted in many DSL applications recently. The DMT systems divide the channel into subchannels and allocate bits and power according to subchannel SNR to maintain an equal error probability across all subchannels. However, the SNR is in general time varying in practical environments. If the SNR of some subchannels deteriorate, the overall performance may be dominated by those subchannels, which may lead to serious performance loss.

To maintain the system performance, bit or power allocation should be adaptively adjusted. A bit adaptation algorithm called bit swap was proposed in [1]. The advantage of bit swap was well explained in [2]. However, the bit swap algorithm does not always give satisfactory performance on the equalization of error probabilities. It is still possible to have a large disparity among subchannel error rates. Moreover, the algorithm may also result in unnecessary bit swaps, *i.e.* bit swap operations that do not improve the error rate.

In this paper, we propose an adaptation algorithm that integrates bit swap and power adaptation to equalize subchannel error rates. The overall transmitted power is preserved after the adjustment. The ADSL standard [4] stipulates that the step size of gain adaptation be 1 dB , which is too large to adequately equalize subchannel error rates. An extended command set was proposed in [3] to adjust the gain factors with a smaller step size. To utilize a finer step size of gain factors, the reserved vendor commands [3] will be used in this work. Experimental results will demonstrate that the subchannel error rates of the proposed algorithm can be more effectively equalized than that of the bit swap algorithm [1]. Moreover, the proposed algorithm always has a faster convergence speed than the bit swap algorithm.

2. BIT AND POWER ALLOCATION

In this section, we will review the symbol error equalization using bit and power allocation in DMT systems.

The block diagram of a DMT transceiver [5] is shown in Fig. 1. The received symbol in the kth subchannel is

$$\hat{X}_k = X_k + Q_k, \quad 0 \le k \le N-1,$$

where X_k is the transmitted symbol, and Q_k is the received noise. For QAM, the bit allocation formula is given by [6]

$$b_k = \log_2\left(1 + \frac{\mathcal{P}_X/\sigma^2_{Q_k}}{\Gamma}\right), \tag{1}$$

where $\mathcal{P}_X$ is the flat input power of X_k, and Γ is called the SNR gap, which depends on the symbol error probability. As given in (1), we see that the bit allocation formula may not lead to an integer solution. Algorithms to obtain integer bit allocation can be found in [7].

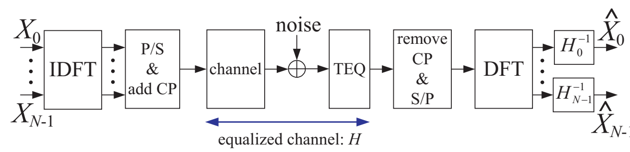

Fig. 1. The block-diagram of a DMT transceiver.

To maintain equal error probabilities for all subchannels under integer bit allocation, the transmitted power may no longer be flat. Let $\widetilde{b}_k$ denote the integer bit rate in the kth subchannel, the transmitted power is given by

$$\mathcal{P}_{X_k} = \left(2^{\widetilde{b}_k} - 1\right)\sigma^2_{Q_k}\Gamma. \tag{2}$$

Since the power allocation is calculated at the receiver, the power table should be sent back to the transmitter. In practice, the receiver does not send the table of power allocation.

Instead, it sends the table of gain factors given by [4]

$$g_k = \sqrt{\mathcal{P}_{X_k}/\mathcal{P}_X}. \tag{3}$$

Based on (1), (2) and (3), we can further obtain

$$g_k = \sqrt{(2^{\widetilde{b}_k} - 1)/\,(2^{b_k} - 1)}. \tag{4}$$

Since there is one-to-one correspondence between the gain factor and the allocated power, power adaptation can be achieved through gain adaptation. We can redraw Fig. 1 and obtain an equivalent block diagram as shown in Fig. 2.

