# Orthogonal Codes for MAI-Free MC-CDMA with Carrier Frequency Offsets (CFO)

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***Abstract*— The performance of MC-CDMA in the presence of carrier frequency offsets (CFO) can be severely degraded due to multiaccess interference (MAI). It is shown in this work that a properly chosen subset of real Hadamard-Walsh or exponential codes can achieve zero MAI in a CFO environment. For a channel of length $L$ and a number $G$ of power of 2 with $G \geq L$, we prove that $1 + \log_2(N/G)$ Hadamard-Walsh codewords or $N/G$ exponential codewords can achieve zero MAI under any CFO level. Simulation results are given to corroborate derived theoretical results and evaluate the performance of MC-CDMA with a variable number of users in a CFO environment.**

## I. Introduction

Multicarrier code division multiple access (MC-CDMA) has emerged as a promising multiaccess technique for high data rate communications. In MC-CDMA, symbols are spread into multiple chips in the frequency domain, which are then modulated onto orthogonal subcarriers. MC-CDMA is inherently more robust to inter symbol interference (ISI) than conventional CDMA system due to the use of the OFDM (Orthogonal Frequency Division Multiplexing) structure. Furthermore, the full diversity gain can be achieved if the maximum ratio combining (MRC) is used in MC-CDMA. However, the multipath and/or the carrier frequency offset (CFO) effects tend to destroy orthogonality among users and lead to MAI. Thus, the performance of MC-CDMA can be greatly degraded. Although multiuser detection (MUD) techniques can mitigate MAI [7], the diversity gain may be sacrificed since MRC is no longer optimally performed. Besides, MUD requires channel state information of all users, which is more difficult to estimate in an MAI environment.

There has been research on MAI suppression using single user detection techniques. For example, the structural differences of interfering users caused by CFO were exploited at the receiver to suppress MAI in [3]. However, this MAI suppression technique imposes a computational burden on the receiver since a DFT (Discrete Fourier Transform) of size larger than $N$ is required due to the oversampling of the received signal in the frequency domain. Moreover, the MAI suppression capability of the resultant MC-CDMA decreases when the frequency shift of the interfering user is small [3]. In [1], groups of users share a set of subcarriers while full frequency diversity of MC-CDMA system is preserved. MAI is only present among users in the same group and is suppressed via simplified multiuser detection. Another way to reduce MAI is achieved by code design while keeping the structure of MC-CDMA unchanged [6]. In [6], a code design method based on real Hadamard-Walsh codes was proposed and shown to achieve zero MAI in a multipath environment in MC-CDMA. The main result in [6] can be stated below. If the length of channel impulse response is $L$, we can partition $N$ Hadamard-Walsh codes into $G$ subsets, where $G$ is a power of 2 and $L \leq G < N$, $N/G$ users will be free from MAI if a proper set of $N/G$ codewords is used in MC-CDMA. Moreover, it was demonstrated that two particular users will not experience any MAI even in the presence of CFO when all $N/G$ users are active.

In this work, we extend the results derived in [6] to a CFO environment. That is, we propose code schemes for MC-CDMA to completely eliminate MAI even in the presence of CFO.

## II. System Model

Suppose that there are $T$ users in an MC-CDMA system. The block diagram of the uplink transmission of the $i$th user is shown in Fig. 1. As shown in the figure, symbol $x_i$ is spread by $N$ codewords in the frequency domain to yield an $N \times 1$ vector:

$$y_i[k] = w_i[k]x_i, \quad 0 \leq k \leq N-1, \tag{1}$$

where $w_i[k]$ is the $k$th component of the $i$th orthogonal code. The spreading code of a user is the same along time. The resulting block of length $N$ is passed through an $N \times N$ IDFT matrix. After the parallel-to-serial conversion, a cyclic prefix is added to mitigate ISI. Then, symbols are fed into the multiple access channel. Since the uplink scenario is considered, it is reasonable to assume that each user experiences a different fading channel with a different amount of CFO.

At the receiver, the cyclic prefix is removed. After the serial-to-parallel conversion, the block is passed through the $N \times N$ DFT matrix. The $k$th component of the DFT output, $\hat{\mathbf{y}}$, can be expressed by

$$\hat{y}[k] = \sum_{j=0}^{T-1} r_j[k] + e[k], \tag{2}$$

where $e[k]$ is the DFT of additive noise, and $r_j[k]$ is the received signal contributed from the $j$th user due to the channel fading and CFO effects. Suppose that user $j$ has a normalized CFO $\epsilon_j$, *i.e.* the actual CFO normalized to the subcarrier spacing. $r_j[k]$ can be written as [5], [6]

$$\begin{aligned} r_j[k] &= \alpha_j \lambda_j[k] y_j[k] \\ &+ \beta_j \sum_{m=0, m\neq k}^{N-1} \lambda_j[m] y_j[m] \frac{e^{-j\pi\frac{m-k}{N}}}{N \sin\frac{\pi(m-k+\epsilon_j)}{N}}, \end{aligned} \tag{3}$$

