Enhanced Performance for an Approximately MAI-Free Multiaccess OFDM Transceiver by Code Selection

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Abstract— **A new code selection scheme is proposed to reduce the MAI of an approximately MAI-free OFDM transceiver that is known as the repetitively coded multicarrier CDMA (RCMC-CDMA) in this work. That is, if we use $M/2$ symmetric codewords or anti-symmetric codewords of the M Hadamard-Walsh codes, the MAI of the system can be greatly reduced. With this code selection scheme, the requirement of a sufficiently large number of subchannels to achieve the MAI-free property can be relaxed so that the approximately MAI-free property still holds when the number of subchannels is relatively small.**

Keywords— **Multiaccess OFDM, RCMC-CDMA, MAI-free, codeword selection, OFDMA.**

I. Introduction

Multicarrier/OFDM/DMT systems have been widely used in communications systems these days, *e.g.* xDSL, WLAN and DVB [1], [2]. Multicarrier systems can deal with the inter-symbol-interference (ISI) problem effectively via a simple transceiver structure using FFT/IFFT and a cyclic prefix (CP). Recently, multiaccess OFDM systems have also received a lot of attention [3], [4]. There are two popular OFDM-based multiaccess schemes that provide attractive solutions to the multiaccess environment. They are the multicarrier CDMA (MC-CDMA) technique [3] and the orthogonally frequency division multiple access (OFDMA) technique [4].

Similar to OFDM, MC-CDMA can also combat the ISI effect caused by frequency-selective fading. However, the capacity of MC-CDMA systems is limited by multiaccess interference (MAI). To suppress the MAI effect, sophisticated multiuser detection (MUD) or signal processing techniques are needed at the receiver end, which implies a higher cost for the deployment of MC-CDMA systems. In contrast, OFDMA systems are MAI-free when there is no carrier frequency offset (CFO). However, OFDMA systems are sensitive to CFO. In the presence of CFO, OFDMA systems are no longer MAI-free. Since every user has his/her own CFO in the multiaccess environment, the problem of CFO estimation and compensation is much more complicated in the OFDMA system than that in the single user OFDM system.

An approximately MAI-free multiaccess OFDM transceiver, called the repetitively coded multicarrier CDMA (RCMC-CDMA), was proposed in [5]. The approximately MAI-free property holds when the number of subchannels, N, is sufficiently large. Unlike conventional MC-CDMA systems, there is no need to suppress MAI in the RCMC-CDMA system. Consequently, the complexity is lower. Furthermore, the RCMC-CDMA system can greatly mitigate the CFO effect that limits the performance of OFDMA systems [6].

In this work, we propose a code selection scheme to further enhance the MAI suppression capability of the RCMC-CDMA system. That is, if we use only $M/2$ symmetric or anti-symmetric codewords of the M Hadamard-Walsh codes in the system, the MAI effect can be greatly reduced. This result can help relax the requirement on the large number of subchannels, N, for the RCMC-CDMA system to have the approximately MAI-free property. Simulation results are given to support the theoretical analysis.

II. RCMC-CDMA System

In this section, we will review the RCMC-CDMA system [5]. The system block diagram is given in Fig. 1, where there are T users. The input of the ith user is an $N \times 1$ vector denoted by $\mathbf{x}_i$. The vector consists of N modulation symbols, *e.g.* PSK or QAM. Each symbol in $\mathbf{x}_i$ is repeated M times and then scaled to form an $NM \times 1$ vector $\mathbf{y}_i$. Let $x_i[k]$ denote the kth symbol of $\mathbf{x}_i$, $0 \le k \le N-1$, and $y_i[l]$ denote the lth symbol of $\mathbf{y}_i$, $0 \le l \le NM-1$. The relation between $x_i[k]$ and $y_i[l]$ is given by