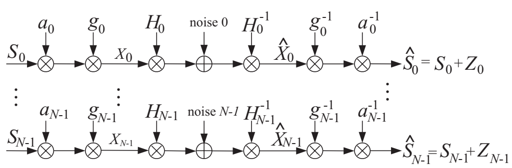

Fig. 2. Equivalent block-diagram of the DMT transceiver.

Let the transmitted symbol in the kth subchannel be denoted by S_k and the corresponding bit rate be denoted by $\widetilde{b}_k$. The minimum distances between the constellation points of S_k are assumed to be the same for all subchannels, denoted by d. The normalization factor a_k, which is used to normalize the power of S_k to $\mathcal{P}_X$, is given by [6]

$$a_k = \sqrt{\frac{6\mathcal{P}_X}{\left(2^{\widetilde{b}_k} - 1\right) d^2}}. \tag{5}$$

At the receiver, the symbol to be detected in the kth subchannel is $\hat{S}_k = S_k + Z_k$, where Z_k is the received noise at the detector input. The variance of Z_k is given by

$$\sigma^2_{Z_k} = a_k^{-2} g_k^{-2} \sigma^2_{Q_k}. \tag{6}$$

It can be verified that, if g_k satisfies (4) and a_k satisfies (5), $\sigma^2_{Z_k}$ will be the same for all subchannels.

The symbol error probability in the kth subchannel can be well approximated [6]

$$P_{e_k} = 4Q\left(\sqrt{\frac{d^2}{2\sigma^2_{Z_k}}}\right), \tag{7}$$

where $Q(x) = \int_x^\infty \frac{1}{\sqrt{2}}\, e^{\frac{-y^2}{2}}\, dy$. Since $\sigma^2_{Z_k}$ is a constant for $0 \le k \le N-1$, P_{e_k} is also a constant and $P_{e_k} = P_e$.

3. REVIEW OF THE BIT SWAP ALGORITHM [1]

The noise environment over xDSL channels is quasi stationary. Even the variances of the received noise at all subchannels are initially equalized, they can change slowly and result in unequal subchannel performance. Thus bit or/and power adaptation is required to equalize the subchannel performance. A bit adaptation scheme called the bit swap algorithm was proposed in [1]. It is described below.

1. The current subchannel noise variances are examined. Suppose that the ith and the jth subchannels have the largest and the smallest noise variances, denoted by $\sigma^2_{Z_i}$ and $\sigma^2_{Z_j}$, respectively. Let $\alpha = \sigma_{Z_i}/\sigma_{Z_j}$.

2. If $\alpha^2 > 2$, the bit rates and the normalization factors should be adjusted by

$$\begin{aligned} \widetilde{b'}_i &= \widetilde{b}_i - 1, \quad \widetilde{b'}_j = \widetilde{b}_j + 1, \\ a'_i &= \sqrt{2}\, a_i, \quad a'_j = a_j/\sqrt{2}. \end{aligned}$$

Under the high bit rate assumption and (5), a_i is scaled up (or down) by a factor of $\sqrt{2}$ when $\widetilde{b}_i$ is increased (or decreased) by one. Let the noise variances at the ith and the jth subchannels after swapping be $\sigma^2_{Z'_i}$ and $\sigma^2_{Z'_j}$, respectively. Let $\alpha' = \sigma_{Z'_i}/\sigma_{Z'_j}$. From (6), we have $\alpha' = \alpha/2$.

