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Complexity Reduction of Maximum-Likelihood Multiuser Detection (ML-MUD) Receivers with Carrier Interferometry Codes in MC-CDMA

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***Abstract*—We propose to use carrier interferometry (CI) codes to reduce the complexity of the maximum-likelihood multiuser-detection (ML-MUD) receiver in MC-CDMA systems with carrier frequency offsets (CFOs). Complexity reduction is achieved by exploiting the sparsity of the cross-correlation matrix of CI codes. It is shown that, for a fully-loaded MC-CDMA with spread gain N over a multipath channel with length L, the complexity of the ML-MUD receiver grows exponentially with $2L-1$ (instead of N), where $L \ll N$ in a typical setting. Moreover, an upper bound for the minimum error probability is derived. Finally, simulation results are given to corroborate theoretical results.**

I. Introduction

Multicarrier code division multiple access (MC-CDMA) [1], [5] has emerged as a promising multiaccess technique for high data rate communications. MC-CDMA is more robust to inter symbol interference (ISI) than conventional CDMA systems due to the use of the orthogonal frequency division multiplexing (OFDM) structure. However, the effects of multipath fading and/or carrier frequency offsets (CFOs) tend to destroy orthogonality among users and lead to multiaccess interference (MAI) and, consequently, the performance of MC-CDMA can be greatly degraded.

Various single user detection [10], [11] and multiuser detection techniques have been proposed to combat MAI. Among them, the maximum likelihood (ML) multiuser detection (MUD) technique [12] has the optimum performance but at the cost of a prohibitive computational complexity that grows exponentially with the number of users. A large amount of efforts has been made to reduce the complexity of the optimum multiuser detector. Cai *et al.* [3] proposed to assign a set of subcarriers to a group of users while preserving the frequency diversity of MC-CDMA as much as possible. With this design, MAI is only present among users in the same group so that it can be suppressed via simplified MUD techniques. A new ML-MUD scheme called sphere decoding was proposed for MC-CDMA, whose complexity is a polynomial function of the user number [2]. However, when the user number is large, the sphere decoding ML algorithm is cumbersome to perform. Moreover, neither of the above techniques have been studied in the presence of CFO.

In this work, we propose to employ carrier interferometry (CI) codes [8] in MC-CDMA systems. By exploiting the sparsity of the cross-correlation matrix of CI codes, we can lower the complexity of ML detectors such that its complexity grows exponentially with the channel multipath length instead of the number of active users. To be more specific, a fully-loaded MC-CDMA system with spread gain N and multipath length L in a CFO environment, each user has only $2(L-1)$ interfering users (rather than other $N-1$ active users). This results in significant complexity reduction in practical situations, where N is much greater than L.

The rest of this paper is organized as follows. The MC-CDMA system model in a CFO environment is presented in Sec. II. We describe the ML detector for MC-CDMA with and without CFO in Sec. III. The sparsity of the cross correlation matrix of CI codes is studied in Sec. IV. An upper bound on the minimum error probability is derived in V. Simulation results are shown in Sec. VI. Finally, concluding remarks are drawn in Sec. VII.

II. System Model

Consider an MC-CDMA system with K users. The block diagram of the uplink transmission of the ith user is shown in Fig. 1. The kth component of the DFT output, $\hat{\mathbf{y}}$, can be written as

$$\hat{y}[k] = \sum_{j=0}^{K-1} r_j[k] + n[k], \qquad (1)$$

where $n[k]$ is the DFT of additive noise, and $r_j[k]$ is the received signal contributed from the jth user due to the channel fading and CFO effects. Suppose that user j has a normalized CFO ϵ_j, *i.e.* the actual CFO normalized by $1/N$ of the overall bandwidth, and $-0.5 \leq \epsilon_j \leq 0.5$. $r_j[k]$ can be written as [7], [10]

$$r_j[k] = \alpha_j \lambda_j[k] y_j[k] + \beta_j \sum_{m=0, m\neq k}^{N-1} \cdot \left\{ \lambda_j[m] y_j[m] g_j(m-k) \right\} \qquad (2)$$

