# Performance Evaluation of Approximately MAI-Free Multiaccess OFDM Transceiver

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*Abstract*— **The performance of an approximately MAI-free OFDM transceiver, called the repetitively coded multicarrier CDMA (RCMC-CDMA), is evaluated and compared with MC-CDMA in this work. In particular, its advantages on the MAI reduction capability and the low implementational complexity are presented. We first demonstrate that RCMC-CDMA has the performance comparable to MC-CDMA using a linear MUD (multiuser detection) MMSE (minimum mean squared errors) detector, while it outperforms MC-CDMA with a MUD decorrelating detector. Then, the complexity of RCMC-CDMA is shown to be lower than that of MUD-based MC-CDMA.**

*Keywords*— **Multiaccess OFDM, MAI-free, RCMC-CDMA, MC-CDMA, multiuser detection.**

## I. Introduction

Multicarrier modulation code division multiple access (MC-CDMA) systems have been proposed for multiaccess transmission recently [1]. As compared with CDMA systems, MC-CDMA can effectively combat the inter symbol interference (ISI) caused by frequency-selective fading. However, the capacity of MC-CDMA system is limited by multiaccess interference (MAI). To suppress the MAI effect, multiuser detection (MUD) is usually used at the receiver end [2]. To perform MUD, the receiver has to know the channel statistic information (CSI) and channel estimation is needed. As the number of active users grows, accurate channel estimation for MC-CDMA systems becomes more difficult due to MAI [3]. Moreover, the complexity of the MC-CDMA system increases as a result of the use of MAI suppression techniques.

An approximately MAI-free multiaccess OFDM transceiver, called the repetitively coded multicarrier CDMA (RCMC-CDMA), was proposed in [4]. The approximately MAI-free property holds when the number of subchannels, $N$, is sufficiently large. Unlike the conventional MC-CDMA system, there is no need to suppress MAI in the RCMC-CDMA system. Consequently, the complexity is lower and channel estimation can be done more easily. Moreover, the RCMC-CDMA system promises the capacity increase as the transmit power increases. This result stands in contrary to the conventional MC-CDMA system, in which increasing the transmit power for one user will also increase the MAI for other users.

Under the assumption that only the receiver knows the CSI, the performance of RCMC-CDMA is evaluated and compared with that of conventional MC-CDMA with MUD in this work. In particular, the following two advantages of the proposed RCMC-CDMA scheme will be demonstrated. First, the RCMC-CDMA outperforms the MC-CDMA with a decorrelating detector and has the performance comparable to MC-CDMA with a linear MMSE (LMMSE) detector. Second, when the number of active users grows or multiple antennas are used, the complexity superiority of RCMC-CDMA over MUD-based MC-CDMA becomes more pronounced.

## II. RCMC-CDMA System Description and Properties

### A. System Description

Fig. 1 shows the block diagram of the RCMC-CDMA system with $T$ users. The transmit side (TX) contains four stages. At the first stage, the input of the $i$th user is an $N \times 1$ vector $\mathbf{x}_i$, consisting of $N$ modulation symbols, *e.g.* PSK or QAM. Each symbol in $\mathbf{x}_i$ is repeated $M$ times and then scaled to form an $NM \times 1$ vector $\mathbf{y}_i$. Let $x_i[k]$ denote the $k$th symbol of $\mathbf{x}_i$, $0 \le k \le N-1$, and $y_i[l]$ denote the $l$th symbol of $\mathbf{y}_i$, $0 \le l \le NM-1$. The relation between $x_i[k]$ and $y_i[l]$ is given by

