Resource Allocation and Minimum Rate for Precoded Non-orthogonal Multiple Access

Chih-Chiang Chen[1], Shang-Ho (Lawrence) Tsai[1], Yuan-Pei Lin[1] and Chia-Hua Lin[2]
Department of Electrical Engineering National Chiao Tung University, Hsinchu, Taiwan[1]
Information and Communications Research Labs, Industrial Technology Research Institute, Hsinchu, Taiwan[2]
E-mails: earl0953.cm03g@nctu.edu.tw and shanghot@alumni.usc.edu

***Abstract*—In this paper, we propose user clustering and power allocation algorithms for a precoded NOMA system. The user clustering for the strong and the weak users, and corresponding power allocation can be optimized disjointedly when SNR is sufficiently high, and this leads to a great reduction in computational complexity. Moreover this proposed power allocation algorithm can maximize sum achievable rate subject to a minimum target rate. Furthermore, from the analytical results, we suggest a reasonable minimum target rate for the weak user so that this rate can always be achieved, *i.e,* with outage probability 1. Using the suggested target rate, the NOMA system can avoid failing in achieving this target rate even if all power is allocated to the weak user. Simulation results corroborate theoretical results.**

I. Introduction

Non-orthogonal multiple access (NOMA) has been widely studied for the fifth generation (5G) wireless communications, because it enhances spectrum efficiency and improves sum achievable rate [1]-[3]. Unlike orthogonal multiple access (OMA), NOMA allows to transmit signals of multiple users in the same time frame and frequency band via different power levels. The superposed signals of multiple users can be decoded using successive interference cancellation (SIC) [1],[3].

The number of transmit antennas is usually smaller than the number of total receive user antennas in NOMA, *i.e.,* an overloaded technique, and this results in multiuser interference. To reduce interference and improve the sum achievable rate, careful resource allocation including user clustering and power allocation shall be applied [3]-[5]. More specifically, the authors in [3] and [4] suggested that users with high channel correlations and high difference of channel gain shall be clustered. In [5], the authors proposed a channel state sorting-pairing algorithm (CSS-PA) based on the results of channel state sorting, which clusters a good channel-condition user with a poor channel-condition user. Furthermore, a power allocation scheme for NOMA systems have been studied to maximize sum-rate while keeping the achievable rate of the weak user be equal to that in conventional case in [3].

To maximize sum rate, the optimal solution for user clustering and power allocation shall be done jointly via exhaustive search. However, this demands huge computational complexity. Moreover, in the NOMA system, a minimum target rate is usually constrained for the weak user in a cluster for achieving a target QoS (quality of service). A reasonable target rate is important, because if the target rate is too high, there is little chance that the weak user can achieve this rate even if all power has been allocated to this weak user. When this happens, some bad situations occur including 1) the system spends a lot of power to the weak user but still the requirement cannot be attained, and this significantly decreases the sum achievable rate because the strong user generally dominates the sum achievable rate but few power has been allocated to him/her; 2) NOMA returns to conventional MU-MIMO due to the failure of achieving the requirement of the minimum target rate. These motivate us to investigate complexity-reduced resource allocation schemes as well as find out reasonable target rate for the weak users.

In this paper, we propose a clustering algorithm for strong and weak users. We notice that the sum rate is usually dominated by the strong users because they do not receive inter- and intra-cluster interference when SIC and block diagonalization precoding are applied. Thus we suggest to select strong users by the semiorthogonal user group (SUS) algorithm [6] to orthogonalize channels. Consequently, high multiuser sum rate of strong users can be achieved. Then the weak users are selected by maximizing the multiuser sum rate, in which both inter- and intra-cluster interference appear. At the first glance, exhaustive search shall be used to select the weak users. In this work, we prove that when SNR is sufficiently high, searching exhaustively for maximizing sum rate can be replaced simply by finding the weak users whose channels have similar directions with those of the strong users, and interestingly this searching is irrelevant to the power allocation scheme. Hence, the optimization problem for user clustering and power allocation can be solved separately. As a result, computational complexity can be significantly reduced. Moreover, subject to a power constraint and a minimum achievable target rate, we propose an optimal power allocation for achieving the maximum sum rate. Furthermore, we provide a closed-form solution for the outage probability of a specific target rate that the weak user can achieve. This solution can be used to suggest a reasonable target rate for the weak user. Since different parameters such as number of total users, number of clusters and SNR affect the reasonable target rate, the proposed solution provides a good reference design in practical precoded NOMA systems.