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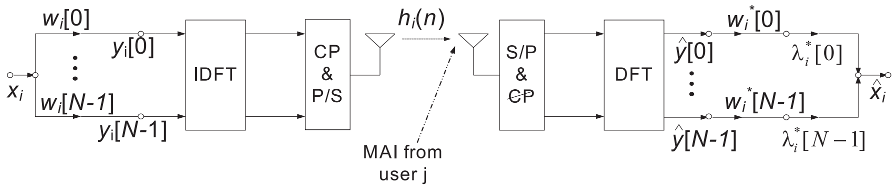


Fig. 1. The block diagram of the uplink transmission of the $i$th user in an MC-CDMA system.

where $\lambda_j[m]$ is the $m$th component of N-point DFT of the channel impulse response of user $j$, and

$$\alpha_j = \frac{\sin \pi\epsilon_j}{N \sin \frac{\pi\epsilon_j}{N}} e^{j\pi\epsilon_j \frac{N-1}{N}} \text{ and } \beta_j = \sin(\pi\epsilon_j) e^{j\pi\epsilon_j \frac{N-1}{N}}.$$

Note that, when there is no CFO, *i.e.* $\epsilon_j = 0$, $r_j[k] = \lambda_j[k]y_j[k]$. However, in the presence of CFO, there are two terms. The first term is $\lambda_j[k]y_j[k]$ distorted by $\alpha_j$ and the second term is the ICI caused by CFO. Finally, the $i$th transmitted symbol is detected by multiplying the received symbol $\hat{y}[k]$ of user $i$ by $w_i^*[k]$ and performing MRC on $\hat{y}[k]w_i^*[k]$, *i.e.,*

$$\begin{aligned} \hat{x}_i &= \sum_{k=0}^{N-1} \hat{y}[k]\lambda_i^*[k]w_i^*[k] \\ &= s_i + \sum_{j=0, j\neq i}^{T-1} MAI_{i\leftarrow j} + \sum_{k=0}^{N-1} e[k]\lambda_i^*[k]w_i^*[k], \end{aligned} \tag{4}$$

where $s_i$ consists of the distorted chip and the ICI caused by CFO for the desired user given by

$$s_i = \sum_{k=0}^{N-1} r_i[k]\lambda_i^*[k]w_i^*[k], \tag{5}$$

and $MAI_{i\leftarrow j}$ is the MAI of user $i$ due to the $j$th user's CFO

$$MAI_{i\leftarrow j} = \sum_{k=0}^{N-1} r_j[k]\lambda_i^*[k]w_i^*[k]. \tag{6}$$

Using Eqs. (3) and (6), we can show that the MAI term is given by

$$MAI_{i\leftarrow j} = MAI_{i\leftarrow j}^{(0)} + MAI_{i\leftarrow j}^{(1)}, \tag{7}$$

where

$$MAI_{i\leftarrow j}^{(0)} = \alpha_j x_j \sum_{k=0}^{N-1} \lambda_j[k]w_j[k]\lambda_i^*[k]w_i^*[k], \tag{8}$$

and

$$MAI_{i\leftarrow j}^{(1)} = \beta_j x_j \sum_{m=0, m\neq k}^{N-1} \lambda_j[m]y_j[m] \left\{ \frac{e^{-j\pi\frac{m-k}{N}}}{N \sin \frac{\pi(m-k+\epsilon_j)}{N}} \lambda_i^*[k]w_i^*[k] \right\}. \tag{9}$$

Eq. (8) can be expressed in matrix form as [6]

$$MAI_{i\leftarrow j}^{(0)} = \alpha_j x_j \mathbf{h}_i^\dagger \underbrace{\mathbf{F}_0^\dagger \mathbf{W}_i^\dagger \mathbf{W}_j \mathbf{F}_0}_{\mathbf{A}_{ij}} \mathbf{h}_j, \tag{10}$$

where $\mathbf{F}_0 = \mathbf{F} \begin{pmatrix} \mathbf{I}_L \\ \mathbf{0} \end{pmatrix}_{N\times L}$, $\quad \mathbf{h}_i = (h_i(0) \cdots h_i(L-1))^T$, and

$$\mathbf{W}_i = \text{diag}(w_i[0]\, w_i[1] \cdots w_i[N-1]),$$

and where $\mathbf{F}$ is the $N \times N$ DFT matrix whose element at the $k$th row and the $n$th column is $[\mathbf{F}]_{k,n} = \frac{1}{\sqrt{N}} e^{-j\frac{2\pi}{N}kn}$. Also, $\dagger$ in Eq. (10) denotes the matrix Hermitian operation. It was shown in [6] that $MAI_{i\leftarrow j}^{(1)}$ given by (9) can be rewritten as

