Reducing Jitter Effects for Multiband Bandpass Sampling Systems

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***Abstract*—Multiband bandpass sampling techniques have been widely used in Soft Define Radio (SDR) systems. The performance of systems using bandpass sampling is highly affected by jitter effects. This work investigates jitter effects for multiband bandpass sampling systems, and proposes a receiver that applies a two-stage method for combating the jitter effect. At the first stage, a maximum likelihood pilot-aided algorithm is used to estimate and compensate the dominated jitter. Then, at the second stage, we exploit the oversampled property of the received signal and propose a decimate-and-average scheme to further mitigate the residual jitter effect. Simulation results show that the proposed system can significantly reduce the jitter effect.**

I. Introduction

Software Define Radio (SDR) receives extensive research attentions in recent years. The motivation of SDR is to digitalize the radio frequency (RF) front-end as much as possible, so as to make the receiver design more flexible. The concept of SDR was introduced firstly by Joe Mitola in the 1992 National Telesystems Conference (NTC-92) [1], where the authors mentioned the advantages of SDR and its challenges. Several new ideas have been proposed for SDR systems, and the most important one may be that such systems waive the use of RF mixers, which is usually called "direct RF sampling". Bandpass sampling is usually used to realize the direct RF sampling Bandpass sampling is a well studied theory in digital signal processing and communication systems [2] and [3]. In general, SDR systems need to deal with signals from multiple frequency bands. Hence, the results of bandpass sampling have been extended to multiband systems, *e.g.,* see [4]-[7]. More specifically, the authors in [4] provided solutions in finding the minimum sampling frequency. [5] considered guard bands and proposed algorithms with relatively low complexity compared to [4]. In [6], a nonuniform bandpass sampling method was proposed for multiband systems. A new set of conditions for aliasing-free bandpass sampling were derived in [7]. Bandpass sampling was also used in multiband OFDM systems to reconstruct the baseband signals in [5].

When applying bandpass sampling techniques in practical systems, the jitter effects should also be considered. In [9] and [10], how aperture jitter affects the performance of bandpass sampling systems was studied. In [11], a pilot aided method to estimate jitter was proposed. Although this is a potential solution to compensate the jitter effect, unfortunately, detailed performance analysis for this method was not provided. Thus one is not able to know how this method performs and what is the residual jitter after compensation. For OFDM-based systems, the jitter effects induce common phase error (CPE) and inter-carrier interference (ICI) in frequency domain. Although the carrier frequency offset (CFO) and time offset (TO) can also lead to CPE and ICI, the CPE and ICI induced by the aperture jitter is more complicated because the values of jitter can be different at individual samples while the CFO and TO may remain unchanged for several OFDM blocks. As a result, the algorithms developed for mitigating CFO and TO effects do not apply for jitter effects.

Therefore, it is important to develop methods and algorithms to mitigate the jitter effects for multiband bandpass sampling systems. Although in [12] and [13] the authors proposed to mitigate the jitter effect via precoding, these methods add redundancies in the transmitted data and thus the actual data rate is reduced. Moreover, these schemes do not consider signals from multiple bands, and do not make good use of the properties of multiband signals. The discussions above motivate us to investigate jitter combating schemes dedicated for multiband bandpass sampling systems.

In this paper, we propose a two-stage scheme to compensate and mitigate the jitter effect for multiband bandpass sampling systems. At the first stage, we propose a maximum likelihood (ML) pilot-aided method to estimate the jitter. The dominated jitter effect can be well compensated when the ML jitter estimate is available. After the compensation, the residual jitter effect can be further mitigated at the second stage. The second stage exploits the oversampled signal property due to the nature of multiband signals, and uses a decimate-and-average method to eliminate the residual jitter effect. From the signal analysis, we show how to reconstruct the transmitted signals at the receiver side. The proposed jitter combating scheme outperforms the precoding schemes proposed in [12] and [13], as observed from the simulation results. These results show that by well using the oversampled property of the received signals, the proposed system can significantly reduce the jitter effects in multiband bandpass sampling systems.