II. System Model and Problem Formulation

A block diagram of the proposed NOMA system is shown in Fig. 1. There are N antennas at base station and K

single-antenna users ($K \geq 2N$). Each one of the N transmit precoding vectors can support two or more users instead of only one user. In this study, we assume that two users are grouped into a cluster for the sake of simplicity. In this case, $2N$ users are selected out of K users, and they are supported by N precoding vectors. In each of the the N clusters, the two users are labeled as strong user and weak user according to their channel gains.

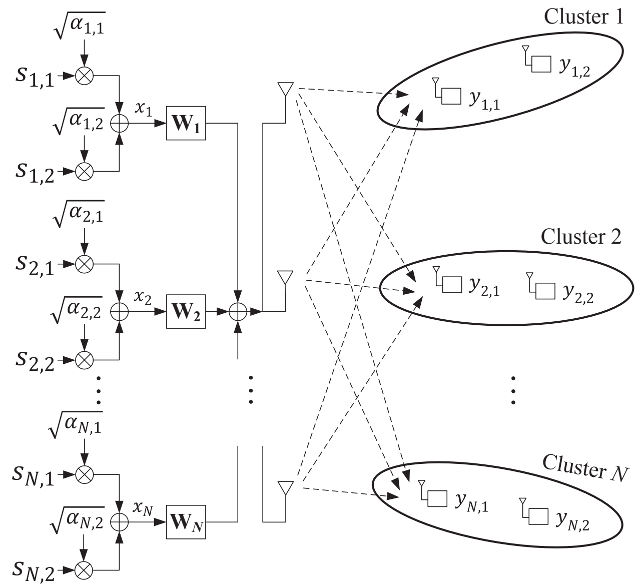

Fig. 1. The proposed downlink precoding NOMA system.

The base station superposes the signals of the two users in the power domain. In the k-th cluster, the signal of the strong user $s_{k,1}$ and that of the weak user $s_{k,2}$ are superposed as

$$x_k = \sqrt{\alpha_{k,1} P_k} s_{k,1} + \sqrt{\alpha_{k,2} P_k} s_{k,2}, \tag{1}$$

where $\mathbb{E}\left\{|s_{k,i}|^2\right\} = 1$ for $i = 1,\, 2$, P_k is transmission power, $\alpha_{k,1}$ and $\alpha_{k,2}$ denotes power allocation factors for the strong user and the weak user, respectively, and $\alpha_{k,1} + \alpha_{k,2} = 1$. In the n-th cluster, the received signals of the strong user and the weak user are defined as $y_{n,1}$ and $y_{n,2}$, respectively, given by

$$y_{n,i} = \mathbf{h}_{n,i} \sum_{k=1}^{N} \mathbf{w}_k x_k + n_{n,i} \quad \text{for} \quad i = 1,\, 2;\; n = 1, \cdots, N, \tag{2}$$

where $\mathbf{w}_k$ is the $N \times 1$ precoding vectors for the k-th cluster, $n_{n,1}$ and $n_{n,2}$ are additive white complex Gaussian noises with zero mean and variances σ_n^2, and $\mathbf{h}_{n,i}$ is the $1 \times N$ channel vector between user i and the base station. We assume that the transmitter knows full channel state information, and the channel is assumed to be Rayleigh fading with zero mean.

Block Diagonalization (BD) is used for the precoding scheme [8], which makes the value of $\mathbf{h}_m \mathbf{w}_n$ be 0 for $m \neq n$, and 1 for $m = n$. In the proposed NOMA system, the channel of the strong users in individual clusters are used to form the precoding vectors [3]. Let the channel matrix $\mathbf{H}$ be consisting of the channels of the strong users as

$$\mathbf{H} = [\mathbf{h}_{1,1}^T \cdots \mathbf{h}_{N,1}^T]^T. \tag{3}$$

The precoding matrix $\mathbf{W}$ is then given by

$$\mathbf{W} = [\mathbf{w}_1 \cdots \mathbf{w}_N] = \mathbf{H}^H (\mathbf{H}\mathbf{H}^H)^{-1} \mathbf{\Lambda}, \tag{4}$$

where $\mathbf{\Lambda}$ is a diagonal matrix to normalize the power of individual columns, and $\mathbf{w}_n$ is the precoding vector for the n-th cluster.