$$MAI_{i\leftarrow j}^{(1)} = \beta_j x_j \sum_{p=1}^{N-1} g_j(-p) \left\{ (\mathbf{h}_i^{(p)})^\dagger \underbrace{\mathbf{F}_0^\dagger (\mathbf{W}_i^{(p)})^\dagger \mathbf{W}_j \mathbf{F}_0}_{\mathbf{C}_{ij}^{(p)}} \mathbf{h}_j \right\}, \tag{11}$$

where

$$g_j(p) = \frac{e^{-j\pi\frac{p}{N}}}{N \sin \frac{\pi(p+\epsilon_j)}{N}}, \tag{12}$$

$$\mathbf{W}_i^{(p)} = \text{diag}(w_i[p] \cdots w_i[N-1]\, w_i[0] \cdots w_i[p-1]), \tag{13}$$

and

$$\mathbf{h}_i^{(p)} = (h_i(0)e^{-j\frac{2\pi 0 p}{N}} \cdots h_i(L-1)e^{-j\frac{2\pi(L-1)p}{N}})^T. \tag{14}$$

## III. Orthogonal Codes for MAI-free MC-CDMA with CFO

### *A. Requirements for MAI-free Codes*

Theoretical requirements for codes to produce an MAI-free MC-CDMA system in the presence of CFO are implied by Eqs. (10) and (11). That is, to have zero MAI in a frequency selective channel with CFO, we demand

$$MAI_{i\leftarrow j}^{(0)} = 0 \text{ and } MAI_{i\leftarrow j}^{(1)} = 0.$$

that $\mathbf{A}_{ij}$ and $\mathbf{C}_{ij}^{(p)}$ must be zero matrices of dimension $L \times L$ for all $i \neq j$ to achieve MAI-free in a CFO environment [6]. Let

$$\mathbf{B}_{ij} = \mathbf{F}^\dagger \mathbf{R}_{i,j} \mathbf{F}, \quad \mathbf{R}_{i,j} = \mathbf{W}_i^\dagger \mathbf{W}_j,$$

$$\mathbf{D}_{ij}^{(p)} = \mathbf{F}^\dagger \mathbf{R}_{i,j}^{(p)} \mathbf{F}, \quad \mathbf{R}_{i,j}^{(p)} = (\mathbf{W}_i^{(p)})^\dagger \mathbf{W}_j.$$

It is well known that $\mathbf{B}_{ij}$ and $\mathbf{D}_{ij}^{(p)}$ are circulant matrices [2]. Therefore, their first columns, *i.e.*, $(b_{i,j}(0) \cdots b_{i,j}(N-1))^T$ and $(d_{i,j}(0) \cdots d_{i,j}(N-1))^T$ are the $N$-point IDFT of $\mathbf{r}_{i,j}$ and $\mathbf{r}_{i,j}^{(p)}$ respectively, where

$$\mathbf{r}_{i,j} = (r_{i,j}[0] \cdots r_{i,j}[N-1])^T, \tag{15}$$

and

$$r_{i,j}^{(p)}[k] = w_i^{(p)}[k]w_j[k], \quad k = 0, 1, \ldots N-1. \tag{16}$$


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Since $\mathbf{A}_{ij}$ and $\mathbf{C}_{ij}^{(p)}$ are $L \times L$ upper left submatrices of $\mathbf{B}_{ij}$ and $\mathbf{E}_{ij}^{(p)}$ respectively, conditions $MAI_{i\leftarrow j}^{(0)} = 0$ and $MAI_{i\leftarrow j}^{(1)} = 0$ are equivalent to

$$\begin{cases} b_{i,j}(n) = 0, & 0 \le n \le L-1, \\ b_{i,j}(N-n) = 0, & 1 \le n \le L-1, \end{cases} \tag{17}$$

and

$$\begin{cases} d_{i,j}(n) = 0, & 0 \le n \le L-1, \\ d_{i,j}(N-n) = 0, & 1 \le n \le L-1, \end{cases} \tag{18}$$

respectively.

Let $w[k]$, $0 \le k \le N-1$, be any codeword of length $N$ and $G < N$ be any integer that divides $N$. For presentation convenience, we define periodic and antiperiodic codewords as follows. We say that the codeword is periodic with period $N/G$ if

$$w[((k + gN/G))_N] = w[k], \tag{19}$$

where $0 \le k \le N-1$, $0 \le g \le G-1$, and $((n))_N$ denotes $n$ modulo $N$. We say the codeword is antiperiodic with antiperiodic $N/G$ if

$$w[((k + N/G))_N] = -w[k]. \tag{20}$$

**Lemma 1:** Let $\mathbf{w}_i = (w_i[0] \cdots w_i[N-1])^T$, be any periodic or antiperiodic codeword with period or antiperiod $N/G$. Then, $\mathbf{w}_i^{(p)}$ is periodic or antiperiodic if $\mathbf{w}_i$ is periodic or antiperiodic, respectively, with the same period or antiperiod $N/G$.