$$y_i[m+kM] = \frac{1}{\sqrt{M}} x_i[k], \quad 0 \le m \le M-1, \qquad (1)$$

where $\frac{1}{\sqrt{M}}$ is included to preserve the power before and after the symbol repetition. At the second stage, each vector $\mathbf{y}_i$ is passed through an $NM \times NM$ diagonal matrix $\mathbf{W}_i$ with its diagonal elements drawn by the $M \times M$ unitary matrix $\mathbf{D}$, *i.e.* $\mathbf{D}^\dagger \mathbf{D} = M\mathbf{I}$, where $\mathbf{D}^\dagger$ is the conjugate-transpose of $\mathbf{D}$ and $\mathbf{I}$ is the $M \times M$ identity matrix. Let $\mathbf{d}_i$ denote the $i$th column of $\mathbf{D}$. Then, $\mathbf{W}_i$ is obtained by repeating $\mathbf{d}_i$ by $N$ times along the diagonal, *i.e.*

$$\mathbf{W}_i = \text{diag}(\mathbf{d}_i^t \ \mathbf{d}_i^t \cdots \ \mathbf{d}_i^t),$$

where $\mathbf{d}_i^t$ denotes the transpose of $\mathbf{d}_i$ and diag($\mathbf{d}^t$) is the function which puts the elements of $\mathbf{d}^t$ on the diagonal. When $M$ is equal to the power of 2, an example for $\mathbf{D}$ is the Hadamard matrix, whose columns form the $M$ Hadamard-Walsh codes [2]. For instance, letting $M = 2$, the second column of the $2\times2$ Hadamard matrix is $(+1\ -1)^t$. If $N = 2$, we have $\mathbf{W}_2 = \text{diag}(+1\ -1\ +1\ -1)$. Since $\mathbf{D}^\dagger \mathbf{D} = M\mathbf{I}$, the diagonal elements of $\mathbf{W}_i$ satisfy the following property:

$$\sum_{m=0}^{M-1} w_i[m+kM] w_j^*[m+kM] = \begin{cases} M, & i = j \\ 0, & i \ne j \end{cases}, \qquad (2)$$

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where $w^*$ is the conjugate of $w$ and $0 \le k \le N-1$. At the third stage, each orthogonally coded vector is passed through the $NM-$point inverse discrete Fourier transform (IDFT) matrix. Finally, at the fourth stage, each transformed vector is converted from parallel to serial (P/S) and the cyclic prefix (CP) of length $\nu - 1$ is added to combat the ISI, where $\nu$ is the maximum length of multipath propagation considered.

The channel path of user $i$ is denoted by $h_i(n)$. For convenience, the elements of $NM$-point DFT of $h_i(n)$ are called "expanded subchannels" and denoted by $\lambda_i[l]$, $0 \le l \le NM-1$, and the elements of the $N$-point DFT of $h_i(n)$ are called "subchannels" and denoted by $\lambda_i^{(N)}[k]$, $0 \le k \le N-1$. Thus, the expanded subchannels with index from $0 + kM$ to $M - 1 + kM$ are actually associated with the $k$th subchannel.

At the receiver (RX) side, a data block of size $N \cdot M + \nu$ is obtained. After removing the CP, each block is converted from serial to parallel (S/P) and then passed through the discrete Fourier transform (DFT) matrix. The DFT output vector is sent to $M$ subbranches. Each subbranch is multiplied by the corresponding $\mathbf{W}_i^*$ and then passed through the block "sum and dump". Let $\hat{\mathbf{y}}_i$ denote the input and $\hat{\mathbf{x}}_i$ denote the output of "sum and dump". The $k$th element of $\hat{\mathbf{x}}_i$ is given by

$$\hat{x}_i[k] = \frac{1}{\sqrt{M}} \sum_{m=0}^{M-1} \hat{y}_i[m + kM], \quad 0 \le k \le N-1, \qquad (3)$$

where $\frac{1}{\sqrt{M}}$ is included so that $\hat{x}_i[k] = x_i[k]$ if the channel is perfect and there is no channel noise.