For the strong user in a specific cluster, the intra-cluster interference from the weak user can be removed by using perfect SIC. Also, the inter-cluster interference does not affect the strong user due to the use of BD precoding. Therefore, the signal $y_{n,1}$ of the strong user in the n-th cluster after SIC can be represented as

$$y_{n,1} = \mathbf{h}_{n,1} \mathbf{w}_n \sqrt{\alpha_{n,1} P_n} s_{n,1} + n_{n,1}. \tag{5}$$

The achievable rate $R_{n,1}$ can be represented by

$$R_{n,1} = \log_2 \left(1 + \frac{|\mathbf{h}_{n,1} \mathbf{w}_n|^2 \alpha_{n,1} P_n}{\sigma_n^2} \right). \tag{6}$$

For the weak user, the received signal can be written as

$$\begin{aligned} y_{n,2} = {} & \underbrace{\mathbf{h}_{n,2} \mathbf{w}_n \sqrt{\alpha_{n,1} P_n} s_{n,1}}_{\text{intra-cluster interference}} + \underbrace{\mathbf{h}_{n,2} \mathbf{w}_n \sqrt{\alpha_{n,2} P_n} s_{n,2}}_{\text{desired signal}} \\ & + \underbrace{\mathbf{h}_{n,2} \sum_{k=1, k \neq n}^{N} \mathbf{w}_k x_k}_{\text{inter-cluster interference}} + n_{n,2}. \end{aligned} \tag{7}$$

The achievable rate of the weak user is given by

$$R_{n,2} = \log_2 \left(1 + SINR_{n,2} \right), \tag{8}$$

where $SINR_{n,2}$ is defined as follows:

$$\frac{|\mathbf{h}_{n,2} \mathbf{w}_n|^2 (1 - \alpha_n) P_n}{|\mathbf{h}_{n,2} \mathbf{w}_n|^2 \alpha_n P_n + \sum_{k=1, k \neq n}^{N} |\mathbf{h}_{n,2} \mathbf{w}_k|^2 P_k + \sigma_n^2}. \tag{9}$$

Let us describe the goals of this study as follows: First, we propose a user clustering algorithm which select the strong and the weak user separately. Then, we introduce a optimal power allocation algorithm for maximizing the sum rate; while it still retains the QoS of the weak user by keeping the achievable rate of the weak user equal to or greater than a given target rate. The problem is formulated as

$$\begin{aligned} & \max_{\alpha_{n,1}, \alpha_{n,2}} (R_{n,1} + R_{n,2}) \\ & \text{subject to } R_{n,1}, R_{n,2} \geq R_t,\; \alpha_{n,1} + \alpha_{n,2} = 1. \end{aligned} \tag{10}$$

where $\alpha_{n,1}$ and $\alpha_{n,2}$ denote the allocated power ratio for the strong user and the weak user in the n-th cluster, respectively, and R_t is a given target rate. Second, we provide a reasonable value of R_t according to various settings of user number, cluster number and SNR value such that the weak user can always achieve this target rate. Please note that if R_t is not properly determined, the weak user may have little chance to achieve this target rate and thus only the strong user attains the QoS requirement. As a result, it returns to conventional multiuser MIMO case instead of NOMA. To make thing worse, the weak user can fail to meet the QoS requirement even if all the power has been allocated to him/her.

III. Proposed Resource Allocations

In this section, the proposed user clustering algorithm is introduced. Then we propose a power allocation algorithm to maximize the sum rate subject to achieving a minimum target rate of the weak user.