**Proof:** By the definition of $\mathbf{w}_i^{(p)}$, we have

$$w_i^{(p)}[k] = w_i[N - p + k]. \tag{21}$$

If $\mathbf{w}_i$ is periodic with period $N/G$, we have

$$\begin{aligned} w_i^{(p)}[((k + gN/G))_N] &= w_i[((N - p + k + gN/G))_N] \\ &= w_i[N - p + k] = w_i^{(p)}[k]. \end{aligned}$$

Similarly, if $\mathbf{w}_i$ is antiperiodic with antiperiod $N/G$, we have

$$\begin{aligned} w_i^{(p)}[((k + N/G))_N] &= w_i[((N - p + k + N/G))_N] \\ &= -w_i[N - p + k] = -w_i^{(p)}[k], \end{aligned}$$

for $0 \le g \le G-1$ and $0 \le k \le N-1$. ■

*B. Hadamard-Walsh Codes*

An $N \times N$ Hadamard matrix $\mathbf{H}_N$ with $N = 2^r$, $r = 1, 2, ...,$ can be recursively generated by the Hadamard matrix of order 2, *i.e.*,

$$\mathbf{H}_N = \mathbf{H}_2 \otimes \mathbf{H}_{N/2} = \begin{pmatrix} \mathbf{H}_{N/2} & \mathbf{H}_{N/2} \\ \mathbf{H}_{N/2} & -\mathbf{H}_{N/2} \end{pmatrix}, \tag{22}$$

where $\otimes$ is the Kronecker product and

$$\mathbf{H}_2 = \begin{pmatrix} +1 & +1 \\ +1 & -1 \end{pmatrix}. \tag{23}$$

Suppose that the channel length is L. We divide $\mathbf{H}_N$ equally into G subsets, where $G = 2^q$ with $q$ being a positive integer and $N > G \ge L$ so that each subset has $N/G$ codewords. The first subset, denoted by $G_0$, has codewords $\{\mathbf{w}_0, \mathbf{w}_1, ..., \mathbf{w}_{\frac{N}{G}-1}\}$. We can further divide codewords in $G_0$ into two disjoint subsets of equal size as

$$G_{00} = \{\mathbf{w}_0, ..., \mathbf{w}_{\frac{N}{2G}-1}\} \text{ and } G_{01} = \{\mathbf{w}_{\frac{N}{2G}}, \cdots, \mathbf{w}_{\frac{N}{G}-1}\}.$$

Then, we have the following properties.

**Lemma 2:** For any codeword $\mathbf{w}_i$ in $G_{01}$ and any codeword $\mathbf{w}_j$ in $G_{00}$, we have $MAI_{i\leftarrow j}^{(0)} = MAI_{i\leftarrow j}^{(1)} = 0$.

**proof:** It was shown in [6] that, if $\mathbf{w}_i$ and $\mathbf{w}_j$ are in two disjoint subsets $G_g$, $g = 0, 1, ...G-1$, $MAI_{i\leftarrow j}^{(0)} = 0$. Since $G_{01}$ and $G_{01}$ are disjoint subsets of $G_0$, we have $MAI_{i\leftarrow j}^{(0)} = 0$. Next, to prove $MAI_{i\leftarrow j}^{(1)} = 0$, we would like to show Eq. (18) is satisfied. Recall $r_{i,j}^{(p)}[k] = w_i^{(p)}[k] w_j[k]$, $k = 0, 1, ...N-1$. By taking the IDFT of $\mathbf{r}_{i,j}^{(p)}$, we have

$$r_{i,j}^{(p)}(n) = \frac{1}{N} \sum_{m=0}^{N-1} r_{i,j}^{(p)}[m] e^{j\frac{2\pi}{N}mn}. \tag{24}$$

Let $m = k + gN/G$, $0 \le k \le N/G - 1$ and $0 \le g \le G-1$, we can rewrite Eq. (24) as

$$r_{i,j}^{(p)}(n) = \frac{1}{N} \sum_{k=0}^{N/G-1} \sum_{g=0}^{G-1} r_{i,j}^{(p)}[k + gN/G] e^{j\frac{2\pi}{N}(k+gN/G)n}. \tag{25}$$