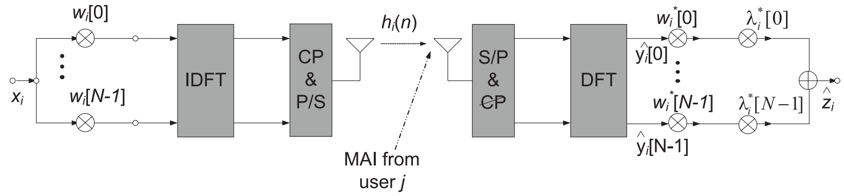

Fig. 1. The block diagram of the uplink transmission of the ith user in an MC-CDMA system.

where $y_i[k] = x_i w_i[k]$, for $0 \leq k \leq N-1$, and $\lambda_j[m]$ is the mth component of N-point DFT of the channel impulse response of user j, and

$$\alpha_j = \frac{\sin \pi\epsilon_j}{N \sin \frac{\pi\epsilon_j}{N}} e^{j\pi\epsilon_j \frac{N-1}{N}},\ \beta_j = \sin(\pi\epsilon_j) e^{j\pi\epsilon_j \frac{N-1}{N}},$$

and

$$g_j(m-k) = \frac{e^{-j\pi\frac{m-k}{N}}}{N \sin \frac{\pi(m-k+\epsilon_j)}{N}}.$$

When there is no CFO (*i.e.* $\epsilon_j = 0$), $r_j[k] = \lambda_j[k] y_j[k]$. Since $\beta_j g_j(0) = \alpha_j$ and by Eq. (2), we have

$$\hat{\mathbf{y}} = \mathbf{C}\mathbf{x} + \mathbf{n}, \quad (3)$$

where $\mathbf{C}$ is an $N \times K$ matrix whose k, jth entry is given by

$$[\mathbf{C}]_{kj} = \beta_j \sum_{m=0}^{N-1} g_j(m-k)\lambda_j[m] w_j[m], \quad (4)$$

and

$$\mathbf{x} = (x_0, ..., x_{K-1})^T, \quad \hat{\mathbf{y}} = (\hat{y}[0], ..., \hat{y}[N-1])^T,$$

and $\mathbf{n} = (n[0], ..., n[N-1])^T$ is circularly symmetric complex Gaussian random vector with zero mean and covariance $\sigma^2\mathbf{I}$. To detect transmitted symbols, one way is to use single user detection techniques such as the maximum ratio combining (MRC). MRC detects the ith transmitted symbol as

$$\begin{aligned} \hat{z}_i &= \sum_{k=0}^{N-1} \hat{y}[k]\lambda_i^*[k] w_i^*[k] \\ &= s_i + \sum_{j=0, j\neq i}^{K-1} MAI_{i\leftarrow j} + \hat{n}_i, \end{aligned} \quad (5)$$

where $\hat{n}_i = \sum_{k=0}^{N-1} n[k]\lambda_i^*[k] w_i^*[k]$ and

$$\begin{aligned} s_i &= \beta_i x_i \sum_{k=0}^{N-1}\sum_{m=0}^{N-1} \\ &\cdot \left\{ g_i(m-k)\lambda_i[m] w_i[m] \lambda_i^*[k] w_i^*[k] \right\} \end{aligned} \quad (6)$$

We see that when $m \neq k$, s_i consists of the intercarrier interference (ICI) caused by CFO for the desired user. It is assumed here that the CFO of each user can be estimated and compensated at the receiver so that there will be no ICI effect. Using Eqs. (2) and (6), the MAI of user i due to the jth user's CFO can be written as

$$\begin{aligned} MAI_{i\leftarrow j} &= \beta_j x_j \sum_{k=0}^{N-1}\sum_{m=0}^{N-1} g_j(m-k) \\ &\cdot \left\{ \lambda_j[m] w_j[m] \lambda_i^*[k] w_i^*[k] \right\}. \end{aligned} \quad (7)$$

Although MRC cannot ensure an MAI-free MC-CDMA system, it does achieve the maximum diversity gain provided by the multipath channel.

