DESIGN OF MAI-FREE MC-CDMA SYSTEMS OVER FREQUENCY-SELECTIVE FADING CHANNELS VIA CODEWORD SELECTION

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ABSTRACT

A scheme to completely eliminate the multiaccess interference (MAI) of the multicarrier code division multiple access (MC-CDMA) system in a frequency-selective fading environment using codeword selection is proposed. We show that, if the codewords of CDMA are chosen to be a proper subset of the Hadamard-Walsh codes, the MC-CDMA system can be MAI-free so that the implementational complexity of the transceiver can be greatly reduced. This MAI-free property is analyzed theoretically and demonstrated via simulation results.

1. INTRODUCTION

Multiaccess interference (MAI) is a major impairment that limits the performance of CDMA-based systems. In a synchronous CDMA (S-CDMA) system where user's timing is aligned within a fraction of chip-time interval, MAI can be greatly reduced by the use of orthogonal codewords [1]. S-CDMA can be used in downlink transmission for large cells such as in the digital cellular IS-95 standard and in both downlink and uplink transmissions in micro cells such as personal communication services (PCS) [1]. However, the orthogonality may be destroyed in a multipath environment. Research has been conducted to use sophisticated multiuser detection (MUD) [2] or signal processing to mitigate MAI. These techniques demand extra complexity in the receiver.

Recently, multicarrier CDMA (MC-CDMA) has been proposed as a promising multiuser technique. MC-CDMA systems can be classified to two types [3]. For the first type, a symbol is transmitted per time slot. It is spread into several chips that are allocated to different subchannels. The number of subchannels is equal to the number of chips. For the second type, several symbols are serial-to-parallel converted and each symbol is spread into several chips. The chips corresponding to the same symbol are allocated to the same subchannel. This is often called the MC-DS-CDMA system. As compared with the conventional CDMA systems, MC-CDMA can effectively combat the inter-symbol-interference (ISI). Moreover, MC-CDMA can fully exploit the frequency diversity gain if the maximum ratio combing (MRC) is used at the receiver. However, the performance of MC-CDMA is still greatly limited by MAI. The MAI-free property is particularly important for mobile communications since it is desirable for the mobile unit to have a simple architecture.

In this work, we propose a codeword selection scheme that can achieve completely MAI-free property for the first type MC-CDMA system in both uplink and downlink transmissions. As a result, the receiver complexity for symbol detection can be greatly reduced since there is no need to use sophisticated MUD or signal processing to eliminate MAI. Moreover, since our code allows the use of MRC and achieves zero MAI at the same time, a full diversity gain of multipath length can be achieved. The main results of our work are given below. Let the spreading factor be $N = 2^{n_s}$ and the multipath length be L. We first show that the MAI-free property for a multipath channel of length L cannot be reached if $N < 2L$. Then, we prove that, if the N Hadamard-Walsh codes are properly divided into $G = 2^{n_g}$ groups so that each group has N/G codewords and if $n_s > n_g > 1$ and $G \geq L$, MAI-free can be achieved using any group of codewords. Simulation results are given to corroborate theoretical results.

2. SYSTEM MODEL

The block diagram of an MC-CDMA system in the uplink direction, *i.e.,* from the mobile station to the base station, is shown in Fig. 1. Note that, even though the analysis is conducted for the uplink direction, the same analysis applies to downlink transmission as well if we set the channel fading of every user to be the same. At each time slot, the input is a modulation symbol. Suppose that there are T users. The symbol from user i is denoted by ${}_i$. In the first stage, ${}_i$ is spread by N chips to form an $N \times 1$ vector, denoted by $\mathbf{y}_i$ and whose kth element is ${}_i[k]$. The relation between ${}_i[k]$ and ${}_i$ is

$$ {}_i[k] = {}_i[k] \ {}_i, \quad 0 \leq k \leq N-1, \tag{1}$$

where ${}_i[k]$ is the kth element of user i's orthogonal code. After spreading, $\mathbf{y}_i$ is passed through the $N \times N$ IDFT matrix. Then, the output is parallel-to-serial (P/S) converted and a cyclic prefix (CP) of length $L-1$ is added, where L is the largest delay spread in the system.

