Reduced Complexity MIMO Equal Gain Precoding

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***Abstract*— We propose a simple search algorithm to reduce the computational complexity and power consumption for equal gain precoder. Simulation results show that the equal gain precoder with the proposed search algorithm can achieve nearly the same performance as that of the Grassmannian precoder. The proposed search algorithm shows several advantages in practical implementation as will be explained later in this paper.**

I. Introduction

Multiple-input and multiple-output (MIMO) techniques which can increase channel capacity without increasing channel bandwidth have shown their promising future in recent years. Among the MIMO techniques, the closed loop precoding (beamforming) that can significantly improve the coverage range and system performance has already been adopted in several communication standards, *e.g.* in MIMO Wi-Fi (IEEE 802.11n) [1] and MIMO Wi-MAX (IEEE 802.16 family) standards [2].

For link modes with one data stream which are important for hand held devices such as cellular phones and PDAs, the optimal precoder that maximizes the receive signal-to-noise ratio (SNR) is to choose the eigenvector corresponding to the maximum eigenvalue of the channel matrix [3]. Although the optimal precoder can achieve the largest SNR value, it demands large amount of feedback information which may not be feasible in rapid changing channel environments. Moreover, it requires eigen decomposition of channel matrix in receiver side, which puts burden in computational complexity and power consumption. Hence, research were conducted to design precoders that use limited feedback information and have reduced computational complexity. In [4], an antenna selection scheme with reduced complexity was proposed for space-time block code (STBC) to give full diversity order. In [5], only channel statistics such as mean and variance are sent back to improve the system performance. In [3], the authors proposed a Grassmannian precoding scheme that constructs the codebook in advance and then determines the best codeword from the codebook according to MIMO channel conditions. The Grassmannian precoding works well in most MIMO channel environments and requires reasonable feedback overhead. However, the optimal codebook is somewhat difficult to construct [3]. The situation becomes more difficult when the number of transmit antennas or the number of quantization bits increases. Some already constructed codebook can be found in [6]. In [7], an equal gain precoder which greatly simplifies the feedback overhead was proposed. However, there is no closed-form solution for this precoder in most MIMO channels. Consequently the exhaustive search, which demands large computational complexity in the receiver, is needed to find the best achievable solution. To reduce the search complexity, the authors in [8] proposed a numerical method to cyclically search the precoding codeword.

In this paper, we find that the equal gain precoder with the exhaustive search can achieve nearly the same performance with the Grassmannian precoder. Furthermore, to overcome the complexity and power consumption issues in the exhaustive search, we propose a simple search algorithm for the equal gain precoder. Let b be the number of quantization bit per transmit antenna, the proposed search algorithm can greatly reduce the search complexity from exponential order in b to linear order in b while its performance can still remain nearly the same with the exhaustive search method. As a result, the equal gain precoder with the proposed search algorithm can achieve nearly the same performance with the Grassmannian precoder. This result leads to the following advantages: 1). The equal gain precoder with the proposed algorithm requires fewer iterations than the Grassmannian precoder to determine the best codeword. The reason is that the search complexity for the proposed algorithm increases linearly in b while that of the Grassmannian precoder increases exponentially in b. Thus, the receiver complexity and power consumption can be greatly reduced. 2). For $b \leq 2$, the proposed scheme does not need multipliers in the transmitter since multiplying a value by ± 1 or $\pm j$ does not need multipliers. 3). In the proposed scheme, the codeword can be easily computed for reasonable large numbers of transmit antennas and quantization bits. On the other hand, the codebook for Grassmannian precoder may not be easy to construct for large numbers of transmit antennas and quantization bits.