II. System Model

Consider a system that thas N_b OFDM signals to be transmitted using different carrier frequencies $f_{c_1}, f_{c_2}, \cdots, f_{c_{N_b}}$. A pilot signal with carrier frequency f_p is also transmitted together with the N_b OFDM signals. Let the baseband signal at the ith band be $x_i(t)$. Let the signal bandwidths of all bands be the same and $T = \frac{1}{BW_i}$. The received multiband RF signal can be written as

$$r(t) = \sum_{m=1}^{N_b} x_m(t) e^{j2\pi f_{c_m} t} + p(t), \tag{1}$$

where $x_m(t)$ is represented as follows (see sampling theory in [2]):

$$x_m(t) = T \cdot BW_m \sum_{n=-\infty}^{\infty} x_m[n] \text{sinc}\left(BW_m(t - nT)\right), \tag{2}$$

and $p(t) = Pe^{j2\pi f_p t}$ is the modulated pilot signal. The received multiband RF signal $r(t)$ is passed through the proposed bandpass sampling receiver shown in Fig. 1.

The received signal $r(t)$ is converted from continuous-time to discrete-time signal using the bandpass sampling techniques. The resulting signal $d[n]$ contains the discrete-time multiband and pilot signals. More specifically, let the sampling frequency of the ADC be f_s, where f_s is chosen by following the results of the bandpass sampling. When jitter effect is considered, the sampling time is no longer equal spaced and there is an uncertain time offset at each sampling interval. That is, when converting the signal from continuous-time domain to discrete-time domain, this effect can be modeled by letting $t = nT_s + \varphi[n]$ and substituting it into (1), where $T_s = 1/f_s = \frac{T}{M}$, M is the oversampling factor, and $\varphi[n]$ is the

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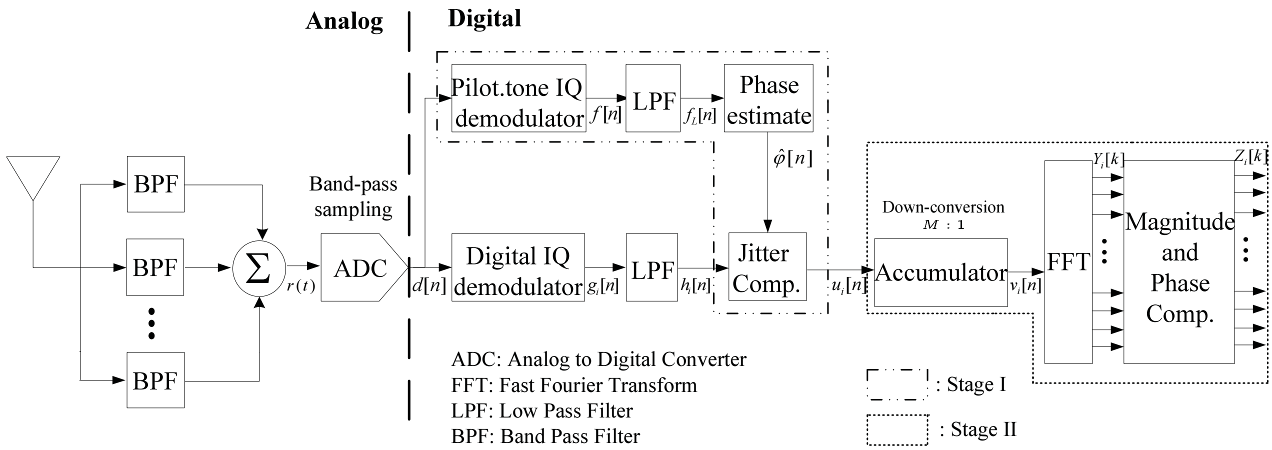


Fig. 1. A block diagram of the proposed receiver system.

time-deviation of the aperture jitter at the n-th sample [10]. The n-th sample affected by the aperture jitter can be represented as

$$\begin{aligned} d[n] &= r(nT_s + \varphi[n]) \\ &= \sum_{m=1}^{N_b} x_m(nT_s + \varphi[n]) e^{j2\pi f_{c_m}(nT_s+\varphi[n])} \\ &\quad + P e^{j2\pi f_p (nT_s + \varphi[n])}. \end{aligned} \tag{3}$$