III. ML Multiuser Detection (ML-MUD)

We consider the ML detection based on the received signal given in Eq. (3) in this section. It is assumed that the receiver has the perfect knowledge of channel coefficients and CFO values. We will examine the multipath and the CFO effects separately.

A. ML-MUD in Multipath Fading Channel

For given transmitted signal $\mathbf{x}$, we would like to maximize the likelihood of the received signal. Since noise $\mathbf{n}$ is Gaussian, the ML estimate can be written as

$$\hat{\mathbf{x}} = \arg\min_{\mathbf{x}} \Omega(\mathbf{x}),$$

where

$$\Omega(\mathbf{x}) = ||\mathbf{C}\mathbf{x}||^2 - 2\Re\{(\mathbf{C}\mathbf{x}, \hat{\mathbf{y}})\},$$

where $\mathbf{C}$ is defined in Eq. (4) in a CFO environment. If there is no CFO, Eq. (4) can be written as

$$[\widetilde{\mathbf{C}}]_{kj} = w_j[k]\lambda_j[k].$$

Thus, we have

$$(\widetilde{\mathbf{C}}\mathbf{x}, \hat{\mathbf{y}}) = \mathbf{x}^\dagger(\widetilde{\mathbf{C}}^\dagger\hat{\mathbf{y}}) = \sum_{j=0}^{K-1} x_j \sum_{k=0}^{N-1} \hat{y}_j[k] w_j^*[k]\lambda_j^*[k].$$

Note that $\sum_{k=0}^{N-1} \hat{y}_j[k] w_j^*[k]\lambda_j^*[k]$ is actually the estimate of the input signal for user j obtained by MRC (*i.e.*, $\hat{z}_j$). On the other hand, $||\widetilde{\mathbf{C}}\mathbf{x}||^2 = \mathbf{x}^\dagger\widetilde{\mathbf{H}}\mathbf{x}$, where $\widetilde{\mathbf{H}} = \widetilde{\mathbf{C}}^\dagger\widetilde{\mathbf{C}}$. Then, the ML optimization problem is equivalent to minimizing

$$\Omega(\mathbf{x}) = \mathbf{x}^\dagger\widetilde{\mathbf{H}}\mathbf{x} - 2\Re\{(\mathbf{x}^\dagger, \hat{\mathbf{z}})\}$$

with respect to $\mathbf{x}$, where $\hat{\mathbf{z}}$ is the output vector of MRC with its ith element given in (5).

We denote the MAI from user j to desired user i without CFO by $\widetilde{MAI}_{i\leftarrow j}$. Then, we have from Eq. (7) that

$$\widetilde{MAI}_{i\leftarrow j} = x_j \sum_{k=0}^{N-1} \lambda_j[k] w_j[k] \lambda_i^*[k] w_i^*[k].$$

It can be easily shown that the (i,j)th component of $\widetilde{\mathbf{H}}$ is

$$[\widetilde{\mathbf{H}}]_{i,j} = \frac{\widetilde{MAI}_{i\leftarrow j}}{x_j}.$$

In fact, $\widetilde{\mathbf{H}}$ can be viewed as the cross-correlation matrix.

B. ML-MUD in Multipath Fading Channel with CFO

We rewrite Eq. (5) in vector format as

$$\hat{\mathbf{z}} = \mathbf{H}\mathbf{x} + \hat{\mathbf{n}},$$

where $\hat{\mathbf{z}} = (\hat{z}_0, ..., \hat{z}_{K-1})^T$, and the i,jth entry of $\mathbf{H}$ is equal to

$$[\mathbf{H}]_{i,j} = \begin{cases} \sum_{k=0}^{N-1} |\lambda_i[k]|^2, & i = j, \\ MAI_{i\leftarrow j}/x_j, & i \neq j. \end{cases}$$

The ML detector has the following form

$$\hat{\mathbf{x}} = \arg\min_{\mathbf{x}} \Omega(\mathbf{x}),$$

where

$$\Omega(\mathbf{x}) = ||\hat{\mathbf{z}} - \mathbf{H}\mathbf{x}||^2 = \sum_{i=0}^{K-1} |\hat{z}_i - \mathbf{h}_i \mathbf{x}|^2,$$

and where $\mathbf{h}_i$ is the ith row of $\mathbf{H}$.