II. System Model

Fig. 1 shows the block diagram of a MIMO precoding system, where the vectors $\mathbf{f}$ and $\mathbf{g}$ are called the precoder and the postcoder, respectively. Let the number of transmit antennas be N_t. At the first stage, one transmit symbol x (can be complex such as QPSK and M-QAM) is multiplied by the $N_t \times 1$ precoding vector $\mathbf{f}$. For equal gain precoding, all

978-1-4244-3828-0/09/$25.00

elements in $\mathbf{f}$ have unit magnitude and phase θ_i. Thus,

$$\mathbf{f} = \left(e^{j\theta_1}/\sqrt{N_t}\ e^{j\theta_2}/\sqrt{N_t}\ \cdots e^{j\theta_{N_t}}/\sqrt{N_t}\right), \tag{1}$$

where multiplying $\frac{1}{\sqrt{N_t}}$ is to normalize the transmit power so that the total power will be kept the same for different N_t. After the precoding, the symbol vector, $\mathbf{s} = (s_1\ s_2\ \cdots s_{N_t})^t$, to be transmitted is given by

$$\mathbf{s} = \mathbf{f}x. \tag{2}$$

Then, $\mathbf{s}$ is transmitted to the MIMO channel.

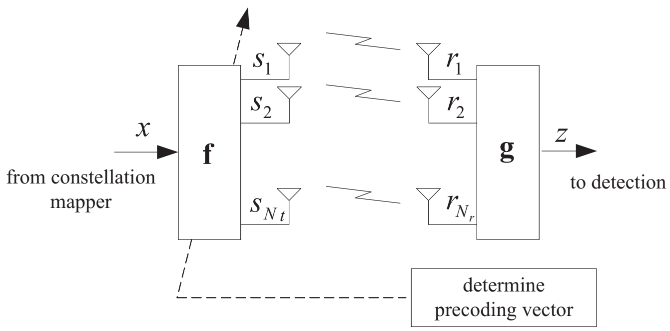

Fig. 1. Block diagram of a MIMO precoder/beamformer.

At the receive side, assuming that there are N_r receive antennas, the receive vector $\mathbf{r} = (r_1\ r_2\ \cdots r_{N_r})^t$ from the MIMO channel is given by

$$\mathbf{r} = \mathbf{H}\mathbf{s} + \mathbf{n}, \tag{3}$$

where $\mathbf{H}$ is a $N_r \times N_t$ channel matrix with its element in the ith row and the jth column being h_{ij}, and $\mathbf{n}$ is a $N_r \times 1$ noise vector. We assume that the elements in the noise vector are complex white Gaussian with variance σ_n^2. $\mathbf{r}$ is multiplied by the $1 \times N_r$ postcoding vector $\mathbf{g}$ to form z, *i.e.*

$$z = \mathbf{g}\mathbf{r} = \mathbf{g}\mathbf{H}\mathbf{f}x + \mathbf{g}\mathbf{n}. \tag{4}$$

In general, we will let the postcoder be $\mathbf{g} = \mathbf{f}^\dagger\mathbf{H}^\dagger$. Thus

$$z = \gamma x + e, \tag{5}$$

where $\gamma = \mathbf{f}^\dagger\mathbf{H}^\dagger\mathbf{H}\mathbf{f}$ is a gain effect (including diversity gain and precoding gain) due to the precoding and $e = \mathbf{f}^\dagger\mathbf{H}^\dagger\mathbf{n}$.

Since $\mathbf{f}^\dagger\mathbf{H}^\dagger\mathbf{H}\mathbf{f}$ is a Hermitian matrix, γ is a nonnegative real value. It can be shown that

$$\gamma = \frac{1}{N_t}\sum_{i=1}^{N_t}\sum_{j=1}^{N_r}|h_{ij}|^2 + \underbrace{\frac{2}{N_t}\sum_{j=2}^{N_t}\sum_{i=1}^{j-1}\Re\left\{\sum_{k=1}^{N_r}h_{ki}^*h_{kj}e^{-j(\theta_i-\theta_j)}\right\}}_{\mu(\boldsymbol{\Theta})}, \tag{6}$$

where $2\mu(\boldsymbol{\Theta})$ is the summation of all off-diagonal elements of $\mathbf{f}^\dagger\mathbf{H}^\dagger\mathbf{H}\mathbf{f}$. It was shown in [7] that in the MISO channel environment, there is a closed-form for the equal gain precoder to achieve its upper bound, which is given by

$$\gamma \quad \leq \quad \frac{1}{N_t}\sum_{i=1}^{N_t}\sum_{j=1}^{N_r}|h_{1i}^*h_{1j}|. \tag{7}$$