From the discussion above, (3) can be approximated as

$$\begin{aligned} d[n] &\approx \sum_{m=1}^{N_b} x_m(nT_s) e^{j2\pi f_{c_m}(nT_s+\varphi[n])} + P e^{j2\pi f_p (nT_s+\varphi[n])} \\ &= \sum_{m=1}^{N_b} \hat{x}_m[n] e^{j2\pi \frac{f_{IF_m}}{f_s} n} e^{j\xi_m[n]} + P e^{j2\pi \frac{f_{IF_p}}{f_s} n} e^{j\xi_p[n]}, \end{aligned} \tag{4}$$

where $\hat{x}_m[n] \equiv x_m(nT_s)$ is the discrete-time signal obtained by sampling the continuous-time signal $x_m(t)$ at every T_s seconds, $\xi_m[n] = 2\pi f_{c_m}\varphi[n]$, $\xi_p[n] = 2\pi f_p \varphi[n]$, and the intermediate frequency (IF) is defined as $f_{IF_m} = f_{c_m} - r_m f_s$, for $r_m \le \lfloor \frac{f_{L_m} - GB_{min}}{2\sum_{m=1}^{m=N_b}(BW_m + GB_m)} \rfloor$, which is determined according to bandpass sampling theorem. In this work, we use the maximum value of r_m so that f_{IF_m} can be as small as possible.

The proposed receiver combats the jitter effect using a two-stage method. At the first stage, the jitter is estimated and compensated using the pilot signal; this can be explained more detailed using Fig. 1. The signal $d[n]$ is demodulated digitally to yield $g_i[n]$ given by

$$\begin{aligned} g_i[n] &= d[n] \times e^{-j2\pi \frac{f_{IF_i}}{f_s} n} \\ &= \sum_{m=1, m\neq i}^{N_b} x_m[n] e^{j2\pi \frac{(f_{IF_m} - f_{IF_i})}{f_s} n} e^{j\xi_m[n]} + \hat{x}_i[n] e^{j\xi_i[n]} \\ &\quad + P e^{j2\pi \frac{(f_{IF_p} - f_{IF_i})}{f_s} n} e^{j\xi_p[n]}. \end{aligned} \tag{5}$$

Then $g_i[n]$ is passed through a low pass filter (LPF) to extract the desired signal given by

$$h_i[n] = \mathrm{LPF}\{g_i[n]\} = \hat{x}_i[n] e^{j\xi_i[n]}. \tag{6}$$

The jitter effect of signal $h_i[n]$ is compensated and the resulting signal is $u_i[n]$. At the second stage, the residual jitter effect is further mitigated using a proposed method that will be described more detailed in the following section.

III. Proposed Methods to Combat Jitter Effects

In this section, we describe the proposed two-stage method for combating the jitter effect.

A. Stage 1: Maximum likelihood (ML) Jitter Estimation and Compensation

Form (4), the main jitter effect is the resulting phase noise $\varphi[n]$. At the first stage, an ML estimator for $\varphi[n]$ is developed using the pilot signal. Then, the jitter effect is compensated by this estimated jitter.

Referring to Fig. 1, the signal $d[n]$ after down conversion at frequency f_{IF_p} can be represented as

$$\begin{aligned} f[n] &= d[n] \times e^{-j2\pi \frac{f_{IF_p}}{f_s} n} \\ &= \sum_{m=1}^{N_b} \hat{x}_m[n] e^{j2\pi \frac{(f_{IF_m} - f_{IF_p})}{f_s} n} e^{j\xi_m[n]} + P e^{j\xi_p[n]}. \end{aligned} \tag{7}$$

After passing through the LPF, the signal becomes

$$f_L[n] = \mathrm{LPF}\{f[n]\} = P e^{j\xi_p[n]}. \tag{8}$$

From (8), the received signal with additive noise can be expressed as

$$y[n] = P e^{j\xi_p[n]} + w[n]. \tag{9}$$

Then we have the following proposition for the ML estimator of $\xi_p[n]$.