The above bit swap algorithm is however not always adequate for the equalization of subchannel noise variances. It has two main drawbacks. First, the performance may be seriously degraded due to unequal error probabilities. Consider the case when the noise variances of all subchannels are equal to σ^2_Z except for the ith subchannel that has a noise variance slightly smaller than 2 σ^2_Z. In this case, bit swapping is not activated. From (7), the symbol error probability of the ith subchannel is $P_{e_i} = 4Q\left(\frac{1}{\sqrt{2}}Q^{-1}\left(\frac{P_e}{4}\right)\right)$. In the ADSL standard [4], $P_e = 10^{-7}$. We have $P_{e_i} = 2.32 \times 10^{-4}$ for the ith subchannel while the symbol error probability of other subchannels is still kept at 10^{-7}. Then, the overall performance will be dominated by P_{e_i}. Second, the two subchannels may still have a large discrepency in terms of noise variances after swapping. In fact, the two subchannels will have the equal noise variance after swapping only when $\alpha = 2$. The worst situation occurs when α is slightly greater than $\sqrt{2}$ (*i.e* 3 dB). After bit swapping, α' is slightly greater than $\frac{\sqrt{2}}{2}$ (-3 dB). In this situation, the error rate improvement due to bit swapping is very limited.

4. GAIN ADAPTATION

In this section, we consider to adjust gain factors g_k to equalize the subchannel error rate under the fixed transmission power constraint. To evaluate the effect of noise equalization after bit and gain adaptation, we define an improvement index $\mathcal{I}$,

$$\mathcal{I} = 20\log_{10}\alpha - |20\log_{10}\alpha'|, \tag{8}$$

where $\alpha = \sigma_{Z_i}/\sigma_{Z_j}$ and $\alpha' = \sigma_{Z'_i}/\sigma_{Z'_j}$. For a given α, the largest value of $\mathcal{I}$ is $20\log_{10}\alpha$ when $\alpha' = 1$. That is, the noise variances in the ith and the jth subchannels are the same after adaptation. For the following sections, we will propose adaptation methods, in which adaptation will be executed only when $\mathcal{I}$ is greater than threshold $\mathcal{I}_{thres}$ to avoid unnecessary adjustment.

4.1. Gain Factor Adjustment

Suppose that the two subchannels that have the largest and smallest noise variances are, respectively, the ith and jth subchannels. Intuitively, we need to divert some of the power of the jth subchannel to the ith subchannel. That is, we need to scale up g_i and scale down g_j. Let the gain factors after gain adaptation be

$$g_i' = g_i\Delta_+ \text{ and } g_j' = g_j/\Delta_-. \tag{9}$$

Note that both Δ^+ and Δ^- are greater or equal to 1. From (6), we have $\alpha'^2 = \alpha^2/(\Delta_+^2\Delta_-^2)$. To have the equal noise variance after adaptation, *i.e.* $\alpha' = 1$, we require

$$\Delta_+^2\Delta_-^2 = \alpha^2. \tag{10}$$

The overall transmission power is unchanged if

$$g_i^2\Delta_+^2 + \frac{g_j^2}{\Delta_-^2} = g_i^2 + g_j^2. \tag{11}$$

Let $\beta = g_j/g_i$. Using (10) and (11), we obtain

$$\Delta_+ = \alpha\sqrt{\frac{1+\beta^2}{\alpha^2+\beta^2}}, \text{ and } \Delta_- = \sqrt{\frac{\alpha^2+\beta^2}{1+\beta^2}}. \tag{12}$$

The improvement index is at its maximum $20\log_{10}\alpha$.

4.2. Gain Factor Adjustment with Constraints

In practical implementations, gain factors are often constrained to be within a range, which will be examined here.

Let the upper and lower bounds of g_k be g_{up} and g_{low} respectively. The maximum of Δ_+ and Δ_- are

$$\Delta_{+,max} = g_{up}/g_i \text{ and } \Delta_{-,max} = g_j/g_{low}.$$

Note that $\Delta_{-,max}$ is the maximum value that g_j can further be scaled down; in addition, if either $\Delta_{+,max}$ or $\Delta_{-,max}$ is smaller or equal to 1, it means there is no room for gain adaptation. Due to these constraints, α' may be larger than 1. Our goal is to find Δ_+ and Δ_- in (9) so that $|20\log_{10}\alpha'|$ can be minimized subject to the condition that the gain factors after adjustment satisfy $g_i' \leq g_{up}$ and $g_j' \geq g_{low}$. Recall that

