Optimal Codeword Design for Precoded UWB (PUWB) Systems

Yu-Hao Chang, Shang-Ho Tsai, Xiaoli Yu and C.-C. Jay Kuo
Department of Electrical Engineering
University of Southern California, Los Angeles, CA 90089-2564, USA
E-mails: {yuhaocha,xiaoliyu}@usc.edu, shanghot@alumni.usc.edu and cckuo@sipi.usc.edu

***Abstract*— A precoding technique applied to symbols transmitted in an ultra-wideband (UWB) system was proposed in [1], which can concentrate the signal power at the receiver to facilitate data detection. The design of the optimal codeword for precoding is investigated in this work. After the problem is formulated, we examine its solution in the full-rank space as well as the reduced-rank subspace to get different tradeoff between detection performance and the amount of feedback messages. Furthermore, we propose two subspace selection schemes that are suitable for different channel conditions. The codeword obtained from this work improves the performance of the result in [1] significantly at the price of a larger amount of feedback messages.**

I. Introduction

There has been growing interest in applying the time-reversal prefiltering (TRP) to the ultra-wideband (UWB) communication system [2], which is also known as pre-RAKE diversity combining. The TRP transmitter prefilters the transmit data with the time-reversed order of the channel impulse response so that the received signal power is well concentrated at the receiver. As compared to the conventional UWB system that employs tens or even hundreds of RAKE fingers at the receiver [3], the number of RAKE fingers required for symbol decoding is greatly reduced in TRP-UWB so that the operating power is saved. This is especially important for mobile devices whose battery power is usually limited.

The TRP-UWB system demands the channel information available to the transmitter. It is however challenging to feedback the entire channel information from the receiver to the transmitter due to the very large number of channel taps in an UWB channel [3]. A novel UWB transceiver system, called the channel-phase-precoded (CPP) UWB, was proposed in [1] to overcome this problem. The CPPUWB transmitter encodes the data symbol with the reversed binary channel phase that takes values of 1 and -1. Consequently, each antipodal phase information of carrierless UWB channel taps is represented by one bit, and the feedback overhead is significantly reduced. The use of the reversed phase codeword leads to concentrated received signal power since all channel taps are coherently combined.

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By exploiting the concentrated power, the CPPUWB system can achieve a higher data rate by shortening the symbol interval of the transmit data while maintaining a tolerable intersymbol interference (ISI) level. Furthermore, it is possible to improve the system performance by selecting the codeword length to maximize the output signal-to-interference ratio (SIR). A fast search algorithm to determine the optimized code length was investigated in [4]. The CPPUWB system with codeword length optimization (CPPUWB/CLO) can reduce the feedback amount furthermore since the optimal code length is usually less than the channel response length. The CPPUWB system is suitable for the case where the feedback channel capacity is parsimonious. In this work, we consider a more generic precoding technique for UWB. That is, the binary codeword to represent the channel phase appears to be restrictive. Instead, we may consider a codeword whose elements can be real numbers in theory (which will be represented by m-bit data in practice), and call the resultant system the precoded ultra-wideband (PUWB) system. The purpose of using multiple bits for each codeword element (rather than 1 bit) is to achieve a higher data detection rate.

After the problem formulation, we first show the construction of the optimal codeword using the channel information to minimize the mean square errors (MSE) at the decoder in a PUWB system. The information is then fed back to the transmitter. This full-rank approach is however expensive in both computational complexity and feedback overhead. To save these costs, the subspace approach (or the reduced-rank algorithm [5]) is explored as an alternative. That is, the signal is projected onto a subspace to reduce its dimension and the optimal codeword is selected from this subspace. The complexity and the feedback overhead can be significantly reduced via the use of the subspace approach. In general, the subspace technique demands the basis information available at the transmitter for codeword reconstruction. However, when the specific basis is used, we show that the receiver only needs to send the channel phase information back to the transmitter for codeword reconstruction.

The rest of this work is organized as follows. The system model is presented in Sec. II. The full-rank and the reduced-rank vector codeword design problems are examined in Sec. III and Sec. IV, respectively. The issue of basis selection is discussed in Sec. V. Simulation results are shown in Sec. VI.