$$y_i[m+kM] = \frac{1}{\sqrt{M}} x_i[k], \quad 0 \le m \le M-1, \qquad (1)$$

where $\frac{1}{\sqrt{M}}$ is included to preserve the power before and after the symbol repetition. For convenience, we call the NM repeated symbols "expanded symbols". Next, each vector $\mathbf{y}_i$ is passed through an $NM \times NM$ diagonal matrix $\mathbf{W}_i$ with its diagonal elements drawn by the $M \times M$ unitary matrix $\mathbf{D}$ (*i.e.* $\mathbf{D}^\dagger \mathbf{D} = M\mathbf{I}$, where $\mathbf{D}^\dagger$ denotes the conjugate-transpose of $\mathbf{D}$ and $\mathbf{I}$ is the $M \times M$ identity matrix). Let $\mathbf{d}_1$, $\mathbf{d}_2$, $\cdots$, $\mathbf{d}_M$ be the M columns of $\mathbf{D}$. Then, the matrix $\mathbf{W}_i$ is formed by repeating $\mathbf{d}_i$ by N times along diagonal, *i.e.* $\mathbf{W}_i = \text{diag}(\mathbf{d}_i^t \; \mathbf{d}_i^t \; \cdots \mathbf{d}_i^t)$, where $\mathbf{d}_i^t$ is the transpose of $\mathbf{d}$. When M is equal to the power of 2, an example for $\mathbf{D}$ is the Hadamard matrix, whose columns form the M Hadamard-Walsh codes. For instance, letting $M = 2$, the second column of the 2×2 Hadamard matrix is $(+1 \; -1)^t$. If $N = 2$, we have $\mathbf{W}_2 = \text{diag}(+1 \; -1 \; +1 \; -1)$. Since $\mathbf{D}$ is unitary, we have the following property

$$\sum_{m=0}^{M-1} w_i[m+kM] w_j^*[m+kM] = \begin{cases} M, & i = j \\ 0, & i \ne j \end{cases}, \qquad (2)$$

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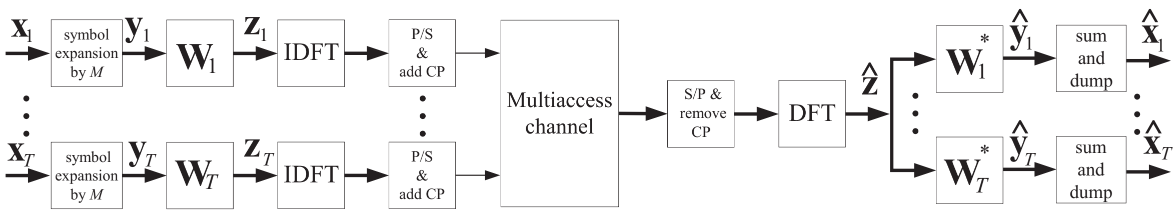

Fig. 1. The block diagram of the RCMC-CDMA system.

where w^* is the conjugate of w and $0 \le k \le N-1$. Afterwards, the coded vector is passed through the $NM-$point inverse discrete Fourier transform (IDFT) matrix. Finally, each transformed vector is converted from parallel to serial (P/S) and the cyclic prefix (CP) of length $\nu - 1$ is added for ISI cancellation, where ν is the maximum length of multipath propagation.

The channel environment can be either wired or wireless. For the wired environment, the channel path is the equivalent channel equalized by the time-domain equalizer (TEQ) [2]. Let the channel path of user i be $h_i(n)$. For convenience, the elements of the NM-point DFT of $h_i(n)$ are called "expanded subchannels" and denoted by $\lambda_i[l]$, $0 \le l \le NM-1$, and the elements of the N-point DFT of $h_i(n)$ are called "subchannels" and denoted by $\lambda_i^{(N)}[k]$, $0 \le k \le N-1$. Thus, the expanded subchannels with index from $0+kM$ to $M-1+kM$ are actually associated with the kth subchannel.

At the receiver side, a data block of size $N \cdot M + \nu$ is received. After removing the CP, the block is converted from serial to parallel (S/P) and then passed through the discrete Fourier transform (DFT) matrix. The DFT output vector is sent to M subbranches, multiplied by $\mathbf{W}_i^*$ and then passed through the block "sum and dump". Let $\hat{\mathbf{y}}_i$ and $\hat{\mathbf{x}}_i$ denote the input and the output of the "sum and dump", respectively. The kth element of $\hat{\mathbf{x}}_i$ is given by

$$\hat{x}_i[k] = \frac{1}{\sqrt{M}} \sum_{m=0}^{M-1} \hat{y}_i[m+kM], \quad 0 \le k \le N-1, \tag{3}$$

where $\frac{1}{\sqrt{M}}$ is included so that $\hat{x}_i[k] = x_i[k]$ if the channel is perfect and there is no channel noise.