The receiver removes the CP and passes each block of size N through the $N \times N$ DFT matrix. Since there are T users, the kth element of the DFT output $\hat{\mathbf{y}}$ is given by [4]

$$\hat{}[k] = \sum_{j=0}^{T-1} {}_j[k] \ {}_j[k] + e[k], \tag{2}$$

where ${}_j[k]$ is the kth component of the N-point DFT of the jth channel path $h_j(n)$, and $e[k]$ is the received noise after DFT. Note that $\hat{\mathbf{y}}$ will be sent to T branches for symbol detection for T users. To detect symbols transmitted by the ith user, $\hat{\mathbf{y}}$ is multiplied by ${}_i^*[k]$ and frequency gain ${}_i^*[k]$, where $*$ denotes complex-conjugate. After frequency gain, the N chips are summed up to form the symbol $\hat{}_i$ given by

$$\hat{}_i = \sum_{k=0}^{N-1} {}_i^*[k] \ {}_i^*[k] \hat{}[k]. \tag{3}$$

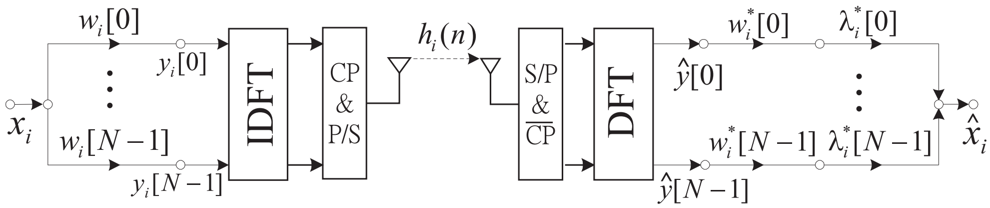

Fig. 1. The block-diagram of an MC-CDMA system.

Note that the path from $\hat{\mathbf{y}}$ to $\hat{\ }_i$ is usually called MRC [3]. From (1), (2) and (3), we have

$$\begin{aligned} \hat{\ }_i &= \ _i \sum_{k=0}^{N-1} | \ _i[k]|^2 + \sum_{k=0}^{N-1} \ _i^*[k] \ _i^*[k] e[k] \\ &+ \underbrace{\sum_{j=0, j\neq i}^{T-1} \ _j \sum_{k=0}^{N-1} \ _i^*[k] \ _i^*[k] \ _j[k] \ _j[k]}_{MAI_{i\leftarrow j}}, \end{aligned} \quad (4)$$

where $MAI_{i\leftarrow j}$ denotes the MAI from user j to user i.

Our goal is to make $MAI_{i\leftarrow j} = 0$. For any target user i, if $MAI_{i\leftarrow j} = 0$, the symbol $\hat{\ }_i$ will be affected only by his/her own transmitted symbols $\ _i$ and the corresponding channel response $\ _i[k]$. Thus, the system can adopt a simple detection scheme without using sophisticated MUD and/or signal processing techniques. When the channel fading is flat, $\ _i[k]$ and $\ _j[k]$ are constants and independent of k. Under this situation, $MAI_{i\leftarrow j} = 0$ if orthogonal codes such as the Hadamard-Walsh code is used. However, in practice, the channel is usually frequency-selective and the orthogonality of codewords can be destroyed. In the next section, we will evaluate $MAI_{i\leftarrow j}$ and seek ways to make it equal to zero under the multipath environment.

3. MAI-FREE MC-CDMA SCHEME

By the definition of DFT, we can expand $MAI_{i\leftarrow j}$ defined in (4) as

$$\begin{aligned} MAI_{i\leftarrow j} &= \ _j \sum_{k=0}^{N-1} \underbrace{\sum_{n,m=0}^{L-1} h_j(n) h_i^*(m) e^{-j\frac{2\pi}{N}k(n-m)}}_{\eta} \\ &\cdot \ _i^*[k] \ _j[k], \end{aligned} \quad (5)$$

where $\ $ is a function of k, i, and j. It can be shown that

$$\begin{aligned} &= \sum_{l=0}^{L-1} h_j(l) h_i^*(l) + \sum_{p=1}^{L-1} \sum_{q=0}^{L-1-p} \Big[h_j(\ + \ ) h_i^*(\ ) e^{-j\frac{2\pi}{N}kp} \\ &+ h_j(\ ) h_i^*(\ + \ ) e^{j\frac{2\pi}{N}kp} \Big]. \end{aligned} \quad (6)$$