Since codewords $\mathbf{w}_i$ and $\mathbf{w}_j$ belong to $G_0$, they are among the first $N/G$ columns of the Hadamard-Walsh matrix and formed by repeating the upper left $N/G \times N/G$ submatrix of $\mathbf{H}_N$ $G$ times. Hence, they are periodic with period $N/G$. By Lemma 1, $\mathbf{w}_i^{(p)}$ is also periodic with period $N/G$. Since the product of two periodic functions whose periods are the same is another periodic function with the same period, we have

$$r_{i,j}^{(p)}[k + gN/G] = r_{i,j}^{(p)}[k]. \tag{26}$$

Then, we can rewrite Eq. (25) as

$$r_{i,j}^{(p)}(n) = \frac{1}{N} \sum_{k=0}^{N/G-1} r_{i,j}^{(p)}[k] e^{j\frac{2\pi}{N}kn} \sum_{g=0}^{G-1} e^{j\frac{2\pi}{G}gn}, \tag{27}$$

where $0 \le k \le N/G - 1$ and $0 \le g \le G-1$. Since

$$\sum_{g=0}^{G-1} e^{j\frac{2\pi}{G}gn} = \begin{cases} G, & n = 0, \pm G, \cdots \\ 0, & \text{otherwise}, \end{cases} \tag{28}$$

we have

$$r_{i,j}^{(p)}(n) = \begin{cases} \frac{G}{N} \sum_{k=0}^{N/G-1} r_{i,j}^{(p)}[k] e^{j\frac{2\pi}{N}kn}, & n = 0, \pm G, \cdots \\ 0, & \text{otherwise}. \end{cases}$$

To prove $r_{i,j}^{(p)}(0) = 0$, we need to show $\mathbf{r}_{i,j}^{(p)}$ has an equal number of 1 and $-1$. In general, $\mathbf{r}_{i,j}^{(p)}$ does not belong to the Hadamard-Walsh matrix whose codewords have an equal number of 1 and $-1$. For example, for $N = 8$, $\mathbf{w}_8^{(1)} \cdot \mathbf{w}_7$ has two $-1$ and six 1, where $\cdot$ denotes the component-wise vector product. However, if $\mathbf{w}_i \in G_{01}$ and $\mathbf{w}_j \in G_{00}$, $\mathbf{r}_{i,j}^{(p)}$ does have an equal number of 1 and $-1$ as show below. According to (22), codewords in $G_{00}$ are the first $N/2G$ columns of $H_N$ and obtained by repeating the $N/2G \times N/2G$ submatrix $2G$ times. Hence, any codeword $\mathbf{w}_j \in G_{00}$ is periodic with period $N/2G$. Similarly, we can show that any codeword $\mathbf{w}_i \in G_{01}$


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is antiperiodic with antiperiodic $N/2G$. By Lemma 1, $\mathbf{w}_i^{(p)}$ is also antiperiodic with the same antiperiod. Therefore, for $0 \le k \le N-1$, we have

$$\begin{aligned} w_i^{(p)}[((k+N/2G))_N]w_j[((k+N/2G))_N] &= \\ -w_i[((N-p+k))_N]\, w_j[k] = -w_i^{(p)}[k]w_j[k]. \end{aligned}$$

Hence, $\mathbf{r}_{i,j}^{(p)}$ is antiperiodic with antiperiod $N/2G$. It can be easily shown that any antiperiodic code has an equal number of $\pm 1$. Thus, $r_{i,j}^{(p)}(0) = \frac{1}{N}\sum_{k=0}^{N-1} r_{i,j}^{(p)}[k] = 0$. ■

Let us give an example with $N = 16$ and $L = 2$. By choosing $G = 2$, $G_{00} = \{\mathbf{w}_0, \mathbf{w}_1, \mathbf{w}_2, \mathbf{w}_3\}$ and $G_{01} = \{\mathbf{w}_4, \mathbf{w}_5, \mathbf{w}_6, \mathbf{w}_7\}$. By Lemma 2, any codeword chosen from $G_{01}$, achieves zero MAI with respect to any codeword from $G_{00}$. On the other hand, we will not have an MAI-free system when both codewords are chosen from $G_{00}$ (or both from $G_{01}$). For instance, $MAI_{2\leftarrow 3} \ne 0$ or $MAI_{4\leftarrow 5} \ne 0$ for $N = 16$ and $L = 2$.

Note that Lemma 2 does not identify a subset of codewords that can be assigned to all active users while keeping the system MAI-free. This choice will be examined in the following theorem. In particular, we want to determine such an MAI-free set from subsets of $G_0$ and specify the number of codewords in the resulting MAI-free set.

**Theorem 1:** For a channel of length $L$ and $G = 2^q \ge L$, there are $1+\log_2(N/G)$ codewords from $N$ Hadamard-Walsh codes that will lead to an MAI-free MC-CDMA system under any CFO level.