IV. Complexity-reduced ML-MUD with CI Codes

In this section, we show how carrier interferometry (CI) codes [8] can reduce the complexity of the ML-MUD receiver significantly. The CI codes of size N have the following form:

$$w_i[k] = e^{j\frac{2\pi}{N}ki}, \quad k, i = 0, 1, \cdots, N-1.$$

There is an attractive MAI-free property associated with CI codewords in a CFO environment, which will greatly simplify the optimum ML detection as stated in the following Theorem.

Theorem: For two CI codewords with indices i and i', where $i, i' = 0, 1, \cdots, N-1$, we have $MAI_{i\leftarrow i'} = 0$ in a CFO environment if $((|i - i'|))_{N-(L-1)} \geq L$, where $((n))_N$ denotes n modulo N and L is the channel length.

Proof: Let $r_{i,i'}^{(p)}[k] = w_i^{(p)}[k] w_{i'}^*[k]$, where $w_i^{(p)}[k] = w_i[((N-p+k))_N]$, for $k = 0, 1, \cdots, N-1$, $p = 0, 1, ...N-1$. It is shown in [10] that, to achieve $MAI_{i\leftarrow i'} = 0$, the first L samples and the last $L-1$ samples of the IDFT of vector $\mathbf{r}_{i,i'}^{(p)} = (r_{i,i'}^{(p)}[0], r_{i,i'}^{(p)}[1], \cdots, r_{i,i'}^{(p)}[N-1])^T$ must be zeros for $p = 0, 1, ...N-1$. By taking the IDFT of $\mathbf{r}_{i,i'}^{(p)}$ for $p = 0, 1, \cdots, N-1$ while neglecting the multiplicative term $\frac{1}{\sqrt{N}}$, we obtain

$$\begin{aligned} r_{i,i'}^{(p)}(n) &= \sum_{k=0}^{N-1} e^{j\frac{2\pi}{N}(N-p+k)i} e^{-j\frac{2\pi}{N}i'k} e^{j\frac{2\pi}{N}kn} \\ &= \begin{cases} Ne^{-j\frac{2\pi}{N}pi}, & ((i - i' + n))_N = 0, \\ 0, & \text{otherwise.} \end{cases} \end{aligned}$$

To meet condition $((|i - i'|))_{N-(L-1)} \geq L$, $|i - i'|$ should take values from $\{L, L+1, \cdots, N-L\}$. Consider the case $i - i' \geq 0$. Under the condition that $i - i'$ is equal to a value in this set, $r_{i,i'}^{(p)}(n) \neq 0$, for $n = N-L, N-L-1, \cdots, L$, by the above equation. On the other hand, if $i - i' \leq 0$, $r_{i,i'}^{(p)}(n) \neq 0$, for $n = N+L, N+L+1, \cdots, 2N-L$. It can be easily shown that $r_{i,i'}^{(p)}(n)$ is periodic with period N. We conclude $r_{i,i'}^{(p)}(n) \neq 0$ for $n = L, L+1, \cdots, N-L$, while $r_{i,i'}^{(p)}(0) = r_{i,i'}^{(p)}(1) = \cdots = r_{i,i'}^{(p)}(L-1) = 0$, and $r_{i,i'}^{(p)}(N-L+1) = r_{i,i'}^{(p)}(N-L+2) = \cdots = r_{i,i'}^{(p)}(N-1) = 0$, for $((|i - i'|))_{N-(L-1)} \geq L$. In other words, $MAI_{i\leftarrow i'} = 0$. ■

Thus, for $K = N$ active users with CI codewords and for $L \leq N/2$, each user has only $2(L-1)$ (instead of $N-1$) interfering users. Therefore, both $\mathbf{H}$ and $\widetilde{\mathbf{H}}$ are sparse matrices. As shown in Fig. 2(a) for $N = K = 16$ and $L = 2$, the non-zero elements (indicated by black squares) of $\mathbf{H}$ (or $\widetilde{\mathbf{H}}$) are concentrated along the three diagonal lines. Elements in the off-diagonal region with $|i-j| \geq L$ are all equal to zero except for the two corners.