According to (6), we can rewrite (5) as

$$\begin{aligned} MAI_{i\leftarrow j} &= \ _j \sum_{l=0}^{L-1} h_j(l) h_i^*(l) \mathcal{O}_1 + \ _j \sum_{p=1}^{L-1} \sum_{q=0}^{L-1-p} \\ &\cdot [h_j(\ + \ ) h_i^*(\ ) \mathcal{O}_2 + h_j(\ ) h_i^*(\ + \ ) \mathcal{O}_3], \end{aligned} \quad (7)$$

where

$$\mathcal{O}_1 = \sum_{k=0}^{N-1} \ _i^*[k] \ _j[k], \quad (8)$$

$$\mathcal{O}_2 = \sum_{k=0}^{N-1} e^{-j\frac{2\pi}{N}kp} \ _i^*[k] \ _j[k], \quad (9)$$

$$\mathcal{O}_3 = \sum_{k=0}^{N-1} e^{j\frac{2\pi}{N}kp} \ _i^*[k] \ _j[k]. \quad (10)$$

Based on (7)-(10), it is clear that if $\mathcal{O}_1 = \mathcal{O}_2 = \mathcal{O}_3 = 0$ simultaneously for $1 \leq \ \leq L-1$, we will achieve $MAI_{i\leftarrow j} = 0$. For convenience, let

$$\ _i^*[k] \ _j[k] = \ _{i,j}[k]. \quad (11)$$

Since the condition $\mathcal{O}_3 = 0$ is equivalent to $\mathcal{O}_3^* = 0$, we can rewrite the three conditions as follows:

$$\begin{cases} \sum_{k=0}^{N-1} \ _{i,j}[k] = 0, \\ \sum_{k=0}^{N-1} e^{-j\frac{2\pi}{N}kp} \ _{i,j}[k] = 0, \quad 1 \leq \ \leq L-1 \\ \sum_{k=0}^{N-1} e^{-j\frac{2\pi}{N}kp} \ _{i,j}^*[k] = 0, \quad 1 \leq \ \leq L-1 \end{cases} . \quad (12)$$

By solving the linear system in (12). It is possible to find codewords that achieve $MAI_{i\leftarrow j} = 0$. Note that if the codewords are real, the first two conditions in (12) are sufficient. Let us use a simple example to illustrate this idea.

Example: Assume that the multipath length $L = 2$. We consider different N in this example. First, let us see if $N = 2$ can achieve $MAI_{i\leftarrow j} = 0$.

N=2. From (12), we have the following conditions.

$$\begin{cases} \ _{i,j}[0] + \ _{i,j}[1] = 0, \\ \ _{i,j}[0] - \ _{i,j}[1] = 0, \\ \ _{i,j}^*[0] - \ _{i,j}^*[1] = 0. \end{cases} \quad (13)$$

Obviously, the only solution to (13) is $\ _{i,j}[0] = \ _{i,j}[1] = 0$. Hence, when $N = 2$, we cannot achieve MAI-free for $L = 2$.

N=3. From (12), we can obtain the following conditions.

$$\begin{cases} \ _{i,j}[0] + \ _{i,j}[1] + \ _{i,j}[2] = 0, \\ \ _{i,j}[0] + (-1+j) \ _{i,j}[1] + (-1-j) \ _{i,j}[2] = 0, \\ \ _{i,j}^*[0] + (-1+j) \ _{i,j}^*[1] + (-1-j) \ _{i,j}^*[2] = 0. \end{cases} \quad (14)$$

By solving the above linear system, the only solution to (14) is $\ _{i,j}[0] = \ _{i,j}[1] = \ _{i,j}[2] = 0$. Hence, when $N = 3$, we cannot reach MAI-free for $L = 2$.