A. User clustering

$2N$ users are clustered in N pairs from total K users. First we use the SUS algorithm in [6] to select the N strong users for the N clusters. Then the N weak users can be determined for the N clusters by maximizing the sum rate in (8) using exhaustive search. When the number K of total users is large, search exhaustively demands huge complexity. To overcome this, we found that the weak users can be selected simply by checking the channel correlation between the strong and the weak users, and is irrelevant to the power allocation. These are introduced in the following proposition:

Proposition 1. *Let the SINR of the weak user in the n-th cluster be in (9). When SNR approaches ∞, if weak users are selected such that $\mathbf{h}_{n,2}$ has similar direction of $\mathbf{h}_{n,1}$ $\forall n$, which implies that $\mathbf{h}_{n,2}$ is highly correlated with $\mathbf{h}_{n,1}$, then the SINR of the weak user in the n-th cluster is maximized and is irrelevant to α_n.*

Proof: For presentation convenience, let $\alpha'_n = 1-\alpha_n$. Because maximizing $SINR_{n,2}$ is equal to minimizing $\frac{1}{SINR_{n,2}}$, the problem becomes

$$\min_{\mathbf{h}_{n,2}} \left(\frac{\alpha_n}{\alpha'_n} + \frac{1}{\alpha'_n} \left[\frac{\sum_{k=1,k\neq n}^{N} |\mathbf{h}_{n,2}\mathbf{w}_k|^2}{|\mathbf{h}_{n,2}\mathbf{w}_n|^2} + \frac{\frac{1}{SNR}}{|\mathbf{h}_{n,2}\mathbf{w}_n|^2} \right] \right), \tag{11}$$

where $SNR = \frac{P_n}{\sigma_n^2}$. When $SNR \to \infty$, (11) becomes

$$\min_{\mathbf{h}_{n,2}} \left(\frac{\alpha_n}{\alpha'_n} + \frac{1}{\alpha'_n} \left[\frac{\sum_{k=1,k\neq n}^{N} |\mathbf{h}_{n,2}\mathbf{w}_k|^2}{|\mathbf{h}_{n,2}\mathbf{w}_n|^2} \right] \right). \tag{12}$$

From (12), the optimization problem is irrelevant to the power allocation α_n. Since the BD scheme is used, $\mathbf{h}_{n,1}\mathbf{w}_k = 0$ for $k \neq n$. Hence, if the weak user is selected such that $\mathbf{h}_{n,2}$ has similar direction of $\mathbf{h}_{n,1}$, it yields $\mathbf{h}_{n,2}\mathbf{w}_k \approx 0$ for $k \neq n$, and $\sum_{k=1,k\neq n}^{N} |\mathbf{h}_{n,2}\mathbf{w}_k|^2 \approx 0$. ■

Using Proposition 1, the proposed algorithm for selecting the weak users is summarized in Algorithm 1.

Algorithm 1 Proposed algorithm for selecting the weak users.

Step 1) Initialization. The set of total users $\mathcal{T}_1 = \{1,\ldots,K\}$, $m=1$, $\mathcal{W}=\emptyset$, $\mathcal{R}_1 = \mathcal{T}_1 - \mathcal{S} - \mathcal{W}$, where $\mathcal{S}$ is the set of the selected strong users, $\mathcal{W}$ is the set of the selected weak users and $\mathcal{R}$ is the set of residual users for selecting the weak users.

Step 2) For each user $k \in \mathcal{R}_m$, select the weak user by

$$\psi(m) = \arg\max_{k\in\mathcal{R}_m} \left\{ \frac{|\mathbf{h}_{m,1}\mathbf{h}_k^H|}{||\mathbf{h}_{m,1}||||\mathbf{h}_k||} \right\}$$
$$\mathcal{W} \leftarrow \mathcal{W} \cup \{\psi(m)\}$$
$$\mathbf{h}_{m,2} = \mathbf{h}_{\psi(m)}$$
$$m \leftarrow m+1$$

If $|\mathcal{W}| < N$, repeat Step 2); otherwise, to the END.