The well-known Viterbi algorithm (VA) can be used to solve the ML optimization problem. Generally speaking, its complexity is proportional to the number of states and the number of transitions per stage. Hence the complexity of ML-MUD is $O(2^N)$ for a general $N \times N$ non-sparse cross-correlation channel matrix and the BPSK modulation. However, by exploiting the sparsity of $\mathbf{H}$, the complexity of VA grows exponentially with $2L-1$ if a proper trellis is defined.

The circulant structure of $\mathbf{H}$ (or $\widetilde{\mathbf{H}}$) suggests the construction of a tail-biting trellis (TBT), *i.e.* a trellis that wraps around itself. The tail-biting trellis has been studied in the area of error correcting codes. A method for trellis construction for a similar matrix structure was proposed in [6], which lends itself to a reduced search Viterbi algorithm. For example by using this method, the trellis state for the example given in Fig. 2 is defined as

$$s[i] = (x_{((i-1))_N}, x_i), \quad 0 \leq i \leq N-1.$$

Fig.2 also illustrates the trellis implied by this state definition for a BPSK alphabet. With the state and trellis definitions, we must solve the ML optimization problem that is equivalent to finding a closed

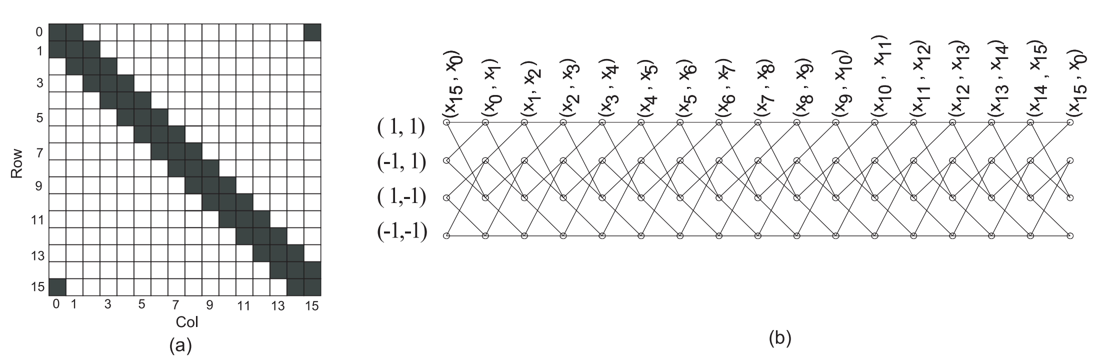

Fig. 2. $N = K = 16$ and $L = 2$. (a) The cross-correlation matrix of CI codewords. (b) The tail-biting trellis.

path with the minimum cost through TBT. Some approximate ML algorithms for decoding TBT have been addressed in [4], [6]. In particular, the approximate ML algorithm proposed in [6], called the Iterative Tail Biting Viterbi Algorithm (ITB-VA), applies VA iteratively around a TBT multiple times without dropping paths that are not closed. This approach is taken in our work.

V. Upper bound for minimum error probability

We can derive an upper bound to the minimum error probability for MC-CDMA with BPSK symbols similar to synchronous CDMA in the AWGN channel [12]. We define E_i to be the set of error vectors that affect the ith user in form of

$$E_i = \{\boldsymbol{e} \in \{-1, 0, 1\}^K, e_i \neq 0\},$$

where $e_i = x_i - \hat{x}_i$. The set of admissible error vectors that are compatible with transmitted vector $\mathbf{x} \in \{-1, 1\}^K$, is defined by

$$A(\mathbf{x}) \quad = \quad \{\boldsymbol{e} \in E,\ 2\boldsymbol{e} - \mathbf{x} \in \{-1, 1\}^K\},$$

where $E = \cup_{i=1}^{K} E_i$ is the set of nonzero error vectors. We can upper bound the error probability for user i, denoted by $P_i(e)$, in the AWGN channel by