### B. Approximately MAI-free property

Referring to Fig. 1, $\hat{y}_j[l]$ contains the faded expanded symbol of $y_j[l]$, interference from other users and the channel noise. More specifically, [5]

$$\begin{aligned} \hat{y}_j[l] &= \lambda_j[l] y_j[l] w_j[l] w_j^*[l] + \sum_{i=1, i\ne j}^{T} \lambda_i[l] y_i[l] w_i[l] w_j^*[l] \\ &\quad + e[l] w_j^*[l], \quad 0 \le l \le NM-1, \end{aligned} \qquad (4)$$

where $e[l]$ is the $l$th component of the $NM$-point DFT of the $NM \times 1$ noise vector. Let $l = m + kM$. From (3) and (4), the $k$th element of $\hat{\mathbf{x}}_j$ is given by

$$\hat{x}_j[k] = S_j[k] + \sum_{i=1, i\ne j}^{T} MAI_{j\leftarrow i}[k] + E_j[k], \qquad (5)$$

where

$$S_j[k] = \frac{1}{\sqrt{M}} \sum_{m=0}^{M-1} \lambda_j[m+kM] y_j[m+kM] w_j[m] w_j^*[m], \quad (6)$$

$$MAI_{j\leftarrow i}[k] = \frac{1}{\sqrt{M}} \sum_{m=0}^{M-1} \lambda_i[m+kM] y_i[m+kM] w_i[m] w_j^*[m], \quad (7)$$

and

$$E_j[k] = \frac{1}{\sqrt{M}} \sum_{m=0}^{M-1} e[m + kM] w_j^*[m + kM]. \qquad (8)$$

Note that we use the notation $MAI_{j\leftarrow i}[k]$ in (7) to denote the MAI at the $k$th symbol of user $j$ contributed by user $i$. If $N$ is sufficiently large, each of the $N$ subchannels can be assumed to be flat [5]. Thus $\lambda_i[m + kM]$ can be approximated by

$$\lambda_i[m+kM] \approx \lambda_i^{(N)}[k], \quad 0 \le m \le M-1, \quad 0 \le k \le N-1, \qquad (9)$$

where $\lambda_i^{(N)}[k]$ is the $k$th element of the $N$-point DFT of the $i$th channel path. Using (1) and the approximation in (9), we can rewrite Eqns. (6) and (7) as

$$\begin{aligned} S_j[k] &\approx \frac{1}{M} \lambda_j^{(N)}[k] x_j[k] \sum_{m=0}^{M-1} w_j[m] w_j^*[m] \\ &= \lambda_j^{(N)}[k] x_j[k] \quad \text{(by (2))}, \end{aligned} \qquad (10)$$

and

$$\begin{aligned} MAI_{j\leftarrow i}[k] &\approx \frac{1}{M} \lambda_i^{(N)}[k] x_i[k] \sum_{m=0}^{M-1} w_i[m] w_j^*[m] \\ &= 0 \quad \text{(by (2))}. \end{aligned} \qquad (11)$$

As given by (11), the MAI is approximately zero. Hence, from (5), (10) and (11), if there is no channel noise, we can approximately reconstruct $x_j[k]$ by multiplying $\hat{x}_j[k]$ by $\left(\lambda_j^{(N)}[k]\right)^{-1}$. An alternative for reconstruction is multiplying $\hat{x}_j[k]$ by $\left(\frac{1}{M}\sum_{m=0}^{M-1} \lambda_j[m + kM]\right)^{-1}$. Since the RCMC-CDMA system is approximately MAI-free, its capacity increases as SNR increases. This result is different from conventional CDMA or MC-CDMA systems, where increasing power for one user will also increase the MAI for other users.

### C. Other properties of RCMC-CDMA

As described in [4], the RCMC-CDMA system has two other nice properties. First, when $N$ is sufficiently large, the RCMC-CDMA system is robust to timing mismatch among users. That is, the RCMC-CDMA system can use the Hadamard-Walsh code in uplink transmission without causing significant MAI even if users are not well synchronized. This result stands in contrary to that of the conventional CDMA or MC-CDMA system, where minor timing mismatch among users will cause great MAI if the Hadamard-Walsh code is used in the uplink transmission. Second, the RCMC-CDMA system can effectively mitigate the carrier frequency offset (CFO) effect. That is, if we use only $M/2$ symmetric or anti-symmetric codewords of the $M$ Hadamard-Walsh codes, the MAI due to the CFO effect can be greatly reduced [6].