B. Power Allocation

After grouping users for all clusters, power allocation shall be applied to maximize the sum rate subject to that the capacity of the weak user achieves R_t. This is introduced in the following proposition:

Proposition 2. *The power ratio α_n for the strong user, which optimizes the objective function in (10), is given by*

$$\alpha_n^* = \frac{1}{2^{R_t}} - \frac{(2^{R_t}-1)(\sigma_n^2 + \sum_{k=1,k\neq n}^{N} |\mathbf{h}_{n,2}\mathbf{w}_k|^2 P_k)}{2^{R_t}|\mathbf{h}_{n,2}\mathbf{w}_n|^2 P_n}. \tag{13}$$

Proof: Omitted due to page limitation. ■

Since the power ratio shall be positive, $\alpha_n^* > 0$ is necessary in (13), the following requirement needs to be satisfied:

$$|\mathbf{h}_{n,2}\mathbf{w}_n|^2 P_n - (2^{R_t}-1)\left[\sigma_n^2 + \sum_{k=1,k\neq n}^{N} |\mathbf{h}_{n,2}\mathbf{w}_k|^2 P_k\right] > 0. \tag{14}$$

When (14) does not hold, $R_{n,2}$ is smaller than R_t even if all transmission power of this cluster is allocated to the weak user. In the following section, we show how to determine a proper R_t via performance analysis so that the probability that (14) holds is almost 1.

IV. Analysis and Minimum Target Rate

Now we derive the probability that the achievable rate of the weak user is equal to or greater than a given target rate. Denote $\mathcal{R}$ as the set of residual users which excludes the strong users. From the discussion in previous section, we shall select the user which maximizes the SINR in (9) from the set $\mathcal{R}$ as the weak user. This is equivalent to minimizing the power ratio for the weak user to achieve the given target rate, as a result, maximizing the sum rate. Thus whether the condition in (14) is satisfied or not is equivalent to that whether the achievable rate of the best selected weak user in (8) can achieve the target rate. We introduce this in the following proposition:

Proposition 3. *Assume that $\sum_{k=1,k\neq n}^{N} |\mathbf{h}_{n,2}\mathbf{w}_k|^2$ and $|\mathbf{h}_{n,2}\mathbf{w}_n|^2$ are independent. Order the SINR such that $\frac{1}{SINR_1} > \frac{1}{SINR_2} > \cdots > \frac{1}{SINR_{K-N}}$, where K is total number of users and N is the number of strong users. The best weak user, whose channel maximizes the SINR, is selected. When SNR approaches ∞, the outage probability that $Pr(R_{n,2} \geq R_t) \equiv Pr'$ for the weak user in the n-th cluster can be expressed as*

$$Pr' = \frac{(K-N)}{\beta((N-1),1)} \int_0^{\frac{1-\alpha_n}{2^{R_t}-1}-\alpha_n} \left\{1 - I_{\frac{x}{x+1}}\right\}^{K-N-1} \cdot x^{N-2}(1+x)^{-N} dx, \tag{15}$$

where $\beta(a,b)$ is the beta function defined by

$$\beta(a,b) = \int_0^1 t^{a-1}(1-t)^{b-1} dt,$$

$I_x(a,b)$ is the regularized incomplete beta function defined by

$$I_x(a,b) = \frac{\beta(x;a,b)}{\beta(a,b)},$$

and $\beta(x;a,b)$ is the incomplete beta function defined by

$$\beta(x;a,b) = \int_0^x t^{a-1}(1-t)^{b-1}dt.$$

Proof: One can show that

$$Pr' = Pr\left(\frac{1}{SINR_{K-N}} \leq \frac{1}{2^{R_t}-1}\right), \tag{16}$$

From (9), Pr' equals to

$$Pr\left(\frac{\frac{1}{SNR}}{|\mathbf{h}_{n,2}\mathbf{w}_n|^2} + \frac{\frac{2(N-1)\sum_{k=1,k\neq i}^{N}|\mathbf{h}_{n,2}\mathbf{w}_k|^2}{2(N-1)}}{\frac{2|\mathbf{h}_{n,2}\mathbf{w}_n|^2}{2}} \leq \frac{(1-\alpha_n)}{2^{R_t}-1}\right). \tag{17}$$

As SNR approaches to ∞, (17) becomes

$$Pr\left(\frac{\frac{2(N-1)\sum_{k=1,k\neq i}^{N}|\mathbf{h}_{n,2}\mathbf{w}_k|^2}{2(N-1)}}{\frac{2|\mathbf{h}_{n,2}\mathbf{w}_n|^2}{2}} \leq \frac{(1-\alpha_n)}{2^{R_t}-1}\right). \tag{18}$$