It can also been shown that for a 2×2 MIMO channel, there is also a closed-form for the equal gain precoder to achieve its upper bound given by

$$\gamma \leq \frac{1}{N_t}\sum_{i=1}^{N_t}\sum_{j=1}^{N_t}\left|\sum_{k=1}^{N_r}h_{ki}^*h_{kj}\right|.$$

For other MIMO channel environment, there is no closed-form to achieve the upper bound performance. Under such situation, the exhaustive search method can be used to achieve the best system performance (worse than the upper bound performance) as follows:

Exhaustive search for equal gain precoder. Let the bit number of each antenna be b (totally bN_t bits). The exhaustive search method computes $\mu(\boldsymbol{\Theta})$ in (6) for all possible phase combinations, and selects the phase combination that maximize $\mu(\boldsymbol{\Theta})$ as the precoder, *i.e.*

$$\widehat{\boldsymbol{\Theta}} = \arg\max_{\text{all possible } \theta_i}\mu(\boldsymbol{\Theta}), \tag{8}$$

where the notation $\widehat{\boldsymbol{\Theta}}$ is to represent quantization effect. The exhaustive search method demands computational effort with order of $\mathcal{O}\left(2^{bN_t}\right)$, whose computations grows exponentially with N_t and b. For large N_t and b, the complexity soon becomes forbidden. To reduce the computational complexity of the exhaustive search while maintaining the system performance, we propose an algorithm whose complexity increases linearly with b.

III. Proposed Algorithm

Let the total bits to represent $\boldsymbol{\Theta}$ be B and define $B = b(N_t-1)$, where b can be regarded as the number of quantization bits for each of the (N_t-1) transmit antennas (the angle of the first transmit antenna can simply be set to 0 to reduce the feedback information [7]). For description convenience, let $\widehat{\boldsymbol{\Theta}}^{(n)}$ be the solution for $b = n$. This algorithm is similar to simultaneously perform binary search for all θ_i described as follows:

Step 1 :
$n = 0$. $\widehat{\boldsymbol{\Theta}}^{(0)} = (0\ 0\ \cdots 0)^t$.

Step 2 :
$n = n + 1$. $\phi_{offset} = \pi/2^{n-1}$.
$\phi_{i,mid} = \hat{\theta}_i^{(n-1)}$, for $1 \leq i \leq N_t$.
$\phi_{i,up} = \hat{\theta}_i^{(n-1)} + \phi_{offset}$, for $1 \leq i \leq N_t$.
$\phi_{i,low} = \hat{\theta}_i^{(n-1)} - \phi_{offset}$, for $1 \leq i \leq N_t$.
Define the set $\Phi_i = \{\phi_{i,mid},\ \phi_{i,up},\ \phi_{i,low}\}$, for $1 \leq i \leq N_t$.

Step 3 :
$\widehat{\boldsymbol{\Theta}}^{(n)} = \arg\max_{\forall\theta_i\in\Phi_i}\mu(\boldsymbol{\Theta})$.
If $n < b$, go to Step 2; otherwise, end this program.

■

For $b = 1$, we notice that the set is simply that $\Phi_i = \{0,\ \pi\}$ since $e^{j\pi} = e^{-j\pi}$ and $\phi_{i,up} = \phi_{i,low}$. The main complexity of this algorithm is in Step 3, which in general requires $3^{(N_t-1)}$

computation-and-comparison to increase one-bit accuracy for all transmit antennas. For b-bit quantization, the total computational complexity is with order of $\mathcal{O}\left(b3^{(N_t-1)}\right)$. Let us see an example to illustrate this algorithm as follows:

Example 1: Proposed search algorithm. Let $N_t = 5$, $N_r = 2$, $b = 4$ and the MIMO channel matrix be

$$\mathbf{H} = \mathbf{H}_R + j\mathbf{H}_I,$$

where

$$\mathbf{H}_R = \begin{pmatrix} 0.979 & -0.362 & -1.778 & 0.896 & -0.509 \\ -0.534 & 1.045 & -1.789 & -0.843 & -0.76 \end{pmatrix},$$

and

$$\mathbf{H}_I = \begin{pmatrix} -0.395 & -1.325 & 0.557 & -1.258 & -0.072 \\ -1.256 & 0.407 & 0.184 & 0.562 & -0.817 \end{pmatrix}.$$