Proposition 1 *Let the real and imaginary parts of each of the received sample of $y[n]$ in (9) be $y_R[n]$ and $y_I[n]$, respectively. Then, the ML estimator for $\xi_p[n] = 2\pi f_p \varphi[n]$ can be shown to be*

$$\hat{\xi}_p[n] = \tan^{-1}\left\{\frac{y_I[n]}{y_R[n]}\right\}. \tag{10}$$

Therefore the ML estimator for the jitter is given by

$$\hat{\varphi}[n] = \frac{\hat{\xi}_p[n]}{2\pi f_p}. \tag{11}$$

When $\hat{\varphi}[n]$ is available, one can compensate the jitter effect. From (6) and (11), the resulting signal can be written as

$$\begin{aligned} u_i[n] &= h_i[n]e^{-j2\pi f_{c_i}\hat{\varphi}[n]} \\ &= \hat{x}_i[n]e^{j2\pi f_{c_i}(\varphi[n]-\hat{\varphi}[n])} \\ &= \hat{x}_i[n]e^{j2\pi f_{c_i}\tilde{\varphi}[n]} \\ &= \hat{x}_i[n]e^{j\tilde{\xi}_i[n]}, \end{aligned} \tag{12}$$

where we call $\tilde{\varphi}[n]$ "residual" jitter, which will be further mitigated at the second stage.

B. Stage 2: Residual Jitter Mitigation

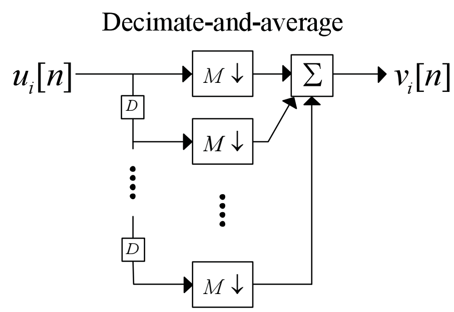


Fig. 2. A block diagram of decimate-and-average.

At the second stage, the residual jitter effect is further mitigated. The jitter noise can be modeled as the Gaussian noise. Because the proposed system is an oversampled system, one can decimate the signal $u_i[n]$ and average the decimated signals to mitigate the jitter effect; we call this procedure "accumulation", which is conducted by a module called "decimate-and-average", shown in Fig. 2. From this figure, the signals before and after the accumulation have the following relationship:

$$\begin{aligned} v_i[n] &= u_i[Mn] + u_i[Mn+1] + \ldots + u_i[Mn+(M'-1)] \\ &= \sum_{r=0}^{M'-1} u_i[Mn+r], \quad 1 \le M' \le M \\ &= \sum_{r=0}^{M'-1} \hat{x}_i[Mn+r]e^{j\tilde{\xi}_i[Mn+r]}, \end{aligned} \tag{13}$$

where M' is the number of accumulated samples. Please note that the rate ratio of $u_i[n]$ and $v_i[n]$ is $M:1$.

Referring to Fig. 1, after decimating and averaging the signal, the signal is passed through an $N-$point FFT (fast Fourier transform) to reconstruct the OFDM symbols. The signal after the FFT can be expressed as

$$\begin{aligned} Y_i[k] &= FFT\{v_i[n]\} \\ &= FFT\{\sum_{r=0}^{M'-1} \hat{x}_i[Mn+r]e^{j\tilde{\xi}_i[Mn+r]}\}. \end{aligned} \tag{14}$$

The signals $Y_i[k]$ with and without jitter effect are discussed in the following two lemmas:

Lemma 1 *Consider the case that there is no aperture jitter. Let* $\hat{X}_{i_r}[k]$*,* $k = 0,\ 1, \cdots N-1$*, be the* $N-$*point FFT of* $\hat{x}_i[Mn+r]$*,* $n = 0,\ 1, \cdots N-1$. *It can be shown that*

$$\hat{X}_{i_r}[k] = X_i[k]e^{j\frac{2\pi rk}{MN}}. \tag{15}$$

Lemma 2 *Consider the case that the system has aperture jitter. Let*

$$\tilde{J}_{i_r}[l-k] = \frac{1}{N}\sum_{n=0}^{N-1} e^{j\tilde{\xi}_i[Mn+r]}e^{j\frac{2\pi n(l-k)}{N}}, \quad l, k = 0,\ 1, \cdots N-1,$$

be the IFFT of the compensated aperture jitter phase noise. Then we have

$$\begin{aligned} Y_i[k] &= \sum_{r=0}^{M'-1}\sum_{l=0}^{N-1} X_i[l]\tilde{J}_{i_r}[l-k]e^{j\frac{2\pi rk}{MN}} \\ &= \sum_{r=0}^{M'-1} X_i[k]\tilde{J}_{i_r}[0]e^{j\frac{2\pi rk}{MN}} \\ &\quad + \sum_{r=0}^{M'-1}\sum_{l=0,l\neq k}^{N-1} X_i[l]\tilde{J}_{i_r}[l-k]e^{j\frac{2\pi rk}{MN}}. \end{aligned} \tag{16}$$