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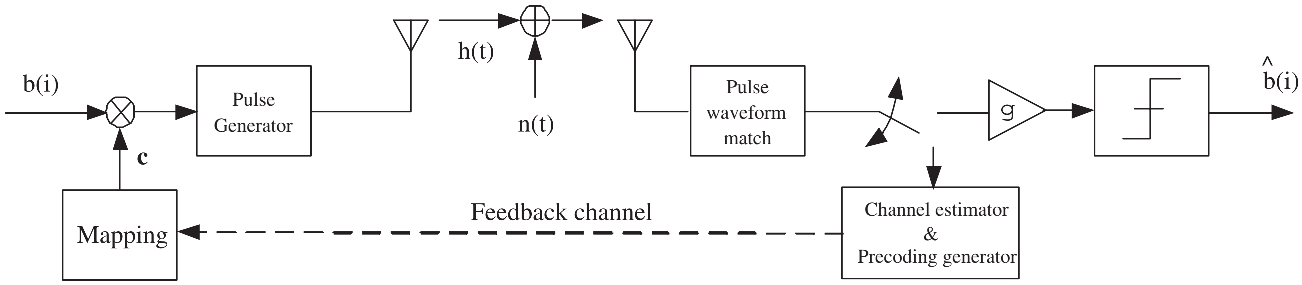

Fig. 1. The block diagram of the proposed PUWB system.

II. System Model

The carrierless tap-delay-line (TDL) channel model given in [6] is adopted here. It can be written as

$$h(t) = \sum_{i=0}^{L-1} h_i \delta(t - i\Delta) = \sum_{i=0}^{L-1} p_i \alpha_i \delta(t - i\Delta), \qquad (1)$$

where $h_i = p_i \alpha_i$, L is the total number of the signal path, $\delta(x)$ is the Dirac delta function of x, Δ is the multipath resolution which is set as the time domain pulse width, $p_i \in \{+1, -1\}$ with equal probability is the ith phase component, and the corresponding amplitude component α_i is modeled as a Rayleigh random variable whose probability density function (PDF) is

$$f_{\alpha_i}(x) = \frac{x}{\sigma_i^2} e^{-x^2/2\sigma_i^2}. \qquad (2)$$

Furthermore, the power of each tap decreases exponentially with respect to its index, *i.e.,*

$$E\{\alpha_i^2\} = 2\sigma_i^2 = \Omega \gamma^i, \qquad (3)$$

where $E\{x\}$ is the expectation of random variable x, Ω is the power of the first tap and $\gamma = e^{-\Delta/\Gamma}$. Four different decay time constants Γ corresponding to four different channel models (CM 1∼CM 4) in [7] were given in [6]. In the current context, the coherent time of the channel is assumed to be long enough so that the channel is invariant during the transmission of one package of data symbols.

The block diagram of the PUWB system is shown in Fig. 1. The ith antipodal data symbol $b(i)$ with power P (*i.e.,* $E\{b(i)^2\} = P$) is encoded by codeword $\mathbf{c} = [c_0, \cdots, c_{L-1}]^T$ and then modulated by pulse waveform $w_s(t)$. Consequently, the transmit signal is of the following form

$$x_s(t) = \sum_{i=-\infty}^{\infty} b(i) \sum_{j=0}^{L-1} c_j w_s(t - j\Delta - iT_s), \qquad (4)$$

where $T_s = M\Delta$ is the symbol interval, which is assumed to be an integer multiple of the pulse width. Note that the main difference between CPPUWB in [1] and the current framework is the contraint on codeword $\mathbf{c} = [c_0, \cdots, c_{L-1}]^T$. Each element of $\mathbf{c}$ takes the value of 1 or -1 in CPPUWB but any real number in PUWB.