The RCMC-CDMA system has the approximately MAI-free property as described below [5]. From Fig. 1, the lth element of $\hat{\mathbf{z}}$ is given by

$$\hat{z}[l] = \sum_{i=1}^{T} \lambda_i[l] z_i[l] + e[l], \quad 0 \le l \le NM-1, \tag{4}$$

where $e[l]$ is the lth element of the DFT of the $NM \times 1$ received noise vector. Note that $z_i[l] = w_i[l] y_i[l]$ and $\hat{y}_j[l] = w_j^*[l]\hat{z}_j[l]$. Thus, from (4), the lth element of $\hat{\mathbf{y}}_j$ can be expressed as

$$\begin{aligned} \hat{y}_j[l] &= \lambda_j[l] y_j[l] w_j[l] w_j^*[l] + \sum_{i=1, i\ne j}^{T} \lambda_i[l] y_i[l] w_i[l] w_j^*[l] \\ &\quad + e[l] w_j^*[l], \quad 0 \le l \le NM-1. \end{aligned} \tag{5}$$

Let $l = m + kM$. From (3) and (5), the kth element of $\hat{\mathbf{x}}_j$ is given by

$$\hat{x}_j[k] = S_j[k] + \sum_{i=1, i\ne j}^{T} MAI_{j\leftarrow i}[k] + E_j[k], \tag{6}$$

where

$$S_j[k] = \frac{1}{\sqrt{M}} \sum_{m=0}^{M-1} \lambda_j[m+kM] y_j[m+kM] w_j[m] w_j^*[m], \tag{7}$$

$$MAI_{j\leftarrow i}[k] = \frac{1}{\sqrt{M}} \sum_{m=0}^{M-1} \lambda_i[m+kM] y_i[m+kM] w_i[m] w_j^*[m], \tag{8}$$

and

$$E_j[k] = \frac{1}{\sqrt{M}} \sum_{m=0}^{M-1} e[m+kM] w_j^*[m+kM]. \tag{9}$$

Note that we use the notation $MAI_{j\leftarrow i}[k]$ in (8) to denote the MAI at the kth symbol of user j contributed by user i. When N is sufficiently large, each of the N subchannels can be assumed to be flat [1], [2]. Thus, the fading of the M expanded subchannels from $\lambda_i[0+kM]$ to $\lambda_i[M-1+kM]$ can be approximated by the kth associated subchannel. That is, $\lambda_i[m+kM]$ can be approximated by

$$\lambda_i[m+kM] \approx \lambda_i^{(N)}[k], \quad 0 \le m \le M-1, \quad 0 \le k \le N-1, \tag{10}$$

where recalled that $\lambda_i^{(N)}[k]$ is the kth element of the N-point DFT of the ith channel path, i.e. the kth subchannel. Using (1) and the approximation in (10), we can rewrite Eqns. (7) and (8) as

$$\begin{aligned} S_j[k] &\approx \frac{1}{M} \lambda_j^{(N)}[k] x_j[k] \sum_{m=0}^{M-1} w_j[m] w_j^*[m] \\ &= \lambda_j^{(N)}[k] x_j[k] \quad \text{(by (2))}, \end{aligned} \tag{11}$$

and

$$\begin{aligned} MAI_{j\leftarrow i}[k] &\approx \frac{1}{M} \lambda_i^{(N)}[k] x_i[k] \sum_{m=0}^{M-1} w_i[m] w_j^*[m] \\ &= 0 \quad \text{(by (2))}. \end{aligned} \tag{12}$$

As given in (12), the MAI is approximately zero. Hence, if there is no channel noise, we can approximately reconstruct $x_i[k]$ by multiplying $\hat{x}_i[k]$ by $\left(\lambda_i^{(N)}[k]\right)^{-1}$. An alternative for reconstruction is multiplying $\hat{x}_i[k]$ by $\left(\frac{1}{M}\sum_{m=0}^{M-1}\lambda_i[m+kM]\right)^{-1}$. Since the RCMC-CDMA system is approximately MAI-free, its capacity increases as SNR increases. This result is very different from the conventional CDMA system, where increasing the power of one user will also increase the MAI for other users. Moreover, the RCMC-CDMA can greatly mitigate the CFO effect which limits the performance in OFDMA system [6].