Since the RCMC-CDMA system is approximately MAI-free, there is no need to use MUD at the receiver side.

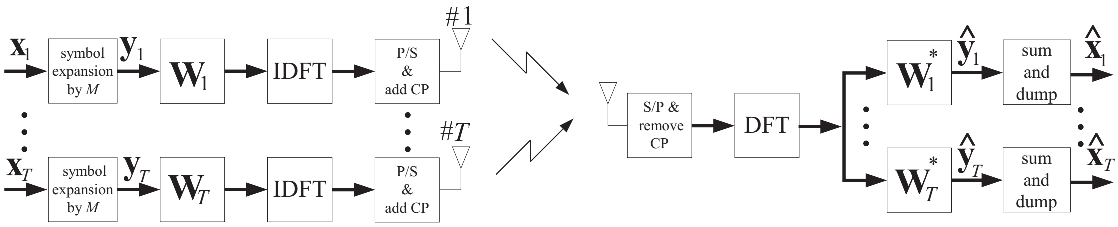


Fig. 1. The block diagram of the RCMC-CDMA system.

Hence, the computational complexity is much lower than that of conventional MC-CDMA with MUD. In the following section, we will discuss how to apply MUD in the MC-CDMA system. Then, we will compare the complexity of the RCMC-CDMA system and that of the conventional MC-CDMA system with MUD.

## III. MUD-based MC-CDMA Systems

For systems with MAI such as CDMA or MC-CDMA, MUD can be adopted to suppress MAI. In this section, we will review the MC-CDMA system and two popular detectors. One is decorrelating detector and the other is the MMSE linear (LMMSE) detector [2].

### A. MC-CDMA System Description

The block diagram of the MUD-based MC-CDMA system with $T$ users is shown in Fig. 2. The transmitter contains three stages. At the first stage, the system spreads a coming symbol $x_i$ of the $i$th user ($1 \le i \le T$) into $M$ chips and modulates a subcarrier with each chip. Thus, the number of chips equals the number of subcarriers. Each chip will be multiplied by an orthogonal code symbol denoted by $w_i[m]$, $0 \le m \le M-1$. At the second stage, the $M \times 1$ chip signal is passed through the $M \times M$ IDFT matrix. Finally, the IDFT output vector is parallel to serial converted and the CP is added to combat ISI.

The receiver contains four stages. First, the CP is removed and the serial data is converted back to the parallel form. Second, the parallel data are passed through the $M \times M$ DFT matrix. At the third stage, the DFT output is divided into $T$ subbranches and each subbranch is passed through the corresponding matched filter. Letting $y_i$ denote the output symbol of the matched filter corresponding to the $i$th user, we have

$$y_i = \sum_{m=0}^{M-1} \left(w_i[m]\lambda_i^{(M)}[m]\right)^* \cdot \left(\sum_{j=1}^{T} \lambda_j^{(M)}[m] w_j[m] x_j + e[m]\right), \tag{12}$$

where $\lambda_j^{(M)}[m]$ is the $m$th element of the $M$-point DFT of the $j$th path, and $e[m]$ is the $M$-point DFT of the received noise vector. Finally, MUD is performed to reconstruct the symbols. Note that the channel information $\lambda_j^{(M)}[m]$ for all users is required to implement the matched filter for the MC-CDMA system due to the multipath propagation. Let $\mathbf{x} = (x_1\ x_2\ \cdots\ x_T)^t$ and $\mathbf{y} = (y_1\ y_2\ \cdots\ y_T)^t$. As given in Eqn. (12), we rewrite the relationship of $x_i$ and $y_i$ using the matrix representation as