At the receiver, the signal is distorted by the multipath channel model and contaminated by the additive white Gaussian noise (AWGN). The receiver digitalizes the received signal by performing the pulse waveform matching and chip-rate sampling and then amplifies the resultant digital signal by a factor of g. The amplifier is used to strike a balance between the channel gain and noise power suppression [8]. As a result, the matrix representation of the discrete received signal can be expressed as

$$\mathbf{r}(i) = g\mathbf{H}\mathbf{c}b(i) + g\mathbf{I}(i) + g\mathbf{n}(i), \qquad (5)$$

where $\mathbf{H}$ is the $(2L-1) \times L$ Toeplitz matrix whose first column contains $\mathbf{h} = [h_0, \cdots, h_{L-1}]^T$ as the first L elements and zero elswhere, $\mathbf{r}(i) = [r_0(i), \cdots, r_{2L-2}(i)]^T$, $\mathbf{I}(i) = [I_0(i), \cdots, I_{2L-2}(i)]^T$ is the interference vector for transmit symbol $b(i)$, and $\mathbf{n}(i) = [n_0(i), \cdots, n_{2L-2}(i)]^T$ is the AWGN vector for the same transmit symbol, and each element of $\mathbf{n}(i)$ has zero mean and covariance $N_0/2$. To decode the ith transmit symbol, the receiver directly applies the decision threshold to $r_{L-1}(i)$, namely,

$$\hat{b}(i) = \mathrm{sign}\{r_{L-1}(i)\}. \qquad (6)$$

It is worthwhile to point out that, when $T_s \geq L\Delta$, the received signal used to decode $r_{L-1}(i)$ contains no ISI. On the other hand, if $T_s < L\Delta$, $r_{L-1}(i)$ contains signals from $b(i)$ as well as $b(i - L_1), \cdots, b(i-1)$ and $b(i+1), \cdots, b(i + L_1)$ where $L_1 = \lfloor (L-1)/M \rfloor$ and $\lfloor x \rfloor$ is the floor function of x. The system performance degrades due to the presence of ISI at $r_{L-1}(i)$.

III. Optimal Codeword Design

To improve the system performance, we consider the optimal codeword design problem in this section. That is, we seek codeword $\mathbf{c}_{opt}$ such that the mean square error (MSE) at the receiver output is minimized. If the channel impulse response is known to the receiver, the receiver can compute the optimal codeword and then send it back to the transmitter. Since only one received sample, *i.e.,* $r_{L-1}(i)$, is used to decode the ith transmit data, the receiver can reduce its sampling rate by taking one sample in every T_s interval in a synchronized environment. Then, we can further simplify (5) to be

$$r_{L-1}(i) = g\mathbf{c}^T \bar{\mathbf{H}} \mathbf{b}(i) + g n_{L-1}(i), \qquad (7)$$

where $\mathbf{b}(i) = [b(i + L_1), \cdots, b(i), \cdots, b(i - L_1)]^T$ and $\bar{\mathbf{H}}$ is an $L \times (2L_1 + 1)$ matrix formed by transposing matrix $\mathbf{H}$ and keeping the $(L + kM)$th column, where $-L_1 \leq k \leq L_1$, in $\mathbf{H}^T$, and removing all other irrelevant columns. Thus, being

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conditioned on one channel realization $\mathbf{h}$, the mean square error (MSE) between $b(i)$ and $r_{L-1}(i)$ can be written as

$$\begin{aligned}\varepsilon(\mathbf{c}, g) &= E\left\{|b(i) - r_{L-1}(i)|^2 \middle| \mathbf{h}\right\} \\ &= E\left\{|b(i) - g\mathbf{c}^T\bar{\mathbf{H}}\mathbf{b}(i) - gn_{L-1}(i)|^2 \middle| \mathbf{h}\right\}. \quad (8)\end{aligned}$$

The optimal codeword, $\mathbf{c}_{opt}$, and the amplifier gain, g_{opt}, are chosen to minimize the value of ε subject to the unit-power constraint on $\mathbf{c}$, *i.e.,*

$$\mathbf{c}_{opt}, g_{opt} = \arg\min_{\mathbf{c},g} \varepsilon(\mathbf{c}, g) \quad \text{s.t.} \quad \mathbf{c}^T\mathbf{c} = 1. \quad (9)$$