III. RCMC-CDMA WITH CODE SELECTION

A. Relation between Fading and Approximately MAI-Free

Since the approxmiation in (10) is used to obtain the approximately MAI-free property, the accuracy of the approximation determines the system performance. That is, the more accurate the approximate is, the less MAI the system will have. To achieve the approximately MAI-free property, we demand that the OFDM-block duration be much longer than the multipath propagation delay. In this situation, the M expanded subchannels associated to each of the N subchannels will experience nearly the same fading. This kind of environment is typical for OFDMA systems in broadband wireless access applications [4].

We show in Fig. 2 (a) the case where the M expanded subchannels of the kth subchannel have the same fading. That is, the subchannel is flat. For this case, the approximation in (10) becomes equality so that there is no MAI. However, the subchannel is in general not flat if the number of multipath propagation, ν, is greater than 1. If $\nu > 1$, a larger value of ν demands a larger value of N to achieve the approximately MAI-free property. Since larger N leads to longer OFDM-block duration, when the system is operated in a fast-varying wireless environment, a longer duration may be greater than the channel coherent time and as a result it destroys the orthogonality. The conflicting requirements on N motivate us to seek another way to resolve the problem. That is, we would like relax the requirement of large N, but keep the approximately MAI-free property of the system.

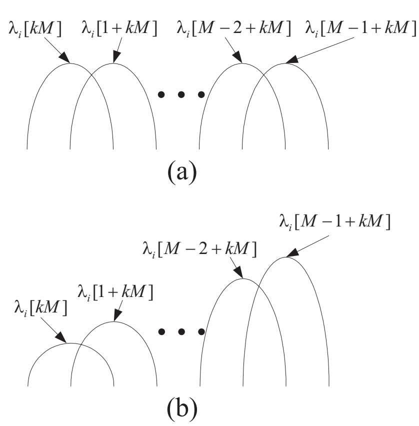

Fig. 2. The fading of the M expanded subchannels for the kth subchannel when the subchannel response is (a) flat and (b) monotonically increasing with the frequency.

For a fixed value of ν, we can find a minimum value of N so that the M expanded subchannels corresponding to each subchannel are monotonically increasing or decreasing. For example, we show in Fig. 2 (b) the case where the expanded subchannels in association with the kth subchannel are monotonically increasing. We will use Fig. 2 (b) to illustrate the concept of MAI reduction. Similar arguments can be applied to the monotonically decreasing case.

As shown in Fig. 2(b), when the number of subchannels, N, increases, the distance between $\lambda_i[M-1+kM]$ and $\lambda_i[kM]$ decreases. Therefore, the assumption that $\lambda_i[kM] \approx \lambda_i[M-1+kM] \approx \lambda_i^{(N)}[k]$ becomes more accurate. That is, when N is sufficiently large, the system is approximately MAI-free. When N is not sufficiently large, we can reduce the MAI effect by properly choosing the orthogonal codewords.

B. Proposed Code Selection Scheme

Suppose that only $M/2$ symmetric or anti-symmetric codewords of the M Hadamard-Walsh codes are used, $MAI_{j\leftarrow i}[k]$ in (8) can be computed as

$$MAI_{j\leftarrow i}[k] = \frac{1}{M}x_i[k]\sum_{m=0}^{M/2-1}\{\lambda_i[m+kM] + \lambda_i[M-m-1+kM]\}\, w_i[m]w_j^*[m], \quad (13)$$

where $0 \le k \le N-1$. It is our claim that $MAI_{j\leftarrow i}[k]$ in (13), which uses the code selection scheme, is much smaller than $MAI_{j\leftarrow i}[k]$ in(8), which uses the M full codewords.

Let us sketch the proof of this claim below. It can be shown that, when only the $M/2$ symmetric or anti-symmetric codewords of the M Hadamard-Walsh codes are used, the $M/2$ selected codewords satisfy the following property:

$$\sum_{m=0}^{M/2-1} w_i[m+kM]w_j^*[m+kM] = \begin{cases} M/2, & i=j \\ 0, & i \neq j \end{cases}. \quad (14)$$

Using (13), (14) and the following assumption

$$\lambda_i[m+kM] + \lambda_i[M-m-1+kM] \approx 2\lambda_i^{(N)}[k], \quad (15)$$

where $0 \le m \le M/2-1$, we achieve $MAI_{j\leftarrow i}[k] \approx 0$ again.