N=4. From (12), we have following conditions.

$$\begin{cases} \ _{i,j}[0] + \ _{i,j}[1] + \ _{i,j}[2] + \ _{i,j}[3] = 0, \\ \ _{i,j}[0] + j \ _{i,j}[1] - \ _{i,j}[2] - j \ _{i,j}[3] = 0, \\ \ _{i,j}^*[0] + j \ _{i,j}^*[1] - \ _{i,j}^*[2] - j \ _{i,j}^*[3] = 0. \end{cases} \quad (15)$$

The solution to the linear system (15) can be written as

$$ {}_{i,j}[0] = - {}_{i,j}[1] = {}_{i,j}[2] = - {}_{i,j}[3]. \tag{16}$$

To meet this condition, we may consider a subset of the Hadamard-Walsh code. For example, consider the following 4×4 Hadamard matrix

$$\mathbf{W}_4 = \begin{pmatrix} +1 & +1 & +1 & +1 \\ +1 & -1 & +1 & -1 \\ +1 & +1 & -1 & -1 \\ +1 & -1 & -1 & +1 \end{pmatrix},$$

whose four columns are denoted by $\mathbf{w}_0$, $\mathbf{w}_1$, $\mathbf{w}_2$ and $\mathbf{w}_3$, respectively. Suppose that we assign two codewords $\{\mathbf{w}_0, \mathbf{w}_1\}$ to two users. The product of the 2 codewords yields

$$ {}_{0,1}[0] = - {}_{0,1}[1] = {}_{0,1}[2] = - {}_{0,1}[3] = 1.$$

Since the condition of (16) is met, the system is completely MAI-free for $L = 2$ and $T = 2$. Similarly, if the other pair of codewords $\{\mathbf{w}_2, \mathbf{w}_3\}$ is used, the system is also MAI-free.

The Example presents a codeword selection scheme to achieve the MAI-free property by solving a system of linear equations. However, as L and N grow, the effort to solve linear equations becomes tremendous. In the following, we will express the problem in matrix representation, which provides more insight into this problem. The conditions given by (12) can be written in matrix form as

$$\mathbf{F}_{L\times N}\boldsymbol{\Phi}_{N\times 1} = \mathbf{F}_{L\times N}\boldsymbol{\Phi}^*_{N\times 1} = \mathbf{0}_{L\times 1}, \tag{17}$$

where $\mathbf{F}_{L\times N}$ is the $L \times N$ matrix with its element defined by

$$[\mathbf{F}_{L\times N}]_{p,q} = e^{-j\frac{2\pi}{N}pq}, \tag{18}$$

$\boldsymbol{\Phi}_{N\times 1}$ is an $N \times 1$ vector of the form:

$$\boldsymbol{\Phi}_{N\times 1} = \left({}_{i,j}[0] \quad {}_{i,j}[1] \ldots \quad {}_{i,j}[N-1]\right)^t, \tag{19}$$

and $\mathbf{0}_{L\times 1}$ is the zero vector of dimension $L \times 1$. In the following, we will prove three lemmas and one theorem to characterize the solution to (17).

Lemma 1: Suppose the multipath length is L and the spreading gain is N. For either real or complex codewords, it demands $N \geq 2L$ to achieve the MAI-free property.

Proof: Let us first prove the lemma for real codewords. From (18), we know that $\mathbf{F}_{L\times N}$ gives the first L rows of the $N \times N$ DFT matrix. Hence, $\mathbf{F}_{L\times N}\boldsymbol{\Phi}_{N\times 1}$ is the first L elements of the N-point DFT of ${}_{i,j}[k]$. To achieve MAI-free, the first L elements of the N-point DFT of ${}_{i,j}[k]$ should be exactly zeros. Let the N-point DFT of ${}_{i,j}[k]$ be denoted by ${}_{i,j}[k]$. The DFT output of a real vector is complex-conjugate symmetric [5], *i.e.*

$$ {}_{i,j}[k] = {}^*_{i,j}[N-k], \quad 1 \leq k \leq \lceil N/2 \rceil - 1, \tag{20}$$

where $\lceil N/2 \rceil$ is the smallest integer that is greater or equal to $N/2$. Since the first L elements of of ${}_{i,j}[k]$ should be exactly zeros to achieve MAI-free, from (20), we demand

$$\begin{cases} {}_{i,j}[l] = 0, & 0 \leq l \leq L-1 \\ {}_{i,j}[N-l] = 0, & 1 \leq l \leq L-1 \end{cases}. \tag{21}$$

If $N \leq 2L - 1$, all elements of ${}_{i,j}[l]$ have to be zeros according to (21). Since this choice does not lead to a valid codeword, we demand $N \geq 2L$.