Eq. (9) can be solved by the method of Lagrange multipliers [8]. After some manipulation, we get

$$\mathbf{c}_{opt} = \frac{1}{g_{opt}}\left(\bar{\mathbf{H}}\bar{\mathbf{H}}^T + \frac{N_0}{2P}\mathbf{I}_L\right)^{-1}\bar{\mathbf{h}} \quad (10)$$

and

$$g_{opt} = \sqrt{\bar{\mathbf{h}}^T\left(\bar{\mathbf{H}}\bar{\mathbf{H}}^T + \frac{N_0}{2P}\mathbf{I}_L\right)^{-2}\bar{\mathbf{h}}}, \quad (11)$$

where $\bar{\mathbf{h}} = [h_{L-1}, \cdots, h_0]^T$ is equal to the reversed order of channel vector $\mathbf{h}$ and $\mathbf{I}_L$ is the identity matrix of size $L \times L$. The corresponding minimized MSE (MMSE) ε_{min} achieved is

$$\varepsilon_{min} = \varepsilon(\mathbf{c}_{opt}, g_{opt}) = P\left(1 - \bar{\mathbf{h}}^T\left(\bar{\mathbf{H}}\bar{\mathbf{H}}^T + \frac{N_0}{2P}\mathbf{I}_L\right)^{-1}\bar{\mathbf{h}}\right).$$

Although the above problem formulation is similar to that in [8], our objective is different. The optimal prefilter was designed to suppress all off-peak signals in [8]. However, it may not minimize the output MSE in our case. In contrast, our optimal codeword is designed to minimize the output MSE.

The bit error rate improves a lot over that of the CPPUWB system by the use of the optimal codeword in the PUWB system. This is confirmed in Sec. VI. However, the complexity required to compute the optimal codeword as given in (10) involves the inversion of an $L \times L$ matrix. Furthermore, since it takes more bits to represent $\mathbf{c}_{opt}$ in PUWB than that for the binary channel phase codeword in CPPUWB, the size of the feedback message increases, too.

IV. Subspace-based Codeword Design

We consider the optimal codeword design problem in a subspace using the reduced-rank algorithm [5] in this section. The codeword design problem can be projected onto a lower dimensional subspace. That is, we choose an $L \times d$ matrix,

$$\mathbf{M}_d = [\mathbf{e}_0, \cdots, \mathbf{e}_{d-1}],$$

whose columns are orthonormal to each other. Then, the optimal codeword in the d-dimensional subspace spanned by $\mathbf{M}_d$ can be found by

$$\mathbf{c}_d = \frac{1}{g_d}\left(\bar{\mathbf{H}}_d\bar{\mathbf{H}}_d^T + \frac{N_0}{2P}\mathbf{I}_d\right)^{-1}\bar{\mathbf{h}}_d, \quad (12)$$

where

$$g_d = \sqrt{\bar{\mathbf{h}}_d^T\left(\bar{\mathbf{H}}_d\bar{\mathbf{H}}_d^T + \frac{N_0}{2P}\mathbf{I}_d\right)^{-2}\bar{\mathbf{h}}_d}, \quad (13)$$

and $\bar{\mathbf{h}}_d = \mathbf{M}_d^T\bar{\mathbf{h}}$ and $\bar{\mathbf{H}}_d = \mathbf{M}_d^T\bar{\mathbf{H}}$ are projections of $\bar{\mathbf{h}}$ and $\bar{\mathbf{H}}$ onto $\mathbf{M}_d$, respectively. Since the size of the square matrix in (12) to be inverted is only $d \times d$, its computational cost is lower.

Once codeword $\mathbf{c}_d$ is found, the receiver can send it back to the transmitter with an overhead lower than sending $\mathbf{c}_{opt}$. After receiving $\mathbf{c}_d$, the transmitter can synthesize the $L \times 1$ codeword $\tilde{\mathbf{c}}_d$ by

$$\tilde{\mathbf{c}}_d = \mathbf{M}_d\mathbf{c}_d. \quad (14)$$

Although $\mathbf{c}_d$ minimizes the output MSE in the subspace spanned by $\mathbf{M}_d$, codeword $\tilde{\mathbf{c}}_d$ in (14) is no longer optimal with respect to the original full-rank space since some useful signal power is dropped by subspace filtering. Furthermore, both the transmitter and the receiver need the information of $\mathbf{M}_d$. It is however costly to send the basis information from the receiver to the transmitter.