$$\mathbf{y} = \mathbf{R}\mathbf{x} + \mathbf{n}, \tag{13}$$

where $\mathbf{R}$ is the cross-correlation matrix with the element in the $i$th row and the $j$th column given by

$$[\mathbf{R}]_{i,j} = \sum_{m=0}^{M-1} \left(\lambda_i^{(M)}[m] w_i[m]\right)^* \left(\lambda_j^{(M)}[m] w_j[m]\right), \tag{14}$$

and $\mathbf{n}$ is the $T \times 1$ noise vector after the matched filter, whose $i$th element is given by

$$n_i = \sum_{m=0}^{M-1} \left(w_i[m]\lambda_i^{(M)}[m]\right)^* e[m]. \tag{15}$$

Note that if the additive noise at the input of the DFT matrix is white and with variance $\sigma^2$. The noise $e[m]$ at the output of the DFT matrix is also white and with variance $\sigma^2$. Then, the $T \times T$ correlation matrix of $n_i$ can be found as

$$E\left\{\mathbf{n}\mathbf{n}^\dagger\right\} = \sigma^2 \mathbf{R}. \tag{16}$$

### B. Multiuser Detectors

We examine two popular multiuser detectors. One is the decorrelating detector and the other is the MMSE linear (LMMSE) detector [2]. The decorrelating detector decorrelates the received vector by multiplying the inverse of the cross-correlation matrix. Hence, the relationship between the input and the output of the decorrelating detector can be expressed by

$$\begin{aligned} \mathbf{x}_{DC} &= \mathbf{R}^{-1}\mathbf{y} \\ &= \mathbf{x} + \mathbf{R}^{-1}\mathbf{n}. \end{aligned} \tag{17}$$

The LMMSE detector minimizes the mean-square error between the transmitted symbol $x_i$ and the reconstructed symbol $\hat{x}_i$, $1 \le i \le T$. Suppose that the additive noise is white and with variance $\sigma^2$. The relationship between the input and the output of the LMMSE detector is given by

$$\mathbf{x}_{MMSE} = (\mathbf{R} + \sigma^2\mathbf{I})^{-1}\mathbf{y}. \tag{18}$$

The derivation for Eqn. (18) can be found in [2]. It is known [2] that, when $\sigma^2 = 0$ in (18), the LMMSE detector becomes the decorrelating detector. In general, the

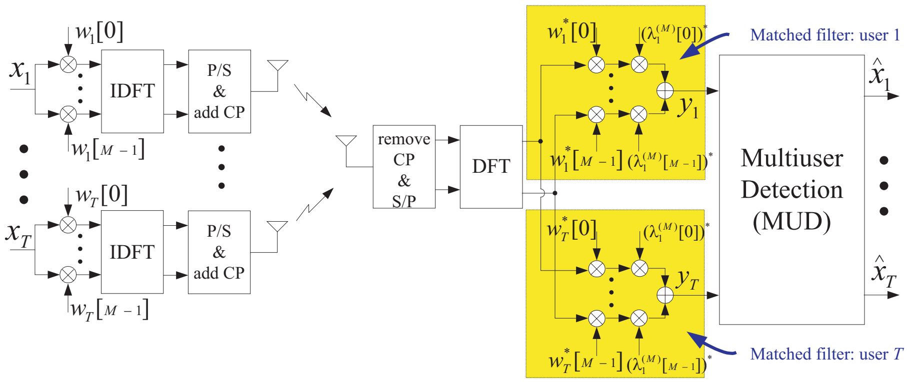


Fig. 2. The block diagram of the MUD-based MC-CDMA system.

LMMSE detector has much better performance than that of the decorrelating detector. However, accurate noise variance is needed in the LMMSE detector, which increases the complexity of the LMMSE detector.