$$\Delta_+^2\Delta_-^2 = \alpha^2/\alpha'^2. \tag{13}$$

Let us consider only the upper bound and ignore the lower bound temporarily. Suppose that the value of α' that gives the minimum of $|20\log_{10}\alpha'|$ is denoted by α'_+. Let $\Delta_+ = \Delta_{+,max}$. We can obtain Δ_- from (11). Furthermore, we can compute α'_+ using (13) as

$$\alpha'_+ = \frac{\alpha}{\beta\Delta_{+,max}}\sqrt{1+\beta^2-\Delta_{+,max}^2}. \tag{14}$$

On the other hand, let us consider only the lower bound and ignore the upper bound. Suppose that the value α' that gives the minimum of $|20\log_{10}\alpha'|$ is α'_-. Let $\Delta_- = \Delta_{-,max}$. We can obtain Δ_+ via (11), and compute

$$\alpha'_- = \alpha\sqrt{\frac{1}{[1+\beta^2]\Delta_{-,max}^2 - \beta^2}} \tag{15}$$

via (13). The value of α' that minimizes $|20\log_{10}\alpha'|$ can be obtained via

$$\alpha'_{min} = \text{MAX}\left\{\alpha'_+,\ \alpha'_-,\ 1\right\}. \tag{16}$$

Given α'_{min}, Δ_+ and Δ_- can be obtained via (11) and (13):

$$\Delta_+ = \frac{\alpha}{\alpha'_{min}}\sqrt{\frac{1+\beta^2}{\frac{\alpha^2}{\alpha'^2_{min}}+\beta^2}} \text{ and } \Delta_- = \sqrt{\frac{\frac{\alpha^2}{\alpha'^2_{min}}+\beta^2}{1+\beta^2}}. \tag{17}$$

The gain adaptation algorithm can be described as follows.

1. This is the same as the first step of the bit swap algorithm. Compute $\alpha = \sigma_{Z_i}/\sigma_{Z_j}$.
2. Calculate α'_+ and α'_- by (14) and (15), respectively, and choose $\alpha'_{min} = \text{MAX}\left\{\alpha'_+,\ \alpha'_-,\ 1\right\}$.
3. Obtain Δ_+ and Δ_- by (17).
4. $g_i' = g_i\Delta_+$ and $g_j' = g_j/\Delta_-$.
5. Calculate the improvement index based on (8). If $\mathcal{I}_{gain} > \mathcal{I}_{thres}$, send the adaptation request to the transmitter. Otherwise, do nothing.

5. PROPOSED BIT SWAP WITH GAIN ADAPTATION (BSGA) ALGORITHM

By using the reserved vendor discretionary commands [4], it is possible to perform the bit swap and the gain adaptation operations simultaneously within one iteration. Hence, we can combine them to further improve the system performance and we call the combined algorithm the BSGA algorithm. The BSGA algorithm is summarized below.

1. Same as the 1st and 2nd steps of the bit swap algorithm.
2. Let the noise variances in the ith and the jth subchannels be $\sigma^2_{\hat{Z}_i}$ and $\sigma^2_{\hat{Z}_j}$, respectively, and $\hat{\alpha} = \sigma_{\hat{Z}_i}/\sigma_{\hat{Z}_j}$. Note that if the bit swap operation is not executed, $\hat{\alpha} = \alpha$.
3. Let T be a threshold to determine whether gain adaptation should be executed at the current iteration. If $\hat{\alpha} > T$, go to Step 1. The reason to have this threshold is that, when $\hat{\alpha}$ is large, we may obtain the solution which $\Delta_- \gg \Delta_+$ from (17). This result could make the smallest noise variance increase dramatically. A reasonable value for T could be $T = 2$.