$$P_i(e) \leq \sum_{\mathbf{e} \in E_i} P\{\Omega(\mathbf{x} - 2\boldsymbol{e}) \leq \Omega(\mathbf{x}), \boldsymbol{e} \in A_i(\mathbf{x})\}, \tag{8}$$

where $A_i(\mathbf{x}) = A(\mathbf{x}) \cap E_i$ and we have used the fact that, if $\mathbf{x} - 2\boldsymbol{e}$ is the most likely vector, it is more likely than $\mathbf{x}$. It can be easily shown that, when no CFO is present, we have

$$\Omega(\mathbf{x} - 2\boldsymbol{e}) - \Omega(\mathbf{x}) = 4\boldsymbol{e}^T \widetilde{\mathbf{H}} \boldsymbol{e} + 4\Re\{\boldsymbol{e}^T \hat{\mathbf{n}}\},$$

We see that the two events $\hat{\mathbf{n}}$ and $\boldsymbol{e} \in \mathbf{A}(\mathbf{x})$ are independent. Extending Eq. (8) to the fading channel, we can express the error probability as

$$\begin{aligned} P_i(e|\widetilde{\mathbf{H}}) \quad &\leq \quad \sum_{\mathbf{e} \in E_i} P\{\Omega(\mathbf{x} - 2\boldsymbol{e}) - \Omega(\mathbf{x}) \leq 0 | \widetilde{\mathbf{H}}\} \\ &\times \quad \Big\{ P\{\boldsymbol{e} \in \mathbf{A}(\mathbf{x})\} \Big\}, \end{aligned}$$

which follows from the fact that the admissibility of $\boldsymbol{\epsilon}$ is independent of $\widetilde{\mathbf{H}}$. For equally likely transmitted bits, we have

$$P\{\boldsymbol{e} \in \mathbf{A}(\mathbf{x})\} = \prod_{i=1}^{K} P\{(x_i - e_i)e_i = 0\} = 2^{-w(e)},$$

where $w(e) = \sum_{i=1}^{K} |e_i|$. To compute $P\{\Omega(\mathbf{x} - 2\boldsymbol{e}) - \Omega(\mathbf{x}) \leq 0 | \widetilde{\mathbf{H}}\}$, we note that, since $\hat{\mathbf{n}}$ is circularly symmetric complex Gaussian random vector with zero mean and covariance $\sigma^2 \widetilde{\mathbf{H}}$,

$$E\{(\Re\{\boldsymbol{e}^T \hat{\mathbf{n}}\})^2\} = \frac{1}{2} E\{\boldsymbol{e}^T \hat{\mathbf{n}} \hat{\mathbf{n}}^\dagger \boldsymbol{e}\} = \frac{1}{2} \sigma^2 \boldsymbol{e}^T \widetilde{\mathbf{H}} \boldsymbol{e}.$$

Thus, the error probability for user i is bounded by

$$P_i(e|\widetilde{\mathbf{H}}) \leq \sum_{\mathbf{e} \in E_i} 2^{-w(e)} Q \left(\frac{\sqrt{2\boldsymbol{e}^T \widetilde{\mathbf{H}} \mathbf{e}}}{\sigma} \right).$$

When CFO is present, we have

$$\Omega(\mathbf{x} - 2\boldsymbol{e}) - \Omega(\mathbf{x}) = 4\boldsymbol{e}^T \mathbf{H}' \boldsymbol{e} + 4\Re\{\boldsymbol{e}^T \mathbf{n}'\},$$

where $\mathbf{H}' = \mathbf{H}^\dagger \mathbf{H}$ and $\mathbf{n}' = \mathbf{H}^\dagger \hat{\mathbf{n}}$. By taking a similar approach, we can show that the upper bound for the error probability is

$$P_i(e|\mathbf{H}) \leq \sum_{\mathbf{e} \in E_i} 2^{-w(e)} Q \left(\frac{\sqrt{2} \boldsymbol{e}^T \mathbf{H}' \mathbf{e}}{\sigma \sqrt{\boldsymbol{e}^T \mathbf{H}^\dagger \widetilde{\mathbf{H}} \mathbf{H} \mathbf{e}}} \right).$$