## IV. Complexity Comparison

Let us compare the complexity of the proposed RCMC-CDMA system and the MUD-based MC-CDMA system. To apply MUD in MC-CDMA, the channel statistical information is demanded for the decorrelating and the LMMSE detectors, and an accurate channel estimation is required for both detectors. Due to the MAI, the performance of the channel estimation in MC-CDMA systems depends on the number of active users. More users leads to worse performance. For a large amount of users, accurate channel estimation becomes difficult in MC-CDMA systems [3]. Considering the MAI suppression for data transmission, MC-CDMA systems may use the MUD techniques. As given by (17) and (18), both detectors deal with the inversion of $M \times M$ Toeplitz matrix, which demands a complexity of $\mathcal{O}(M^2)$, when a fast algorithm is used. Note that the processing gain $M$ grows with the number of active users.

Since the multiple-antenna technique becomes popular these days, let us consider the MUD complexity in this situation as well. If $M_T$ transmit antennas and $M_R$ receive antennas are used, we may treat this situation as $M_T \cdot M$ users with single transmit antennas with $M_R$ receive antennas. Since the performance of channel estimation depends on the number of active users, the use of multiple transmit antennas will deteriorate the channel estimation accuracy. Moreover, we need the complexity of $\mathcal{O}\left((M_T \cdot M)^2 M_R\right)$ for the MUD detectors in the MIMO environment.

As to the RCMC-CDMA system, channel estimation is easy to perform due to its approximately MAI-free property. If the number of subchannels, $N$, is sufficiently large, the performance of channel estimation is nearly unaffected by the number of active users so that we can estimate the channel information for an individual user without worrying about MAI. In other words, the estimation algorithm under the single user environment can be adopted, and there is no need to apply the MUD techniques. The major computational module in the RCMC-CDMA system is the $NM$-point FFT/IFFT. Since the FFT algorithm has been widely used, the implementation of the RCMC-CDMA system is relatively straightforward.

## V. Simulation Results

In this section, we evaluate the performance of the MC-CDMA system with MUD and the RCMC-CDMA systems without MUD. Simulation parameters are chosen as follows. For fair comparison, the spreading gains and the chip durations of the MC-CDMA and the RCMC-CDMA systems are set to be the same and thus the two systems demand the same bandwidth. We assume that the users in both systems are well synchronized and the Hadamard-Walsh codes are used. It is also assumed that the receivers of both systems know the channel statistic information exactly. Let $T = M = 8$, *i.e.* a fully loaded situation. The channel has two taps. Each is an i.i.d. (independently identically distributed) complex Gaussian random variable with the unit variance. The overall channel variance is normalized such that $E\left\{\sum_{n=0}^{\nu-1} |h(n)|^2\right\} = 1$. The modulation is BPSK and flat bit allocation is used for all subchannels. The Monte Carlo method is used to run more than $10^6$ bits for all 8 users totally. Two linear MUD techniques, *i.e.* the decorrelating detector and the LMMSE detector [2], are used for comparison.

First, let us consider the single-input and single-output (SISO) case. Fig. 4 gives the performance comparison between the RCMC-CDMA (with $N = 8$, 16 and 32) and the conventional MC-CDMA with a matched filter (MF) alone, a decorrelating detector and an LMMSE detector [2]. Note that matched filtering was performed before the decorrelator and the LMMSE detector in Fig. 2. We see from the figure that MC-CDMA with the LMMSE detector has much better performance than MC-CDMA with the matched filter alone or the decorrelator. We see that the performance of MC-CDMA with LMMSE is close to that of the single user in the presence of flat fading channels [7]. That is, the MAI is almost completely suppressed by the LMMSE detector.

As shown in Fig. 4, we see that RCMC-CDMA out-

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| | MC-CDMA + MUD | RCMC-CDMA |
|---|---|---|
| TX | $O(M \log M)$ for $M$-point IFFT | $O(NM \log NM)$ for $NM$-point IFFT |
| RX | ● $O(M \log M)$ for $M$-point FFT<br>● Channel estimation with MAI<br>● $O((M_T M)^2 M_R)$ for MUD<br>● Extra complexity for noise variance estimation if MMSE linear detector is used | ● $O(NM \log NM)$ for $NM$-point FFT<br>● Channel estimation with negligible MAI. Hence, algorithm for single user environment can be used |

Fig. 3. Summary of complexity comparison for MC-CDMA and RCMC-CDMA systems, where TX and RX mean the transmitter and the receiver, respectively.