Although the assumptions of both (10) and (15) lead to the approximately MAI-free property, the assumption (15) is much more accurate than the assumption (10) due to the following reason. We see from Fig. 2 and (15) that the largest expanded subchannel fading will be added to the smallest one, the second largest expanded subchannel fading will be added to the second smallest one, and so on. This makes the summation of every two-term pair be very close to $2\lambda_i^{(N)}[k]$. Thus, for a fixed N that makes the M expanded subchannel fading associated to any subchannel monotonically increasing or decreasing, the approximation in (15) is much more accurate than that in (10). Let us give an example for the proposed code selection scheme.

Example: Let $M = 8$. The 8×8 Hadamard matrix is given as follows.

$$\mathbf{D} = \begin{pmatrix} +1 & +1 & +1 & +1 & +1 & +1 & +1 & +1 \\ +1 & -1 & +1 & -1 & +1 & -1 & +1 & -1 \\ +1 & +1 & -1 & -1 & +1 & +1 & -1 & -1 \\ +1 & -1 & -1 & +1 & +1 & -1 & -1 & +1 \\ +1 & +1 & +1 & +1 & -1 & -1 & -1 & -1 \\ +1 & -1 & +1 & -1 & -1 & +1 & -1 & +1 \\ +1 & +1 & -1 & -1 & -1 & -1 & +1 & +1 \\ +1 & -1 & -1 & +1 & -1 & +1 & +1 & -1 \end{pmatrix}.$$

We can either choose the column set $\{\mathbf{d}_1, \mathbf{d}_4, \mathbf{d}_6, \mathbf{d}_7\}$, which consists of symmetric vectors, or the column set $\{\mathbf{d}_2, \mathbf{d}_3, \mathbf{d}_5, \mathbf{d}_8\}$, which consists of anti-symmetric vectors, for the four codewords of four users. Although the code selection scheme decreases the number of users from M to $M/2$, it relaxes the requirement of a large N value to achieve the approximately MAI-free property.

IV. Simulation Results

In this section, we provide simulation results to demonstrate that N can be greatly relaxed to achieve the approximately MAI-free property if the proposed code selection scheme is used. The simulation was conducted under the following parameter setting. The expansion factor is chosen to be $M = 16$. The BPSK modulation is used. Each tap of channel path $h_i(n)$, $0 \leq n \leq \nu - 1$, is an i.i.d. (independently identically distributed) complex Gaussian random variable with the unit variance. The overall channel variance is normalized to 1, *i.e.* $E\left\{\sum_{n=0}^{\nu-1} |h_i(n)|^2\right\} = 1$. In practical OFDMA systems, the commonly used NM value is 2048 [4], which is corresponding to a reasonable OFDM-block duration time so that the channel coherent time is much larger than the block duration. Here, we let NM not to exceed 2048 to achieve the same purpose. The Monte Carlo method is used to run for more than 1000 OFDM blocks per user. The total MAI is obtained as follows. For the kth symbol of a target user, we accumulate the MAI contributed from all N symbols of the other $T-1$ users. The procedure is repeated and, then, the MAI power is averaged for k from 0 to $N-1$. That is, the total MAI is obtained by averaging $\frac{1}{N}\sum_{k=0}^{N-1}\left|\sum_{i=1, i\neq j}^{T} MAI_{j\leftarrow i}[k]\right|^2$ for 1000 OFDM blocks. Let the number ν of the multi-path propagation be 4. The total MAI power (in dB) of each user for $N = 16$, 32, 64 and 128 is shown in Fig. 3, where the 16 circled curves are for a system using the M full codewords, the 8 squared curves are for a system using the first $M/2$ codewords of the M Hadamard-Walsh codes and the 8 triangulated curves are for a system using the $M/2$ symmetric codewords of the Hadamard-Walsh codes. Please note that the performance of each individual user is very similar so that these curvers are clustered to result in three bold curves visually.