**proof:** We form subsets $G_0$, $G_{00}$ and $G_{01}$ as described above. To build an MAI-free subset, we must choose only one of the codes from $G_{01}$. To determine the remaining codes from $G_{00}$, we divide $G_{00}$ into two subsets $G_{000}$ and $G_{001}$, each of $N/4G$ codes, by following the same procedure. Then, we can choose one from $G_{001}$, since it can be proved by arguments similar to that in Lemma 2 that any codeword from $G_{001}$ is MAI-free from any codeword in $G_{000}$. By repeating this procedure, we can obtain an MAI-free set. Since the division of a subset generates one codeword to be included in the MAI-free codeword set in each stage, we have $\log_2(N/G)$ codes in the MAI-free set. Furthermore, each of the two subsets has only one codeword in the last stage, we can add both codewords ($\mathbf{w}_0$ and $\mathbf{w}_1$) to the MAI-free set. Thus, the total number of MAI-free codewords is $1 + \log_2(N/G)$. ■

### *C. Exponential Orthogonal Codes*

Since a relatively small number of users can be MAI-free in a channel with CFO using Hadamard-Walsh codes, we look for other codes for this purpose. In this section, we study the exponential codes of size $N$, which is of the following form

$$w_i[k] = e^{j\frac{2\pi}{N}ki}, \quad k, i = 0, 1, ..., N-1. \tag{29}$$

Then, the MAI-free property of this code can be stated below.

**Theorem 2:** Let the channel length be $L$ and $G = 2^q \ge L$. There exists $N/G$ exponential codewords such that the corresponding MC-CDMA is MAI free in a CFO environment.

**Proof:** Consider two codewords with indices $i$ and $i'$. If we let $i - i' = mG$, $m = 1, 2, ...$, then, for $k = 0, 1, ...N-1$ and $g = 0, 1, ...G-1$, we have

$$\begin{aligned} r_{i,j}[k+gN/G] &= e^{j\frac{2\pi}{N}(k+gN/G)(i-i')} = \\ e^{j\frac{2\pi}{N}k(i-i')}e^{j\frac{2\pi}{G}g(i-i')} &= e^{j\frac{2\pi}{N}k(i-i')} = r_{i,j}[k], \end{aligned} \tag{30}$$

and

$$\begin{aligned} r_{i,j}^{(p)}[k+gN/G] = e^{j\frac{2\pi}{N}(N-p)i}e^{j\frac{2\pi}{N}(k+gN/G)(i-i')} = \\ e^{j\frac{2\pi}{N}(N-p+k)i}e^{-j\frac{2\pi}{N}(k)i'}e^{j\frac{2\pi}{G}g(i-i')} = r_{i,j}^{(p)}[k]. \end{aligned} \tag{31}$$

By using the same procedure as Lemma 2, we can show

$$r_{i,j}^{(p)}(n) = \begin{cases} \frac{G}{N}\sum_{k=0}^{N/G-1} r_{i,j}^{(p)}[k]e^{j\frac{2\pi}{N}kn}, & n = 0, \pm G, ... \\ 0, & \text{otherwise}, \end{cases} \tag{32}$$

and

$$r_{i,j}(n) = \begin{cases} \frac{G}{N}\sum_{k=0}^{N/G-1} r_{i,j}[k]e^{j\frac{2\pi}{N}kn}, & n = 0, \pm G, ... \\ 0, & \text{otherwise}. \end{cases} \tag{33}$$

Furthermore, for $i \ne j$, we have

$$r_{i,j}(0) = \sum_{k=0}^{N-1} r_{i,j}[k] = \sum_{k=0}^{N-1} e^{j\frac{2\pi(i-j)}{N}k} = 0, \tag{34}$$

and similarly, $r_{i,j}^{(p)}(0) = 0$. Thus, Eqs. (17) and (18) hold. Since there are $N/G$ codewords such that $i - i' = mG$, $m = 1, 2, ...$, the total number of MAI-free codewords from $N$ exponential codes is $N/G$. ■

The exponential codes are especially valuable as training sequences in a MIMO-OFDM system as they can decouple inter-antenna interference in a CFO-free channel [4]. Due to Theorem 2, we can use exponential codes as training sequences for multi-user MIMO-OFDM systems in a CFO environment to eliminate both inter-antenna interference and MAI.

## IV. Simulation Results

The Monte Carlo simulation was conducted to corroborate theoretical results derived in the last section. In the simulation, channel taps were generated as independently identically distributed (i.i.d.) random variables of unit variance. Every user had his/her own CFO value, and the worst case was considered. That is, every user was randomly assigned by a CFO value of either $\epsilon$ or $-\epsilon$. The MAI power was normalized by $\sum_{k=0}^{N-1}|\lambda_i[k]|^2$ since the desired signal was scaled by the same amount. In all examples, we suppose $N = 16$, $L = 2$ and the CFO value is fixed to be $\pm 0.1$.