From Lemmas 1-2, the signal after the IFFT has magnitude distortion and phase rotation compared to the transmitted signal. We introduce how to compensate the magnitude distortion and phase rotation as follows: First, from (16), when there is no jitter, $\tilde{J}_{i_r}[l-k]$ is one for $l = k$, and is zero for $l \neq k$. Thus, to reconstruct $X_i[k]$, one can divide $Y_i[k]$ by $\sum_{r=0}^{M'-1} e^{j\frac{2\pi rk}{MN}}$. Using the following formula for series:

$$\begin{aligned} \sum_{r=0}^{M'-1} e^{j\frac{2\pi rk}{MN}} &= 1 + e^{j\frac{2\pi k}{MN}} + \ldots + e^{j\frac{2\pi(M'-1)k}{MN}} \\ &= \frac{1-e^{-j\frac{2\pi M'k}{MN}}}{1-e^{j\frac{2\pi k}{MN}}} \\ &= \frac{\sin(\frac{\pi M'k}{MN})}{\sin(\frac{\pi k}{MN})}e^{j\frac{\pi(M'-1)k}{MN}}, \end{aligned} \tag{17}$$

the symbols after the compensation can be expressed as

$$\begin{aligned} Z_i[k] &= \frac{Y_i[k]}{\frac{\sin(\frac{\pi M'k}{MN})}{\sin(\frac{\pi k}{MN})}e^{j\frac{\pi(M'-1)k}{MN}}} \\ &= \frac{\sin(\frac{\pi k}{MN})}{\sin(\frac{\pi M'k}{MN})}Y_i[k]e^{-j\frac{\pi(M'-1)k}{MN}}. \end{aligned} \tag{18}$$

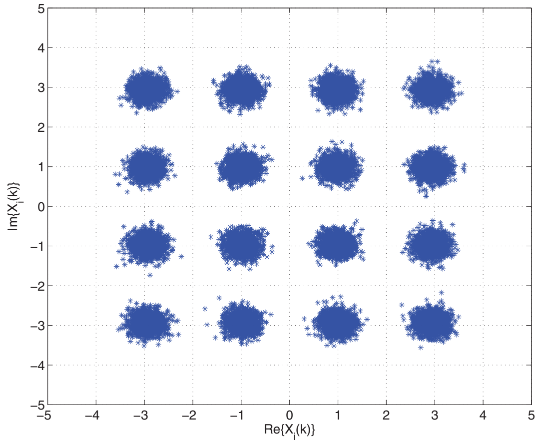
Substituting (16) into (18) yields

$$\begin{aligned} Z_i[k] &= \frac{\sin(\frac{\pi k}{MN})}{\sin(\frac{\pi M'k}{MN})}\sum_{r=0}^{M'-1}\sum_{l=0}^{N-1} X_i[l]\tilde{J}_{i_r}[l-k]e^{j\frac{2\pi rk}{MN}}e^{-j\frac{\pi(M'-1)k}{MN}} \\ &= X_i[k]\frac{\sin(\frac{\pi k}{MN})}{\sin(\frac{\pi M'k}{MN})}\sum_{r=0}^{M'-1}\tilde{J}_{i_r}[0]e^{-j\frac{\pi k(M'-2r-1)}{MN}} \\ &\quad + \frac{\sin(\frac{\pi k}{MN})}{\sin(\frac{\pi M'k}{MN})}\sum_{r=0}^{M'-1}\sum_{l=0,l\neq k}^{N-1} X_i[l]\tilde{J}_{i_r}[l-k]e^{-j\frac{\pi k(M'-2r-1)}{MN}}. \end{aligned} \tag{19}$$