The 16 circled, 8 squared and 8 triangulated curves are averaged and redrawn in Fig. 4 to show the averaged total MAI for all users. We see that when N increases, the MAI decreases. This is reasonable, since a larger N gives a better approximation to lead to approximately MAI-free. However, for the system using the M full codewords with a smaller value of N such as $N = 16$ and $N = 32$, the MAI is greater than -13 dB. This MAI level is usually a non-negligible interference source that limits the system performance. When only the first $M/2$ codewords of the Hadamard-Walsh codes are used (not the code selection scheme), we see the performance has a reasonable improvement around 5 dB since the user number is decreased from 16 to 8. However, the improvement is not good enough. Let us see the proposed code selection scheme. When only $M/2$ symmetric codewords of the Hadamard-Walsh codes are used, we see that the system outperforms the one using full codewords by 17 dB when $N = 16$ and 35 dB when $N = 128$. The performance has improved dramatically. Moreover, we see that the system using symmetric codewords with $N = 16$ can achieve comparable performance of that using full codewords with $N = 128$. That is, by using symmetric codewords, N can be greatly relaxed while the system still owns the approximately MAI-free property.

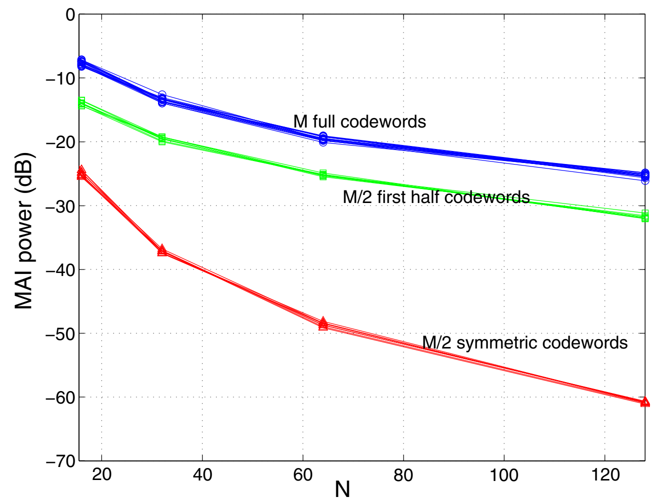

Fig. 3. The total MAI of individual user as a function of N with $\nu = 4$ with different code selection schemes.

The down-triangulated curve in Fig. 5 shows the system using only the $M/2$ anti-symmetric codewords of the Hadamard-Walsh codes. When compared to the up-triangulated curve in Fig. 4, we see that the symmetric and the anti-symmetric codewords have similar performance.

Let ν increase from 4 to 8. The average MAI performance with $\nu = 8$ is shown in Fig. 6. We have similar observations. That is, the use of symmetric codewords greatly outperforms the system with either the full M codewords or the first $M/2$ codewords. When $N = 16$, the MAI for symmetric codewords is around -10 dB which is not a negligible interference source when compared with the unit transmit power of BPSK. This is reasonable because $N = 16$ is not large enough for $\nu = 8$ so that the expanded subchannel fading within each subchannel are monotonically increasing

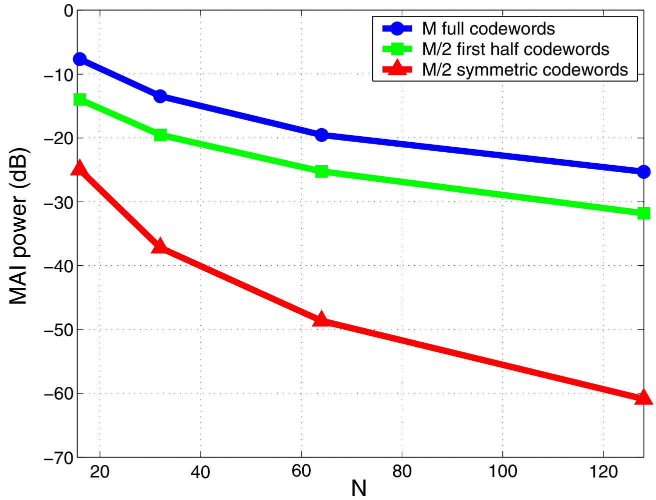

Fig. 4. The averaged total MAI as a function of N with $\nu = 4$ with different code selection schemes, including the $M/2$ symmetric codewords.