To address this problem, we use a specific basis that can be computed easily with very little feedback information. Let us first rewrite $\bar{\mathbf{h}}$ as

$$\bar{\mathbf{h}} = \bar{\mathbf{P}}\bar{\mathbf{a}}, \quad (15)$$

where $\bar{\mathbf{P}} = \text{diag}[p_{L-1}, \cdots, p_0]$ is a diagonal matrix whose ith diagonal component is the phase of the $(L-i)$th path takes values of $+1$ or -1 and $\bar{\mathbf{a}} = [\alpha_{L-1}, \cdots, \alpha_0]^T$ is the reversed order of the amplitude vector. Let $\mathbf{R}_{\bar{\mathbf{a}}}$ denote the autocorrelation of $\bar{\mathbf{a}}$, *i.e.,*

$$\begin{aligned}\mathbf{R}_{\bar{\mathbf{a}}} = E\{\bar{\mathbf{a}}\bar{\mathbf{a}}^T\} &= \Omega\begin{bmatrix}\beta^{2(L-1)} & \alpha\beta^{2L-3} & \cdots & \alpha\beta^{L-1} \\ \alpha\beta^{2L-3} & \beta^{2(L-2)} & \cdots & \alpha\beta^{L-2} \\ \vdots & \vdots & \ddots & \vdots \\ \alpha\beta^{L-1} & \alpha\beta^{L-2} & \cdots & 1\end{bmatrix} \\ &= \mathbf{E}_{\bar{\mathbf{a}}}\mathbf{\Lambda}_{\bar{\mathbf{a}}}\mathbf{E}_{\bar{\mathbf{a}}}^T, \quad (16)\end{aligned}$$

where $\alpha = \pi/4$, $\beta = \gamma^{1/2} = e^{-\Delta/2\Gamma}$,

$$\mathbf{\Lambda}_{\bar{\mathbf{a}}} = \text{diag}[\lambda_0, \ldots, \lambda_{L-1}]$$

is the diagonal matrix of L eigenvalues with $\lambda_0 \geq \cdots \geq \lambda_{L-1}$, and $\mathbf{E}_{\bar{\mathbf{a}}} = [\tilde{\mathbf{e}}_0, \cdots, \tilde{\mathbf{e}}_{L-1}]$ whose columns are the corresponding eigenvectors. The above equation gives the eigen-decomposition of $\mathbf{R}_{\bar{\mathbf{a}}}$. The specific basis set $\mathbf{M}$ can be expressed as

$$\mathbf{M} = [\mathbf{e}_0, \cdots, \mathbf{e}_{L-1}] = [\bar{\mathbf{P}}\tilde{\mathbf{e}}_0, \cdots, \bar{\mathbf{P}}\tilde{\mathbf{e}}_{L-1}], \quad (17)$$

where $\mathbf{e}_m = \bar{\mathbf{P}}\tilde{\mathbf{e}}_m$, $0 \leq m \leq L-1$. $\mathbf{M}_d$ collects d out of L column vectors from $\mathbf{M}$, *i.e.,*

$$\mathbf{M}_d = [\mathbf{e}_{i_0}, \cdots, \mathbf{e}_{i_{d-1}}],$$

where $i_0 < \cdots < i_{d-1}$.

We proceed with the following three steps to construct $\mathbf{M}_d$.

1) The L-bit channel phase information is fed back from the receiver to the transmitter to get diagonal matrix $\bar{\mathbf{P}}$ as shown in (15).
2) Δ is specified as the system parameter and Γ can be determined owing to an excellent ranging capability of the UWB signal [9]. Thus, the transmitter can synthesize correlation matrix $\mathbf{R}_{\bar{\mathbf{a}}}$ and decompose it for $\tilde{\mathbf{e}}_i$, $0 \leq i \leq L-1$.

3) Indices of d selected columns should be sent from the receiver to the transmitter.

The selection of d from L basis vectors will be detailed in the next section.

V. Subspace Selection Schemes

The choice of d basis vectors, $\mathbf{e}_{i_0}, \cdots, \mathbf{e}_{i_{d-1}}$, for subspace-based codeword design plays an important role in the ultimate system performance. In this section, we propose two subspace selection schemes.