The unconditional BEP for user i can be obtained by

$$P_i(e) = \int_0^\infty P_i(e|\mathbf{H}) P\{\mathbf{H}\} d_{\mathbf{H}}. \tag{9}$$

The upper bound for synchronous DS-CDMA in the AWGN channel was made tighter in [12] by eliminating the so-called *decomposable* error vectors from the summation. This result was extended to the fading channel case in [13] by expressing channel matrix $\mathbf{H}$ as $\mathbf{A}^\dagger \mathbf{R} \mathbf{A}$ where $\mathbf{A}$ and $\mathbf{R}$ only contain fading coefficients and cross-correlation coefficients, respectively, and whereby allowing the set of *indecomposable* errors to be independent of fading [13]. However, we are not able to separate fading coefficients and cross-correlation in our system to make the bound tighter.

VI. Simulation Results

The Monte Carlo simulation was conducted to corroborate derived theoretical results. Every user was randomly assigned by a CFO value of ϵ or $-\epsilon$. To compute the analytical upper bound for BEP, the Monte Carlo integration method in [9] was used.

Fig. 3 shows the upper bound to BEP as a function of the SNR value, E_b/N_0, under the setting of $N = 8$, $L = 2$, $K = 8$ for both zero CFO and CFO= ± 0.3 cases. The upper bound curves in each case are plotted against their corresponding simulated BEP. To shorten simulation time, only the BEP for the first user was computed. We see that the upper bound is not very tight particularly for the system with CFO. The reason is that the decomposable error sequences could not be identified and discarded in the presence of fading channels, unlike the asynchronous DS-CDMA case in [13].

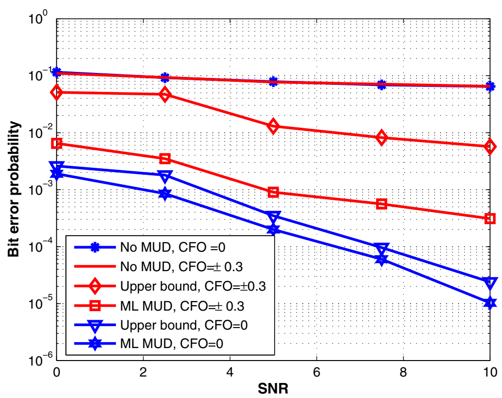

Fig. 3. Upperbound and simulation BER $N = K = 8$, $L = 2$, $CFO = 0, \pm 0.3$.

Fig. 4 shows significant performance improvement of the ML detector, where performance of MC-CDMA with ML-MUD is compared to MC-CDMA with the single user MRC detection. The system parameters were $N = 16$, $L = 2$, $K = 16$. As compared with Fig. 3 with $N = 8$, we see that ML performs better since there were more pairwise MAI-free users. Separate simulations were performed to acquire the BEP performance for $CFO = 0$ and $CFO = \pm 0.3$. We see that the BEP achieved by ML for both systems is very low when SNR is close to 10 dB. We also see that in both figures the ML detector performs better without CFO.

VII. Conclusion

The performance of MC-CDMA in a CFO environment with ML-MUD and CI codewords was studied. The sparse cross-correlation matrix of CI codewords can be used to reduce the complexity of ML-MUD. Simulation results were given to corroborate derived theoretical results.

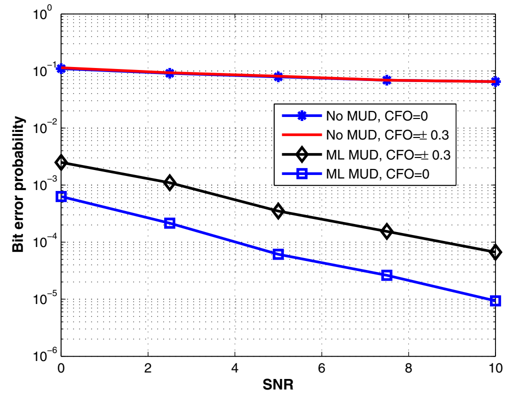

Fig. 4. BEP versus SNR $N = 16$, $L = 2$, $CFO = 0, \pm 0.3$.

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