Because we select the maximum SINR to calculate the outage probability, we need to determine the PDF of left-hand side of (18). Thus, (18) becomes

$$\int_0^{\frac{1-\alpha_n}{2^{R_t}-1}-\alpha_n} f_{K-N,K-N}(x)dx$$
$$= \int_0^{\frac{1-\alpha_n}{2^{R_t}-1}-\alpha_n} (K-N)\{1-F(x)\}^{K-N-1}f(x)dx \tag{19}$$

From [9], we know that if the channel $\mathbf{h}_{n,2} \in \mathbb{C}^{1\times N}$ has complex Gaussian distribution $\mathcal{CN}(\mathbf{0},\mathbf{I}_N)$ and the precoding vector $\mathbf{w}_n$ is a unit vector, then in (18) $|\mathbf{h}_{n,2}\mathbf{w}_n|^2 \sim \chi_2^2/2$ and $\sum_{k=1,k\neq n}^{N}|\mathbf{h}_{n,2}\mathbf{w}_k|^2 \sim \chi_{2(N-1)}^2/2$, where χ_2^2 and $\chi_{2(N-1)}^2$ denote chi-square distribution with degree of freedom 2 and $2(N-1)$, respectively. We assume that $|\mathbf{h}_{n,2}\mathbf{w}_n|^2$ and $\sum_{k=1,k\neq n}^{N}|\mathbf{h}_{n,2}\mathbf{w}_k|^2$ are independent in (18). $f(x)$ is the F distribution with degrees of freedom $2(N-1)$ and 2. Thus, (19) becomes

$$\int_0^{\frac{1-\alpha_n}{2^{R_t}-1}-\alpha_n} (K-N)\{1-F(x;2(N-1),2)\}^{K-N-1}$$
$$\cdot \quad f(x;2(N-1),2)dx. \tag{20}$$

Manipulating (20) leads to the equation in (15). ∎

V. Simulation Results

Simulation results are provided to show the advantages of the proposed power allocation, and the accuracy of the theoretical results. Let $N=2$ in these examples.

Experiment 1. The accuracy and usage of the theoretical results. In this experiment, we show the outage probability in Proposition 3 as a function of power ratio α for various target rates. These theoretical results are compared to the complementary cumulative distribution function (CCDF) from the histogram of the proposed power allocation in Proposition 2. Note that Proposition 3 is an analytical result, and the proposed power allocation in Proposition 2 is an instantaneous result. We run several different realizations, obtain the histogram, and calculate the CCDF from the histogram.

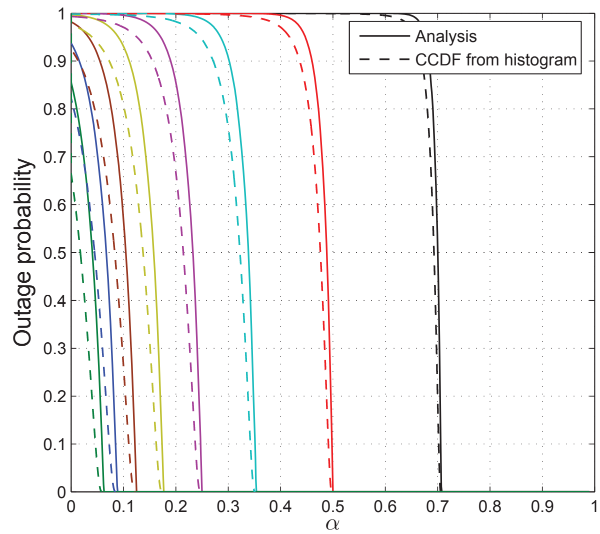

Fig. 2. The outage probability and CCDF results for cluster 1; $R_t = [4,\ 3.5,\ 3,\ \cdots, 1,\ 0.5]$ from left to right, SNR = 20 dB and $K=32$.