A. Scheme A: Sequential Greedy Search Algorithm

Generally speaking, the optimal signal subspace that minimizes the output MSE demands an exhaustive search among all possible subspaces whose cardinality is $\begin{pmatrix} L \\ d \end{pmatrix}$. Since L is usually a large number, the associated computational cost of exhaustive search is too high to be attractive. A suboptimal basis selection scheme with less complexity is more desired. A greedy search algorithm is proposed below.

To find the first basis vector, we search all possible basis vectors and pick up the one that provides the minimum MSE, *i.e.,*

$$i_0 = \arg\min_{i \in \mathcal{I}_0} P\left(1 - a^{(i)}\left(||\mathbf{f}^{(i)}||^2 + \frac{N_0}{2P}\right)^{-1} a^{(i)}\right), \quad (18)$$

where $\mathcal{I}_0 = \{0, \cdots, L-1\}$, $a^{(i)} = \mathbf{e}_i^T \bar{\mathbf{h}}$, $\mathbf{f}^{(i)} = \bar{\mathbf{H}}^T \mathbf{e}_i$ and $||\mathbf{x}||$ is the 2-norm of vector $\mathbf{x}$. Next, if m ($m < d$) basis vectors are already selected and $\mathbf{M}_m = [\mathbf{e}_{i_0}, \cdots, \mathbf{e}_{i_{m-1}}]$. Let $\mathbf{e}_{i_m}$ be the next basis vector to add so that the corresponding basis set is $\mathbf{M}_{m+1} = [\mathbf{M}_m, \mathbf{e}_{i_m}]$. The MSE in the $(m+1)$-dimensional subspace is shown in (19).

Note that (19) can be further simplified by applying the block matrix inversion formula [10] as

$$\varepsilon_{min}^{(m+1)} = \varepsilon_{min}^{(m)} - PC_{i_m}\zeta_{i_m}, \quad (20)$$

where

$$C_{i_m} = \Bigg(||\mathbf{f}^{(i_m)}||^2 + \frac{N_0}{2P} - \mathbf{f}^{(i_m),T}\bar{\mathbf{H}}_m^T\left(\bar{\mathbf{H}}_m\bar{\mathbf{H}}_m^T + \frac{N_0}{2P}\mathbf{I}_m\right)^{-1}\bar{\mathbf{H}}_m\mathbf{f}^{(i_m)}\Bigg)^{-1},$$

and

$$\zeta_{i_m} = \left(\mathbf{f}^{(i_m),T}\bar{\mathbf{H}}_m^T\left(\bar{\mathbf{H}}_m\bar{\mathbf{H}}_m^T + (N_0/2P)\mathbf{I}_m\right)^{-1}\bar{\mathbf{h}}_m - a^{(i_m)}\right)^2.$$

It can be easily shown that both C_{i_m} and ζ_{i_m} are non-negative, and $PC_{i_m}\zeta_{i_m}$ can be viewed as the gain coming from adding one more basis vector $\mathbf{e}_{i_m}$. Hence, to minimize $\varepsilon_{min}^{(m+1)}$, the $(m+1)$th basis vector can be chosen from the remaining index set such that the product of C_{i_m} and ζ_{i_m} is maximized, *i.e.,*

$$i_m = \arg\max_{i \in \mathcal{I}_m} C_i\zeta_i, \quad (21)$$

where $\mathcal{I}_m \in \{0, \cdots, L-1\}/\{i_0, \cdots, i_{m-1}\}$ is the remaining index set. By following the above procedure, we can get one more basis vector each time until the complete set of d basis vectors is selected.

B. Scheme B: Subspace Formed by Leading Columns

For a very low SNR channel, we have the following special case.

Proposition 1: When the channel SNR is asymptotically low, we can minimize the average MSE by taking d column vectors out of $\mathbf{M}$ as $i_m = m$, $0 \le m \le (d-1)$. In words, $\mathbf{M}_d$ is composed of the first d column vectors in $\mathbf{M}$.