By using the proposed search algorithm, we can obtain

$$\widehat{\mathbf{\Theta}}^{(1)} = \pi(0\ 0\ 1\ 0\ 1),\ \widehat{\mathbf{\Theta}}^{(2)} = \pi(0\ \tfrac{1}{2}\ 1\ 0\ 1),$$

$$\widehat{\mathbf{\Theta}}^{(3)} = \pi(0\ \tfrac{1}{2}\ 1\ \tfrac{1}{4}\ \tfrac{3}{4}),\ \widehat{\mathbf{\Theta}}^{(4)} = \pi(0\ \tfrac{3}{8}\ \tfrac{7}{8}\ \tfrac{1}{8}\ \tfrac{5}{8}).$$

Note that for $b = 4$, the solution is $\widehat{\mathbf{\Theta}}^{(4)}$. In addition, we will also obtain the solutions for all cases of $b < 4$, *i.e.* $\widehat{\mathbf{\Theta}}^{(1)}$, $\widehat{\mathbf{\Theta}}^{(2)}$, and $\widehat{\mathbf{\Theta}}^{(3)}$ during the search procedure.

IV. Simulation Results

In the following simulation, the elements of $\mathbf{H}$ are i.i.d. complex Gaussian with zero mean and unit variance. Except Example 4, the modulation level is BPSK.

Example 2: Comparison of exhaustive search and proposed search. We would like to see the performance of the exhaustive search and the proposed search algorithm used in the equal gain precoder. For the proposed algorithm, the number of total quantization bits is $B = b(N_t - 1)$ while that of the exhaustive search is $B = bN_t$. Fig. 2 shows the comparison for these two search algorithm. We see that the proposed algorithm can achieve nearly the same performance with the exhaustive search. This shows the advantage of the proposed algorithm in complexity and power consumption over the exhaustive search since it only needs $b3^{(N_t-1)}$ iterations while the exhaustive search needs 2^{bN_t} iterations.

Example 3: Comparison of various precoders. We compare the performance of the following precoders: STBC with antenna selection [4], equal gain (EG) precoder with the proposed search algorithm, Grassmannian percoder [3],[6], and the optimal precoder. For EG in MISO channels, we simply use the closed-form solution; for MIMO channels, the proposed search algorithm is used.

Fig. 3 shows the performance comparison in 4T1R (4-transmit and 1-receive) and 4T2R channel environments. We observe that the EG precoder (6-bit) and the Grassmannian percoder (6-bit) have nearly the same performance, and outperform the antenna selection by around 3 dB in 4T1R case and around 2.2 dB in 4T2R case. The optimal precoder without quantization outperforms the EG and Grassmannian precoders by 1.1 dB in 4T1R case and by 1 dB in 4T2R case.

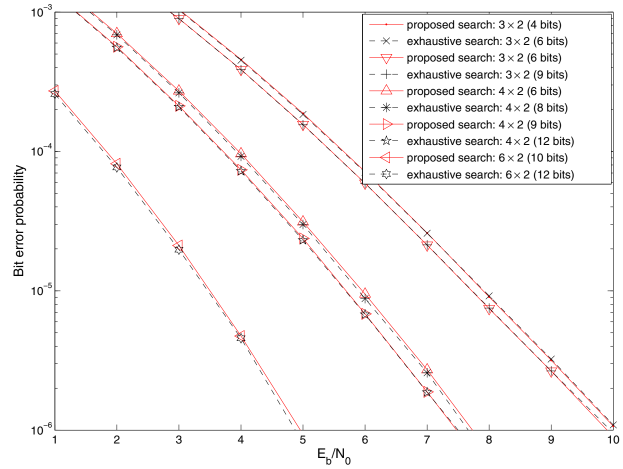

Fig. 2. BEP performance for equal gain precoder with the exhaustive search and the proposed search algorithm.