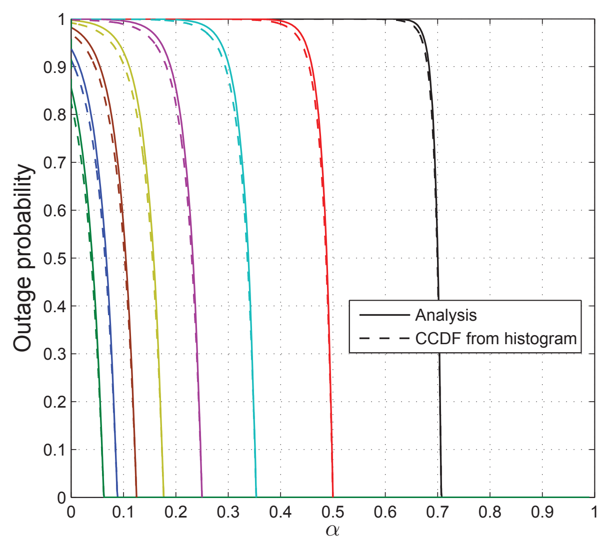

Fig. 3. The outage probability and CCDF results for cluster 1; $R_t = [4,\ 3.5,\ 3,\ \cdots, 1,\ 0.5]$ from left to right, SNR = 50 dB and $K=32$.

Let $K=32$. Fig. 2 and 3 show the results for SNR = 20 dB and 50 dB, respectively, for cluster 1. The solid curves and and the dashed curves represent, respectively, the analytical and the simulation results. From Fig. 2 there is a gap between the analytical and the simulation results due to low value of SNR and the independency between $|\mathbf{h}_{n,2}\mathbf{w}_n|^2$ and $\sum_{k=1,k\neq n}^{N}|\mathbf{h}_{n,2}\mathbf{w}_k|^2$. However, the gap becomes quite small when the value of SNR increases from 20 dB to 50 dB, observed from Fig. 2 and 3.

Let $K=128$. Fig. 4 shows the results for SNR = 50 dB for

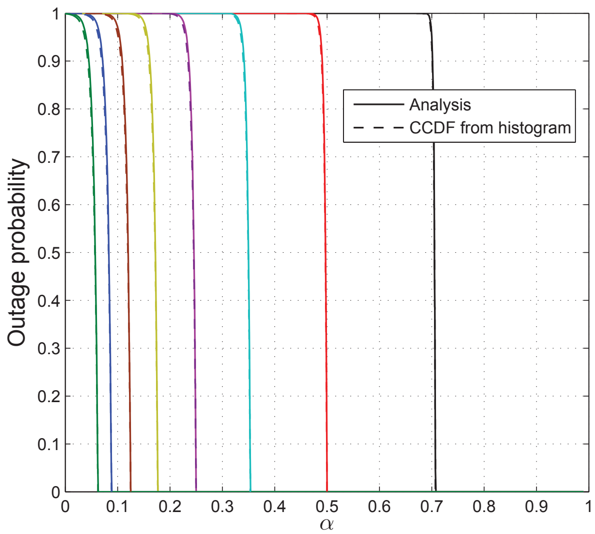

Fig. 4. The outage probability and CCDF results for cluster 1; $R_t = [4,\ 3.5,\ 3,\ \cdots, 1,\ 0.5]$ from left to right, SNR = 50 dB and $K = 128$.

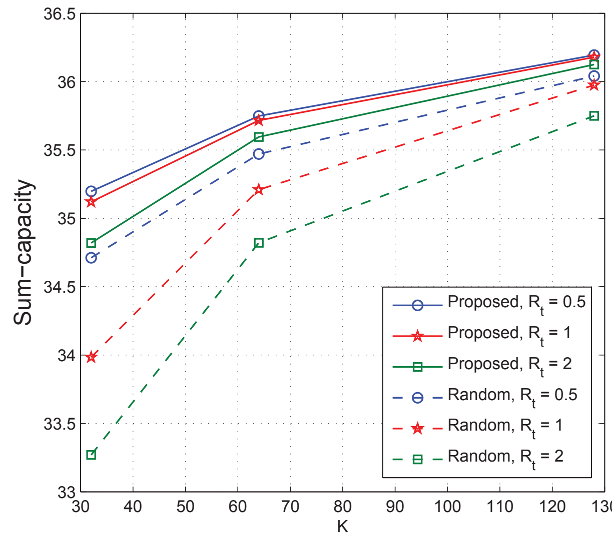

Fig. 5. Comparison of the sum-rate between proposed power allocation scheme and randomly generated scheme for $N = 2$ and SNR = 50 dB.

cluster 1. Comparing this figure to Figs. 2 and 3, the analytical and simulation results are quite matched. These figures show the accuracy of the proposed performance analysis.