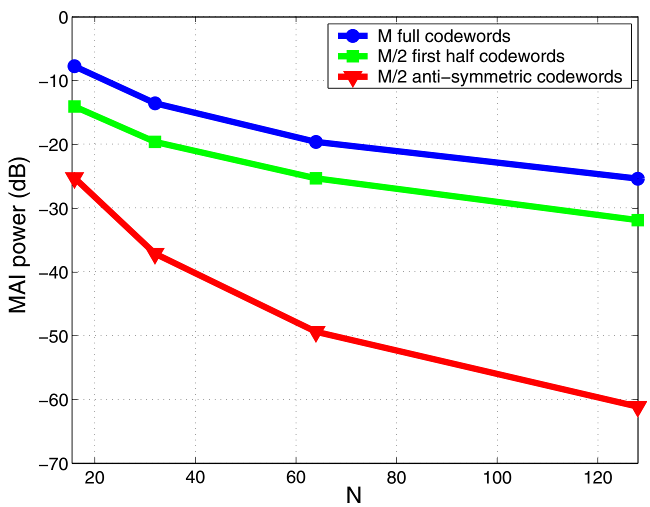

Fig. 5. The averaged total MAI as a function of N with $\nu = 4$ with different code selection schemes, including the $M/2$ anti-symmetric codewords.

or decreasing. Fig. 7 gives the performance when $\nu = 12$, a long propagation delay situation. From the figure, we see that the use of symmetric codewords with $N = 64$ can still reduce the MAI to be less than -25 dB.

V. Conclusion

We proposed a code selection scheme to further reduce the MAI in the RCMC-CDMA system. It was shown that, when the number ν of multipath propagation increases, the number N of subchannels should be increased accordingly to achieve the approximately MAI-free property. By using the code selection scheme, the N value to achieve the approximately MAI-free property can be greatly relaxed.

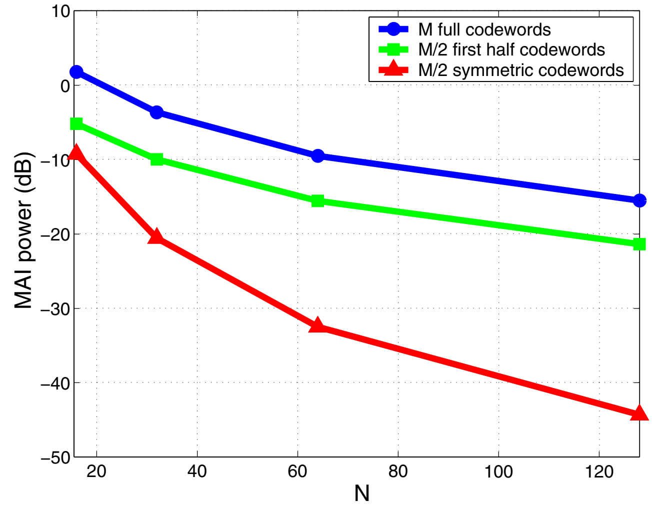

Fig. 6. The averaged total MAI power as a function of N for $\nu = 8$ with different code selection schemes, including the $M/2$ symmetric codewords.

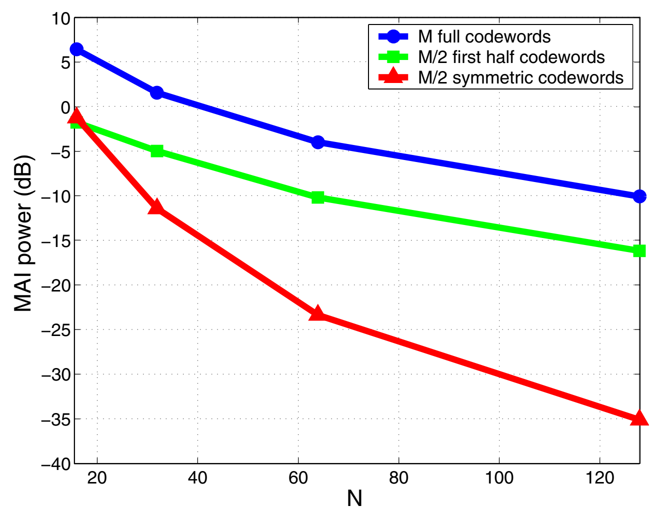

Fig. 7. The averaged total MAI power as a function of N for $\nu = 12$ with different code selection schemes, including the $M/2$ symmetric codewords.

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