For the case of complex codewords, the third condition in (12) has to be satisfied as well. That is,

$$\sum_{k=0}^{N-1} e^{-j\frac{2\pi}{N}k(N-l)} {}_{i,j}[l] = 0, \quad 0 \leq l \leq L-1.$$

Thus, ${}_{i,j}[N-l]$ is zero for $0 \leq l \leq L-1$. Together with the first two conditions in (12), the complex codewords should meet the same conditions in (21) to achieve MAI-free. Hence, we demand $N \geq 2L$ for complex codewords as well. ■

Next, we show that a proper subset of Hadamard-Walsh codes can achieve the MAI-free conditions in (17) as follows.

Proposed code selection scheme: Suppose $N = 2^{n_s}$ and $G = 2^{n_g}$, where $n_s > n_g + 1 > 1$. Let the Hadamard matrix of order N be denoted by $\mathbf{W}_N$, whose columns form the N Hadamard-Walsh codes. Starting at codewords index 0, every consecutive N/G codewords define a group so that we have G groups with N/G codewords in each group. Moreover, group 0 is defined to be the one that contains column 0 of $\mathbf{W}_N$, which is the all one code and denoted by $\mathbf{w}_0$.

Lemma 2: Suppose the proposed code selection is used. Then, the product of any two codewords in the same group is equal to one of the N/G-1 codewords of group 0, where $\mathbf{w}_0$ is excluded.

Proof: Before we proceed to the proof, let us recall a well known property of the Hadamard matrix [6]. A Hadamard matrix $\mathbf{W}_N$ of order $N = 2^p$, $= 1, 2, \cdots$, can be recursively defined using the Hadamard matrix of order 2, *i.e.*

$$\mathbf{W}_N = \mathbf{W}_2 \otimes \mathbf{W}_{N/2} = \begin{pmatrix} \mathbf{W}_{N/2} & \mathbf{W}_{N/2} \\ \mathbf{W}_{N/2} & -\mathbf{W}_{N/2} \end{pmatrix}, \tag{22}$$

where $\otimes$ is the Kronecker product [6] and

$$\mathbf{W}_2 = \begin{pmatrix} +1 & +1 \\ +1 & -1 \end{pmatrix}.$$

Let us first prove that the product of any two codewords within group 0 is again a codeword within group 0. According to (22), the $N/G \times N/G$ upper left submatrix of $\mathbf{W}_N$ is an $N/G \times N/G$ Hadamard matrix. Thus, the product of any two columns of this submatrix is again a column within this submatrix (see [7]). Since the codewords in group 0 are the first N/G columns of $\mathbf{W}_N$, which is obtained by repeating the $N/G \times N/G$ submatrix by G times along the column. Hence, the product of any codewords in group 0 remains to be a codeword within group 0.

Next, let us prove that the product of any two codewords within the same group rather than group 0 is a codeword within group 0. Recall that ${}_i[k]$ is the kth element of the ith codeword. It can also be used to denote the kth element of the ith column of $\mathbf{W}_N$, *i.e.* the element at the kth row and the ith column. According to (22), we have the following property

$$\begin{cases} {}_i[k] = {}_{i+N/2}[k], & 0 \leq k \leq N/2-1, \\ {}_i[k] = - {}_{i+N/2}[k], & N/2 \leq k \leq N-1. \end{cases} \tag{23}$$

From (23), we see that the product of any two columns within the last half $N/2$ columns is equal to the product of the two corresponding columns within the first half $N/2$ column, *i.e.*

$$ {}_i[k] \quad {}_j[k] = {}_{i+N/2}[k] \quad {}_{j+N/2}[k], \quad 0 \leq i,j \leq N/2-1. \tag{24}$$

This can be viewed as dividing N codewords into two groups. The first $N/2$ half codewords forms group 0 and the last $N/2$ half codewords forms group 1. Using (24), the product of any two codewords in group 1 is equal to a codeword in group 0. Using a recursive procedure, we can divide N codewords into $G = 2^{n_g}$ groups and show the product of any two codewords within the same group is a codeword of group 0. ■

Lemma 3: Excluding $\mathbf{w}_0$, the N-point DFT of any of the codewords in group 0, denoted by $\mathcal{W}_i[f]$ with $0 \leq f \leq N-1$ and $1 \leq i \leq N/G - 1$, has the following property:

$$\mathcal{W}_i[0] = \mathcal{W}_i[1] = \ldots = \mathcal{W}_i[G-1] = 0. \tag{25}$$