These results also suggest reasonable minimum target rate for the weak user. For instance, in Fig. 4, when the given target rate is 0.5, this target rate is achievable because it corresponds to a positive power allocation solution; in this example, α shall be around 0.68 indicated by both the analytical and CCDF results (see the first right curves). On the other hand, in Fig. 3, when the given target rate is 4, this target rate is NOT always achievable because it corresponds to a negative power allocation solution if one would like the outage probability be 1 (see the first left curves).

Experiment 2. Sum rate of proposed power allocation. In this experiment, we demonstrate the advantage of the proposed power allocation in Proposition 2. Let SNR = 50 dB. Random power allocation is used for comparison purpose. For the random power allocation, α is set to achieve the target rate according to the result in previous example; for instance, uniformly distributed in [0 0.68] for $K = 128$ and $R_t = 0.5$.

Fig. 5 shows the sum-rate as a function of number of total users K for these two power allocation schemes. Observe that the proposed power allocation outperforms the random allocation scheme for all simulation settings. The advantage is more pronounced when K is small than it is large.

Acknowledgment

The authors would like to thank the support from the Industrial Technology Research Institute (ITRI) and the Ministry of Science and Technology (MOST) Taiwan.

VI. Conclusion

We have proposed a complexity-reduced algorithm to cluster the strong and the weak users separately. The SUS algorithm has been applied to select the strong users. Then the weak users who have similar channel directions with the strong users shall be selected and the selection is irrelevant to power allocation thanks to the results in Proposition 1. In addition, an optimal power allocation solution for maximizing sum rate subject to a minimum target rate has been proposed in Proposition 2. Finally from the analytical results, we have derived a closed-form expression for the outage probability that the weak user achieves a given target rate in Proposition 3. Using this proposition, a reasonable minimum target rate can be obtained so that the QoS can always be attained; meanwhile it avoids allocating all the power to the weak user but still failing to achieve the QoS.

References

[1] Y. Saito, Y. Kishiyama, A. Benjebbour, T. Nakamura, A. Li, and K. Higuchi, "Non-orthogonal multiple access (NOMA) for cellular future radio access," in *Proc. IEEE Veh. Technol. Conf.*, Jun. 2013, pp. 1–5.

[2] Z. Ding, F. Adachi, H. V. Poor, "The application of MIMO to nonorthogonal multiple access," *IEEE Trans. Wireless Commun.*, vol. 15, pp. 537–552, Jan. 2016.

[3] B. Kim, S. Lim, H. Kim, S. Suh, J. Kwun, S. Choi, C. Lee, S. Lee and D. Hong, "Non-orthogonal multiple access in a downlink multiuser beamforming system," in *Proc. IEEE MILCOM*, Nov. 2013.

[4] J. Kim, J. Koh, J. Kang, K. Lee, and J. Kang, "Design of user clustering and precoding for downlink non-orthogonal multiple access (NOMA)," in *Proc. IEEE MILCOM*, Oct. 2015, pp. 1170–1175.

[5] H. Zhang, D. K. Zhang, W. X. Meng and C. Li, "User pairing algorithm with SIC in non-rrthogonal multiple access system," in *Proc. IEEE ICC*, 2016, pp. 1–6.

[6] T. Yoo and A. Goldsmith, "On the optimality of multiantenna broadcast scheduling using zero-forcing beamforming," *IEEE J. Select. Areas Commun.*, vol. 24, pp. 528–541, Mar. 2006.

[7] A. J. Goldsmith, "The capacity of downlink fading channels with variable-rate and power," *IEEE Trans. Vehic. Technol.*, vol. 46, pp. 569–580, Aug. 1997.

[8] Q. H. Spencer , A. L. Swindlehurst and M. Haardt, "Zero-forcing methods for downlink spatial multiplexing in multi-user MIMO channels," *IEEE Trans. Signal Process.*, pp. 461–471, 2004.

[9] A. M. Mathai and S. B. Provost, *Quadratic Forms in Random Variables.* Marcel Dekker, 1992.