**Example 1.** The values of $MAI_{i\leftarrow j}$ power for all Hadamard-Walsh codewords in $G_0$ are tabulated in Table I. When the MAI value is below $-290$ dB, it is equivalent to zero numerically. We have several interesting observations from Table I. First, users with codewords $\mathbf{w}_0$ and $\mathbf{w}_1$ will be mutually MAI-free with other users. This result is not a surprise since it was already shown in the proof of Theorem 1. Second, there is no MAI among any two users, if one uses a codeword from $G_{01} = \{\mathbf{w}_4, \mathbf{w}_5, \mathbf{w}_6, \mathbf{w}_7\}$ while the other from $G_{00} = \{\mathbf{w}_0, \mathbf{w}_1, \mathbf{w}_2, \mathbf{w}_3\}$. This result validates Lemma 2. Third, if users use codewords from $G_{01}$ (or from $G_{00}$), MAI may not be zero. For example, we see $MAI_{4\leftarrow 6} = -27.6$ dB and $MAI_{5\leftarrow 7} = -45.0$ dB. Finally, we observe


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$\{\mathbf{w}_0, \mathbf{w}_1, \mathbf{w}_2, \mathbf{w}_4\}$ or $\{\mathbf{w}_0, \mathbf{w}_1, \mathbf{w}_3, \mathbf{w}_7\}$ provides two subsets of MAI-free codewords in the presence of CFO. This confirms our claim that there are $1 + \log_2(16/2) = 4$ users that will be mutually MAI-free.

TABLE I

$MAI_{i \leftarrow j}$ POWER $(dB)$ AS A FUNCTION OF HADAMARD-WALSH CODEWORDS IN $G_0$.

| | $\mathbf{w}_0$ | $\mathbf{w}_1$ | $\mathbf{w}_2$ | $\mathbf{w}_3$ | $\mathbf{w}_4$ | $\mathbf{w}_5$ | $\mathbf{w}_6$ | $\mathbf{w}_7$ |
|---|---|---|---|---|---|---|---|---|
| $\mathbf{w}_0$ | × | -324 | -324 | -325 | -325 | -328 | -326 | -328 |
| $\mathbf{w}_1$ | -325 | × | -325 | -324 | -328 | -324 | -328 | -326 |
| $\mathbf{w}_2$ | -325 | -325 | × | -30.1 | -326 | -328 | -324 | -327 |
| $\mathbf{w}_3$ | -325 | -325 | -30.1 | × | -328 | -326 | -328 | -324 |
| $\mathbf{w}_4$ | -324 | -329 | -326 | -328 | × | -33.1 | -27.6 | -53.6 |
| $\mathbf{w}_5$ | -328 | -324 | -328 | -326 | -33.1 | × | -54.25 | -45.0 |
| $\mathbf{w}_6$ | -326 | -329 | -324 | -328 | -27.6 | -54.2 | × | -33.1 |
| $\mathbf{w}_7$ | -328 | -326 | -328 | -323 | -53.6 | -45.0 | -33.1 | × |

**Example 2.** Here we focus on the performance of exponential codes. The MAI power in the unit of dB between users with different codewords is shown in Table II. According to Theorem 2 in Sec. III, users with even indexed codewords, *i.e*, $\{\mathbf{w}_0, \mathbf{w}_2, \mathbf{w}_4, \mathbf{w}_6, \mathbf{w}_8, \mathbf{w}_{10}, \mathbf{w}_{12}, \mathbf{w}_{14}\}$ are mutually MAI-free. This is illustrated in the top half of Table II. We also observe that even though users with odd-indexed codewords *i.e.*, $\{\mathbf{w}_1, \mathbf{w}_3, \mathbf{w}_5, \mathbf{w}_7, \mathbf{w}_9, \mathbf{w}_{11}, \mathbf{w}_{13}, \mathbf{w}_{15}\}$ may have MAI with even-indexed codewords, this occurs sparsely. For example, codeword $\mathbf{w}_0$ has strong MAI with two codewords $\mathbf{w}_1$ and $\mathbf{w}_{15}$, codeword $\mathbf{w}_2$ has strong MAI with two codewords $\mathbf{w}_1$ and $\mathbf{w}_3$, etc.

TABLE II

$MAI_{i \leftarrow j}$ POWER $(dB)$ AS A FUNCTION OF EXPONENTIAL CODEWORDS.