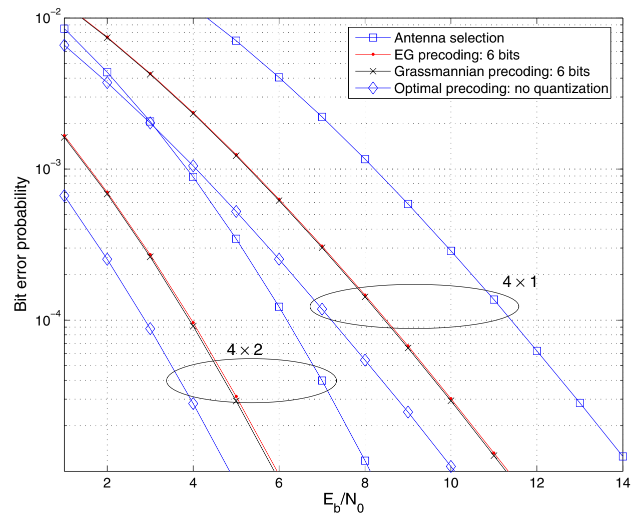

Fig. 3. BEP performance for different precoders in 4T1R and 4T2R channel environments.

Let us increase the receiver antenna number to three. Fig. 4 shows the performance comparison in a 4T3R channel. We again see that for the same number of quantization bits, the EG precoder with the proposed algorithm and the Grassmannian percoder have nearly the same performance. Figs. 3 and 4 show the implementation superiority of the proposed algorithm explained as follows:

First let the total bits be B and $b = B/(N_t - 1)$. Although the two systems have nearly the same performance for the same B, the EG with the proposed algorithm needs $b3^{(N_t-1)}$ iterations to determine the precoder while the Grassmannian needs $2^{b(N_t-1)}$ iterations to determine the precoder. Hence, when b increases, the iteration number grows exponentially for Grassmannian precoder while that grows linearly for EG precoder. In this example, $b = 2$ and $N_t = 4$. Thus, the EG needs 54 iterations while the Grassmannian needs 64 iterations (codebook size is 64 for Grassmannian precoder).

Moreover, in the transmitter side, the Grassmannian precoder requires more computational effort and power consumption than the EG with the proposed algorithm. The reason is that the proposed algorithm is equal gain. Hence it only involves phase rotation and does not require magnitude multiplication. For instance, for $b = 1$, x is multiplied by ± 1, and for $b = 2$, x is multiplied by ± 1 or $\pm j$. Such operations do not need multipliers and can greatly reduce the complexity. That is, for $b \leq 2$, the proposed method only needs operations of sign exchange, or exchange of real part and imaginary part. We see from Figs. 3 and 4 that the performance of the EG and the Grassmannian precoders with $b = 2$ are satisfactory in 4T1R and 4T2R channel environments. On the other hand, since Grassmannian precoding is not equal gain, complex multiplications are required.

Finally, we see from Fig. 4 that as b increases, the EG with the proposed algorithm can still be applied easily. We plot the performance curves using $B = 9$ and 12. The performance can be further improved around 0.3 dB as B increases from 6 to 12. For the Grassmannian precoder, however, $B = 9$ and 12 corresponds to the codebook sizes of 512 and 4096, respectively, which may not be easy to construct as mentioned in [3]. This shows the advantage in the codebook construction of the EG precoder with the proposed search algorithm.

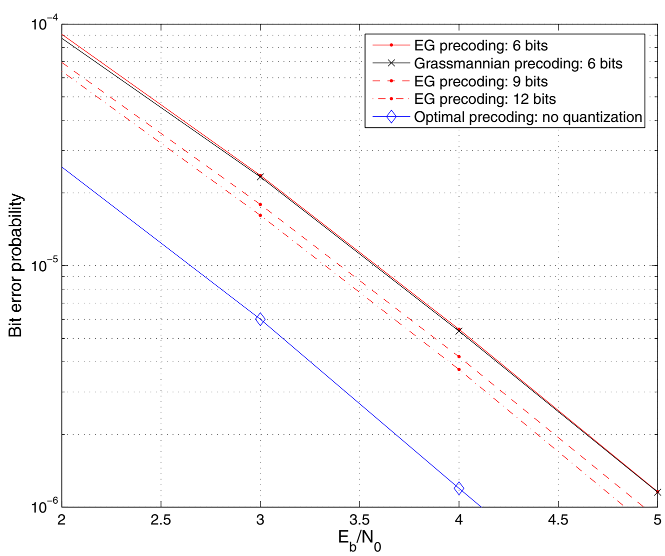

Fig. 4. BEP performance for equal gain, Grassmannian and the optimal precoders in a 4T3R channel environment.