4. If $\hat{\alpha} > 1$, it means that the noise variance of the ith subchannel is still greater than that of the jth subchannel, and we can conduct Steps 1 to 4 of the gain adaptation algorithm. Otherwise, it means that the noise variance of the ith subchannel is less than that of the jth subchannel after bit swapping. Then, we should regard j and i as the subchannel indices that have the maximum and minimum noise variances, respectively, and conduct Steps 1 to 4 of the gain adaptation algorithm accordingly.

5. Calculate the improvement index $\mathcal{I}_{bit+gain}$ using (8). If $\mathcal{I}_{bit+gain} > \mathcal{I}_{thres}$, send the adaptation request to the transmitter. Otherwise, do nothing.

The integrated BSGA algorithm always has better performance than the bit swap (BS) algorithm alone [1], since it can perform gain adaptation furthermore to reduce the dynamic range of the noise variance. Moreover, the BSGA algorithm has a faster convergence speed than the BS algorithm alone as demonstrated in the next section.

6. SIMULATION RESULTS

We give an example to compare the performance of the BS algorithm [1] and the proposed BSGA algorithm in this section. The parameters of the experiment were chosen as follows. $N = 256$. The initial g_k is uniformly distributed within [-1.5 dB 1.5 dB]. This can be justified by the following reason. Since the SNR required to add or remove one bit is about 3 dB under the high bit rate assumption, the initial gain factors tend to have a saw-tooth shape with approximately a 3 dB peak-to-peak dynamic range [7]. The upper bound and the lower bound of gain factors were chosen to be 0.002 and 8, respectively; the step size was 0.002 [4]. The threshold $\mathcal{I}_{thres}$ was set to 0.1 dB and let $T = 2$. Initially, the noise at the detector input was -140 dBm/Hz AWGN, and then an ADSL-FEXT noise of one disturber in CSA #6 loop was added. The power spectral density of the combined noise is indicated by the dash-dot line in Fig. 3.

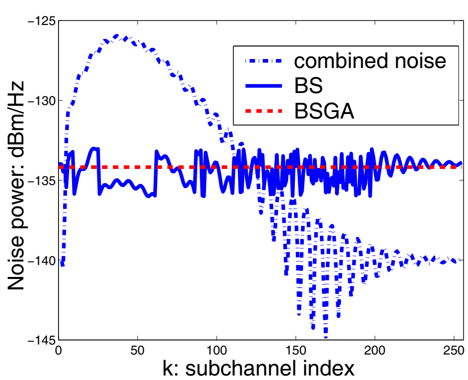

Fig. 3. Comparison of the steady state noise variances.

The steady noise variances obtained by the two algorithms (*i.e.* BS from [1] and the proposed BSGA) are shown in Fig. 3. We see that the steady noise variances by BS alone have a 3 dB dynamic range. In contrast, the proposed BSGA algorithm has a dynamic range around 0 dB. It is clear that the proposed BSGA algorithm equalizes the noise variance more effectively than the BS algorithm in the steady state.

Fig. 4 shows the convergence speed of the two algorithms in the subchannel error rate equalization process. The vertical axis is the value of α as defined in Step 1 of the BS algorithm. We see that the BSGA algorithm needs fewer iterations to make α minimized than the BS algorithm. We see from Fig. 4 that, for $\alpha = 3$dB, the proposed algorithm outperforms the bit swap algorithm by around 50 iterations.

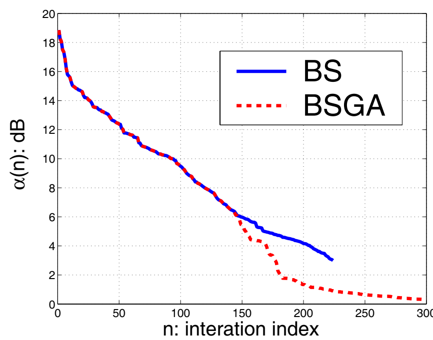

Fig. 4. The distance between the maximum and the minimum noise variance as a function of the iteration number.

7. REFERENCES

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