Proof: For the DC term that $f = 0$, it is easy to see $\mathcal{W}_i[0] = \sum_{k=0}^{N-1} {}_i[k] = 0$ since there are equal positive ones and minus ones for any codewords excluding $\mathbf{w}_0$. Let us consider the case $f \neq 0$. Since the codewords in group 0 are the first N/G columns of $\mathbf{W}_N$, they are formed by repeating the upper left $N/G \times N/G$ submatrix of $\mathbf{W}_N$ by G times in the column. Since the codewords are repeated G times in the time domain, it will follow $G - 1$ zeros in the frequency domain as defined in (25) [5]. From (25) and Lemma 2, we have the following property

$$ {}_{i,j}[0] = {}_{i,j}[1] = \ldots = {}_{i,j}[G-1] = 0, \tag{26}$$

where ${}_{i,j}[k]$ is the kth element of the N-point DFT of the product of any two codewords in the same group. ■

The following theorem is the direct consequence of results in (17) and Lemmas 1-3.

Theorem: Let the multipath length be L. Suppose the proposed code selection is used with $G \geq L$. Then, if the codewords in any one group are used in the MC-CDMA system, the system is completely MAI-free.

Note that we did not constrain the multipath coefficient throughout the derivation. Thus, once $G \geq L$, MAI-free can be achieved for arbitrary multipath coefficients.

4. SIMULATION RESULTS

In this section, we provide simulation results to confirm the theoretical derivation given in the previous section. The simulation was conducted with the following parameter setting. The size of the Hadamard-Walsh codes is $N = 64$. The transmit power has an unit variance. The multipath length is L, where each tap is an i.i.d. random variable with an unit variance. We will evaluate the $MAI_{i \leftarrow j}$ defined in (5).

For $L = 2$, one realization of $|MAI_{i \leftarrow j}|$ as a function of user indexes i and j is shown in Fig. 2. We see that there are two zones where the MAI is completely zero. That is, the zone with codewords from 0 to 31, and the zone with codewords from 32 to 63. The diagonal terms have the peak value since it is the reconstructed desired signal power. It is clear that the system is MAI-free if we use either one of the two groups of codewords.

For $L = 4$, the performance is shown in Fig. 3. Again, we observe 4 zones where the MAI effect is zero. That is, codewords 0-15, 16-31, 32-47 and 48-63. Hence, we can use any one of the 4 groups to achieve the MAI-free property.

5. CONCLUSION AND FUTURE WORK

A codeword selection scheme was proposed to completely eliminate the MAI effect of the MC-CDMA system. The MAI-free property can greatly simplify the implementational complexity of the transceiver. This MAI-free property was proved and confirmed via simulation. We will study how the MAI-free property is affected by the carrier frequency offset (CFO) in the near future.

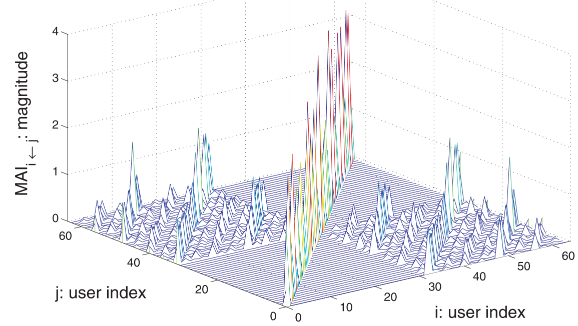

Fig. 2. $|MAI_{i \leftarrow j}|$ as a function of user indexes ($N = 64$, $L = 2$).

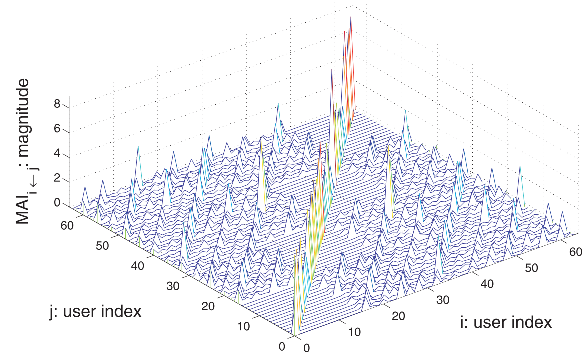

Fig. 3. $|MAI_{i \leftarrow j}|$ as a function of user indexes ($N = 64$, $L = 4$).

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