| | $\mathbf{w}_0$ | $\mathbf{w}_2$ | $\mathbf{w}_4$ | $\mathbf{w}_6$ | $\mathbf{w}_8$ | $\mathbf{w}_{10}$ | $\mathbf{w}_{12}$ | $\mathbf{w}_{14}$ |
|---|---|---|---|---|---|---|---|---|
| $\mathbf{w}_0$ | × | -317 | -306 | -303 | -309 | -301 | -294 | -295 |
| $\mathbf{w}_2$ | -317 | × | -317 | -302 | -308 | -309 | -297 | -291 |
| $\mathbf{w}_4$ | -306 | -317 | × | -306 | -306 | -302 | -298 | -297 |
| $\mathbf{w}_6$ | -303 | -302 | -306 | × | -311 | -302 | -298 | -297 |
| $\mathbf{w}_8$ | -310 | -308 | -305 | -311 | × | -305 | -298 | -299 |
| $\mathbf{w}_{10}$ | -300 | -308 | -303 | -302 | -304 | × | -296 | -293 |
| $\mathbf{w}_{12}$ | -295 | -297 | -298 | -303 | -298 | -297 | × | -302 |
| $\mathbf{w}_{14}$ | -296 | -291 | -297 | -301 | -299 | -293 | -302 | × |
| $\mathbf{w}_1$ | -18.6 | -19.0 | -309 | -302 | -311 | -304 | -296 | -292 |
| $\mathbf{w}_3$ | -306 | -18.4 | -12.9 | -306 | -306 | -301 | -300 | -293 |
| $\mathbf{w}_5$ | -305 | -309 | -16.2 | -19.2 | -304 | -304 | -297 | -304 |
| $\mathbf{w}_7$ | -309 | -299 | -301 | -19.6 | -16.3 | -299 | -298 | -308 |
| $\mathbf{w}_9$ | -304 | -304 | -306 | -302 | -16.6 | -15.3 | -296 | -292 |
| $\mathbf{w}_{11}$ | -293 | -304 | -304 | -297 | -302 | -18.2 | -11.1 | -296 |
| $\mathbf{w}_{13}$ | -297 | -303 | -306 | -305 | -307 | -300 | -19.9 | -19.0 |
| $\mathbf{w}_{15}$ | -15.3 | -297 | -298 | -303 | -307 | -302 | -300 | -17.6 |

**Example 3.** In this example, we evaluate the system performance when the number of users goes beyond that maximum number of MAI-free codewords as specified in Theorems 1 and 2. The total MAI power for user $i$ from all other users, denoted by $\overline{MAI}_i$, is calculated by $\frac{1}{\sum_{k=0}^{N-1}|\lambda_i[k]|^2}\left|\sum_{j=0,j\neq i}^{T-1} MAI_{i\leftarrow j}\right|^2$. The average MAI power of the system is the averaged value of $T$ MAI values, $\frac{1}{T}\sum_{i=0}^{T-1}\overline{MAI}_i$, where $T$ is the number of active users in the system.

The code priority for Hadamard-Walsh codes is given below. First, we choose the codeword set $A = \{\mathbf{w}_0, \mathbf{w}_1, \mathbf{w}_2, \mathbf{w}_4\}$, which have zero MAI as shown in Example 1. Then, we add more codewords as the number of users increases. First we add the remaining code in $G_{00}$ *i.e.* $\mathbf{w}_3$. Then, we see from Table I that we should choose from $G_{01} = \{\mathbf{w}_4, \mathbf{w}_5, \mathbf{w}_6, \mathbf{w}_7\}$. After including $\mathbf{w}_5$, $\mathbf{w}_6$, $\mathbf{w}_7$ in A, we add codewords $\mathbf{w}_8$, $\mathbf{w}_9$, $\mathbf{w}_{10}$ and $\mathbf{w}_{11}$ to A.

Next, we consider the code priority for exponential codes. First, we choose the set of exponential codes with odd indices, $\{\mathbf{w}_1, \mathbf{w}_3, \mathbf{w}_5, \mathbf{w}_7, \mathbf{w}_9, \mathbf{w}_{11}, \mathbf{w}_{13}, \mathbf{w}_{15}\}$ as the main set since they are mutually MAI-free according to Theorem 2. To increase the user capacity of MC-CDMA system, we add some of the codes with even indices. From Table II, we see that any user with even-indexed codeword has zero mutual MAI from 6 users with odd-indexed codewords. Thus, we add $\mathbf{w}_4, \mathbf{w}_6, \mathbf{w}_8$, and $\mathbf{w}_{10}$.

We plot the average MAI according the code priority described above in Fig 2. When the MC-CDMA has a light load (*i.e.* with less than 5 users), both codes give an excellent MAI free performance. When the number of users is between 5 and 8 The exponential codes clearly outperform the Hadamard-Walsh codes. Finally, when the user number is 9 or above, both codes give similar MAI performance again.

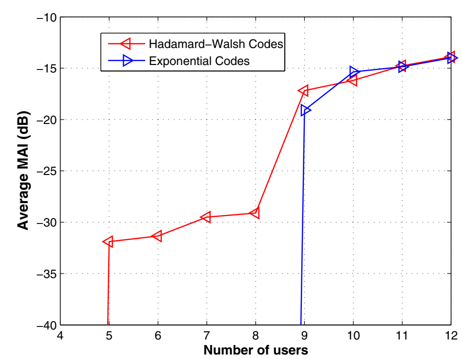


Fig. 2. Average MAI as a function of the number of users with $N = 16$, $L = 2$ and $CFO = \pm 0.1$.

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