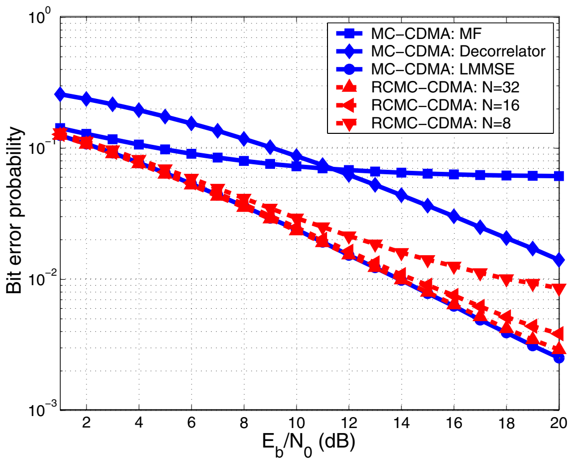


Fig. 4. Performance comparison of MC-CDMA with MUD and RCMC-CDMA without MUD (1 receive antenna).

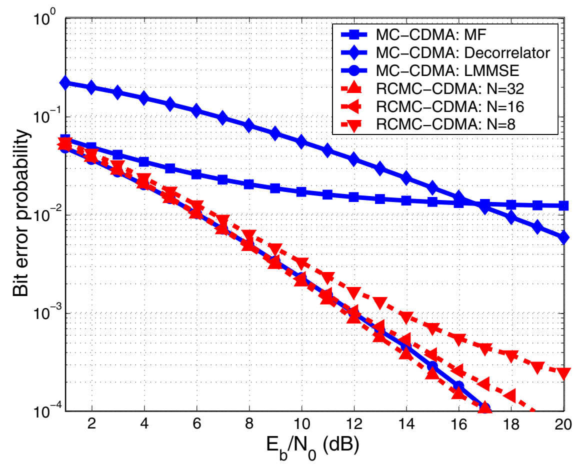


Fig. 5. Performance comparison of MC-CDMA with MUD detectors and RCMC-CDMA without MUD (2 receive antennas with MRC).

performs MC-CDMA with the matched filter alone or the decorrelator for the three tested $N$ values (i.e. $N = 8$, 16 and 32). When $N = 32$, the performance of the RCMC-CDMA system is nearly the same as that of the MC-CDMA with the LMMSE detector. That is, when $N = 32$, all sub-channels can be viewed as nearly flat and the MAI effect is negligible. When $N < 32$, the performance degrades since the MAI effect in the RCMC-CDMA system still has an impact. The performance depends on the approximation accuracy in (9). A larger $N$ value leads to better performance in MAI removal.

Next, let us consider an example with 1 transmit and 2 receive antennas with maximum ratio combining (MRC), *i.e.* the $1 \times 2$ single-input multiple-output (SIMO) system. The performance is shown in Fig. 5. We see again that RCMC-CDMA outperforms MC-CDMA with the matched filter or the decorrelator. When $N = 32$, the performance of RCMC-CDMA is nearly the same as that of MC-CDMA with the LMMSE detector. These two curves are close to that of a single user in a $1 \times 2$ flat fading channel [7]. The result is reasonable due to the approximately MAI-free property of the RCMC-CDMA system.

## VI. Conclusion

The performance and the computational complexity of the RCMC-CDMA system and the MUD-based MC-CDMA system were compared in this work. As the number of active users increases or the antenna diversity is used, the complexity superiority of RCMC-CDMA over MUD-based MC-CDMA becomes more pronounced. Moreover, experimental results showed that RCMC-CDMA have comparable performance as that of MC-CDMA with LMMSE.

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