Example 4: Proposed algorithm with large N_t. Let us see the performance comparison in a relatively large number of transmit antennas, *i.e.* $N_t = 8$. Let the modulation level be 16-QAM. Fig. 5 shows the performance of the EG precoder with the proposed algorithm, and the optimal precoder without quantization in 8T2R and 8T3R channel environments. We see that for $B = 21$, the gap between EG precoder and the optimal precoder is around 1 dB in 8T2R case and is around 0.75 dB in 8T3R case. These small gaps again show that satisfactory performance performance can be achieved by using phase information alone.

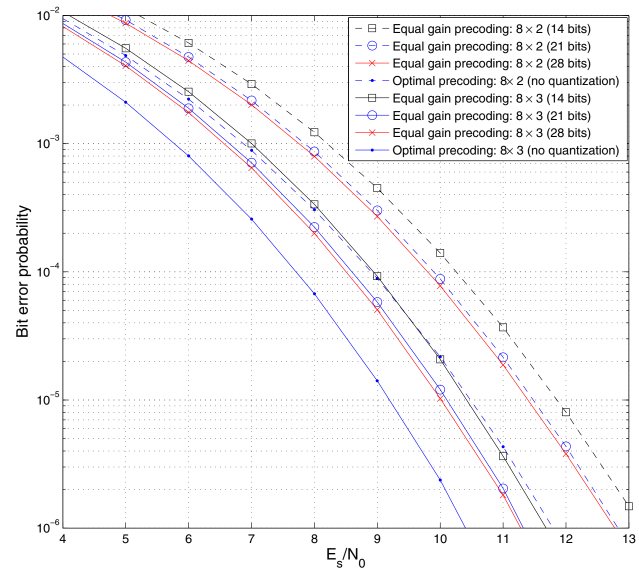

Fig. 5. BEP performance for equal gain precoding and the optimal precoding in 8T2R and 8T3R channel environments.

V. Conclusion

This paper proposed a reduced-complexity codeword search algorithm for equal gain precoder. Simulation results show that the proposed search can achieve nearly the same performance as the exhaustive search algorithm and the Grassmannian precoder. The proposed scheme can greatly reduce the complexity, power consumption and the feedback overhead.

References

[1] *Draft standard for Information Technology-Part 11: Wireless LAN Medium Access Control (MAC) and Physical Layer (PHY) specifications (IEEE 802.11n/D2.0),* The IEEE Inc., Feb. 2007.

[2] *IEEE Standard for Local and Metropolitan Area Networks- Part 16: Air Interface for Fixed and Mobile Broadband Wireless Access Systems (IEEE 802.16e),* The IEEE Inc., Feb. 2006.

[3] D. J. Love, R. W. Heath and T. Strohmer "Grassmannian beamforming for multiple-input multiple-output wireless systems," *IEEE Trans. Inform. Theory,* vol. 49, pp. 2735-2747, Oct. 2003.

[4] D. A. Gore, A. J. Paulraj "MIMO Antenna Subset Selection with Space-time Coding," *IEEE Trans. Signal Processing,* vol. 50, pp. 2580-2588, Oct. 2002.

[5] E. Visotsky and U. Madhow "Space-time transmit precoding with imperfect feedback," *IEEE Trans. Inform. Theory,* vol. 47 pp. 2632-2639, Sep. 2001.

[6] http://cobweb.ecn.purdue.edu/∼djlove/grass

[7] D. J. Love and R. W. Heath Jr. "Equal gain transmission in multiple-input multiple-output wireless systems," *IEEE Trans. Communications,* vol. 51, pp. 1102-1110, July 2003.

[8] X. Zheng, Y. Xie, J. Li and P. Stoica "MIMO transmission beamforming under uniform elemental power constraint," *IEEE Trans. Signal Processing,* vol. 55, pp.5395-5406, Nov. 2007.