Proof: As the channel SNR goes low asymptotically, the optimal codeword in (12) converges to the normalized version of $\bar{\mathbf{h}}_d$ so that minimized MSE becomes

$$\varepsilon_{min}^{(d)} \simeq P\left(1 - \frac{2P}{N_0}\bar{\mathbf{h}}_d^T\bar{\mathbf{h}}_d\right). \quad (22)$$

By averaging $\varepsilon_{min}^{(d)}$ over all possible channel realizations, we get the averaged MSE as

$$\begin{aligned}\bar{\varepsilon}_{min}^{(d)} &= E_{\mathbf{h}}\left\{\varepsilon_{min}^{(d)}\right\} \simeq P\left(1 - \frac{2P}{N_0}E_{\mathbf{h}}\left\{\bar{\mathbf{h}}_d^T\bar{\mathbf{h}}_d\right\}\right) \\ &= P\left(1 - \frac{2P}{N_0}tr\left\{E_{\mathbf{h}}\left\{\bar{\mathbf{h}}_d\bar{\mathbf{h}}_d^T\right\}\right\}\right), \quad (23)\end{aligned}$$

where $tr(\mathbf{A})$ is the trace operator for matrix $\mathbf{A}$. By substituting $\bar{\mathbf{h}}_d = \mathbf{M}_d^T\bar{\mathbf{h}}$ into (23) and after some manipulations, we can get

$$\begin{aligned}\bar{\varepsilon}_{min}^{(d)} &\simeq P\left(1 - \frac{2P}{N_0}tr\left\{\tilde{\mathbf{M}}_d^T\mathbf{R}_{\bar{\mathbf{a}}}\tilde{\mathbf{M}}_d\right\}\right) \\ &= P\left(1 - \frac{2P}{N_0}\sum_{m=0}^{d-1}\lambda_{i_m}\right)\end{aligned}$$

where $\tilde{\mathbf{M}}_d = [\tilde{\mathbf{e}}_{i_0}, \cdots, \tilde{\mathbf{e}}_{i_{d-1}}]$. Since $\lambda_0 \ge \cdots \ge \lambda_{L-1}$, we can select $i_m = m$ to minimize the averaged MSE, $\bar{\varepsilon}_{min}^{(d)}$. ■

By Proposition 1, we should compute the suboptimal codeword in the subspace spanned by the first d specific bases when the SNR is asymptotically low. It is interesting to consider the system performance when another basis set is applied, say, the standard basis. This is stated in the following proposition.

Proposition 2: When the standard basis is applied to the subspace filtering in (12) for a channel with asymptotically low SNR, the output MSE is no less than

$$P\left(1 - \frac{2P}{N_0}\sum_{m=L-d}^{L-1}\bar{\lambda}_m\right),$$

where $\bar{\lambda}_m$ is the average power of the mth element in $\bar{\mathbf{h}}$, *i.e.,* $\bar{\lambda}_m = E\{|h_{L-m}|^2\}$.

Proof: The proof is similar to that in Proposition 1 and, thus, omitted here. ■

Proposition 3: If $d < L$, the use of the specific basis for subspace filtering specified in Proposition 1 achieves lower MSE than the standard basis given in Proposition 2.

Proof: Using Proposition 3 in [11], we have

$$\sum_{m=0}^{d-1}\lambda_m > \sum_{m=L-d+1}^{L}\bar{\lambda}_m.$$

Therefore, the claim is true. ■

$$\varepsilon_{min}^{(m+1)} = P\left(1-\left(\mathbf{M}_{m+1}^T\bar{\mathbf{h}}\right)^T\left(\left(\mathbf{M}_{m+1}^T\bar{\mathbf{H}}\right)\left(\mathbf{M}_{m+1}^T\bar{\mathbf{H}}\right)^T+\frac{N_0}{2P}\mathbf{I}_{m+1}\right)^{-1}\left(\mathbf{M}_{m+1}^T\bar{\mathbf{h}}\right)\right)$$
$$= P\left(1-\left[\begin{array}{c}\bar{\mathbf{h}}_m\\ a^{(i_m)}\end{array}\right]^T\left[\begin{array}{c|c}\bar{\mathbf{H}}_m\bar{\mathbf{H}}_m^T+\frac{N_0}{2P}\mathbf{I}_m & \bar{\mathbf{H}}_m\mathbf{f}^{(i_m)}\\ \hline \mathbf{f}^{(i_m),T}\bar{\mathbf{H}}_m^T & \|\mathbf{f}^{(i_m)}\|^2+\frac{N_0}{2P}\end{array}\right]^{-1}\left[\begin{array}{c}\bar{\mathbf{h}}_m\\ a^{(i_m)}\end{array}\right]\right) \quad (19)$$