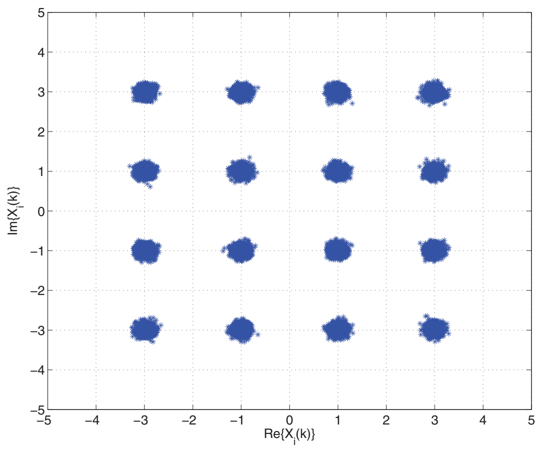
Equation (19) shows the resulting signal of the proposed system. Later this equation will be used to compute the MSE of the transmitted and received signals.

IV. Simulation Results

Example 1: MSE for overall system (with both Stages 1 and 2). Considering the system performance with both Stage 1 and Stage 2 jitter combating methods and letting $M = M' = 7$. Fig. 3 shows the symbol constellation diagrams with $\sigma_\varphi^2 = (10\text{ ps})^2$ conducted in AWGN with SNR fixed at 20 dB. One can see that when the proposed Stage 1 and Stage 2 jitter combating methods are used together, the jitter effect can be greatly reduced.



(a) With Stage 1 jitter compensation.



(b) With Stage 1 jitter compensation and Stage 2 jitter mitigation.

Fig. 3. Comparison of the symbol constellation with σ_{φ}^2=(10 ps)^2 and SNR= 20 dB: overall system

Example 2: Comparison of proposed and other jitter mitigation methods We compare the proposed jitter combating methods with other existing methods via precoding [12], [13]. Letting SNR= 30 dB, Fig. 4 shows the MSE as a function of jitter in terms of phase noise variance σ_{φ}^2. Observe that the precoding method and the proposed method can significantly reduce the MSE due to the jitter effect. In addition, the proposed scheme outperforms the precoding scheme. The reason is that the proposed method has advantages in multiband systems, in which the received signals are oversampled. By exploiting the properties of oversampled signals, the proposed system can more effectively mitigate the jitter effect.

V. Conclusion

In this paper, we have proposed a receiver that uses a two-stage method to mitigate the jitter effect for multiband bandpass sampling systems. At the first stage, the dominated jitter effect is estimated and compensated using the proposed ML pilot-aided algorithm. Then the residual jitter effect is further mitigated at the second stage by the proposed decimate-and-average method. From the signal analysis, we have shown how to reconstruct the transmitted signal by compensating the magnitude distortion and phase rotation. Moreover the proposed system has been shown to outperform other jitter combating methods via precoding when they are used in multiband bandpass sampling systems. The reason is that the proposed system well exploits the oversampled property of the received signals, and average the jitter noise. As a result, the jitter effect can be greatly reduced by the proposed system.

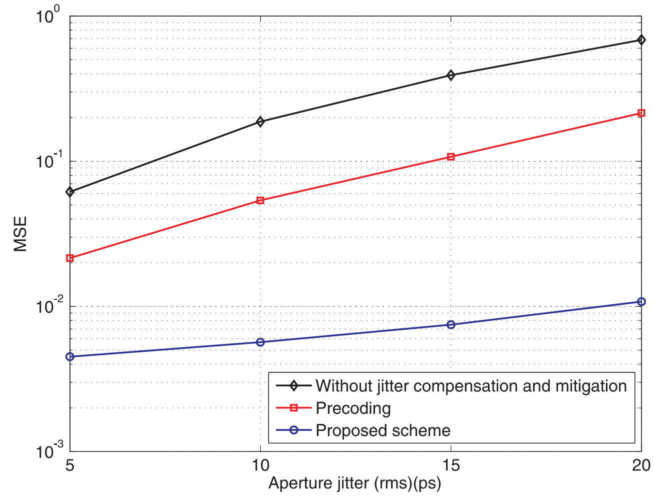


Fig. 4. MSE comparison for various methods.

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