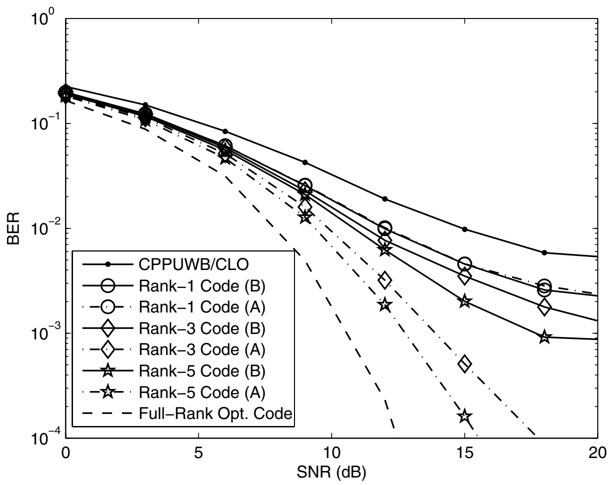

Fig. 2. The BER as a function of the SNR value for different codewords under different subspace ranks (1, 3 and 5) and selection schemes (A and B).

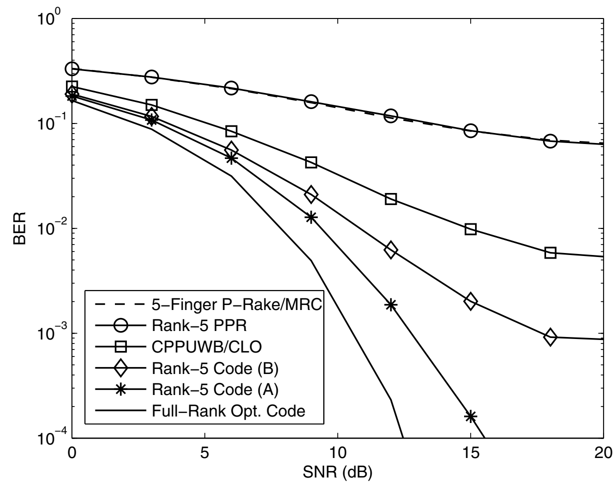

Fig. 3. The BER as a function of the SNR value for the conventional Rake receiver and various precoding schemes.

VI. Simulation Results

We test the performance of the subspace-based codeword with different dimensions of the subspace and various input SNR. The system parameters are chosen as follows: $\Delta = 1$ ns, $\Gamma = 20.5$ ns (CM3), $L = 120$, $Ts = 10$ ns. The reported results are the average over 1000 independent channel realizations. In the first simulation, we evaluate the system performance in terms of the bit error rate (BER) and consider subspaces of rank 1, 3 and 5. The performance curve of the CPPUWB/CLO is also shown to demonstrate the performance improvement by the use of the codeword in Fig. 2. The BER performance of CPPUWB/CLO is the worst. The performance gap between different ranks and different subspace selection schemes becomes wider as SNR becomes larger. Furthermore, the two curves corresponding to the two rank-1 codewords almost overlap with each other. In other words, we can always select the first column $\mathbf{e}_0$ as the first basis vector without search to save the computational cost.

Next, the proposed precoding schemes are compared with the partial Rake (P-Rake) receiver [12] using maximal ratio combining (MRC) and the partial pre-Rake (PPR) in [13] for the BER performance in Fig. 3. Again, the performance curves of CPPUWB/CLO and the full-rank code are plotted as benchmarks. It is observed that the performance gap between 5-finger P-Rake/MRC and Rank-5 PPR is almost indistinguishable. By utilizing the second order channel statistics, the proposed reduced-rank precoding scheme outperforms PPR greatly at the cost of slightly increased feedback burden and more computational complexity for codeword construction.

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