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# Transmit Antenna Selection with Linear Precoding in MIMO Multiuser Systems

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***Abstract*—In this paper, we propose a low-complexity transmit antenna selection algorithm for MIMO multiuser channels. This algorithm greedily finds the optimal transmit antenna subset in base station. When the transmit power for individual data streams is equal, the proposed algorithm can be applied to the zero-forcing and the MMSE precoding schemes to further simplify the computational complexity. Simulation results show that the proposed low-complexity suboptimal antenna selections can achieve very close performance with the optimal antenna selection scheme in [6]. As a result, a good trade-off between complexity and performance is attained by the proposed antenna selection schemes.**

## I. Introduction

Multiple-input multiple-output (MIMO) techniques are widely employed in transceiver to achieve high rate transmission in wireless communication systems. In the downlink broadcast channels (BCs), the base station transmits signals to several mobile stations in the same frequency band simultaneously; the desired signals for one user, however, leads to interference to other users and such interference degrades system performance seriously. Hence, this research aims to mitigate interference and maximize the transmission capacity for MIMO-BCs recently, *e.g.* see [3]-[6].

Dirty paper coding (DPC) is shown to achieve the sum capacity of MIMO-BCs in [1]. However DPC may be somewhat difficult to implement in practical systems due to the highly complicated coding procedure. To overcome the high complexity problem, several practical near-DPC techniques based on the concept of precoding are proposed for reasonable tradeoffs between complexity and performance. An alternative suboptimal linear precoding strategy is beamforming (BF) [4], [5] which can improve the quality of service (QoS) of MIMO-BC channels with reduced complexity when compared to DPC. In [5], the authors design beamforming vectors from an orthogonal codebook to maximize the received signal to interference plus noise ratio (SINR) for each data stream when the number of receive antennas is greater than the number of transmit antennas. Such beamforming scheme is also showed to be asymptotically optimal when the number of users goes to infinity. However, in environments with finite users, orthogonal beamforming cannot eliminate the interference completely; the residual interference hence limits the capacity. This problem cannot be solved even if we increase the transmit power. Several solutions are proposed for this issue. For instance, zero-forcing (ZF) precoding in [4] chooses the precoding vectors to avoid interference among different user streams. Linear minimum mean-square error (MMSE) precoding scheme in [2] tries to find a good tradeoff between noise and interference.

In addition to the above precoding methods, selection diversity is another popular solution to increase channel gain and decrease co-channel interference in MIMO channels. For instance, multiuser diversity can be obtained in multiuser environments, as introduced in [3]. Hence, when the number of users is larger than the number of transmit antennas, user selection is needed in this case. Multiuser diversity can be obtained by choosing suitable users with favorable channel conditions to improve system throughput *e.g.* see [4], [5]. Another example is transmit antenna selection in multiuser environments. That is, we can design a base station whose number of transmit antennas is larger than its number of RF components; an appropriate antenna set can then be selected to achieve selection diversity, *e.g.* see [6]. The proposal in [6] can attain selection diversity as well as reduce implementational cost since the RF components are actually much more expensive than antennas. However, to achieve the highest sum throughput in [6], exhaustive search is needed and the corresponding computational complexity grows exponentially with the numbers of antennas or users. As antenna or user number grows, the complexity soon becomes prohibiting. This motivates us to explore some other suboptimal solutions with low complexity to conduct transmit antenna selection in MIMO multiuser channels.

In this paper, we propose a low-complexity antenna selection algorithm that greedily finds the optimal transmit antenna subset in base station. When the transmit power for individual data streams is equal, the proposed algorithm can be applied to the zero-forcing and the MMSE precoding schemes to further simplify the computational complexity. Simulation results show that the proposed low-complexity suboptimal antenna selections can achieve very close performance with the optimal exhaustive search in [6]. As a result, a good trade-off between complexity and performance is attained by the proposed antenna selection schemes.

## II. System Model and Precoding strategies

A MIMO-BC with single base station which can support $K$ users is depicted in Fig. 1. The base station is equipped with $M_T$ antennas, $M_S$ RF chains, where $M_T \geq M_S \geq K$; that is, $M_S$ branches are selected from $M_T$ antennas to transmit data for $K$ users. The $k$-th user has $M_{R,k}$ receive antennas,

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$k = 1, 2, \cdots, K$. Let $\mathbf{x}_k$ be the $M_{R,k} \times 1$ transmitted symbol vector to $k$-th user with $\mathbb{E}\{\mathbf{x}\mathbf{x}^H\} = \mathbf{I}_{M_{R,k} \times M_{R,k}}$, and first multiply $\mathbf{P}_k$, which represents a $M_{R,k} \times M_{R,k}$ power allocated matrix with diagonal terms $\sqrt{P_{k,1}}, \cdots, \sqrt{P_{k,M_{R,k}}}$. Then, pass to $M_S \times M_{R,k}$ precoding matrix $\mathbf{W}_k$ and send to transmit antennas. All channels are assumed to be quasi-static and flat with Rayleigh distribution and $\mathbf{H}_k$ denotes the $M_{R,k} \times M_S$ channel transfer matrix of chosen transmit antenna set from BS to the $k$-th user. The received signal $\mathbf{y}_k$ for the $k$-th user is given by

$$\mathbf{y}_k = \mathbf{H}_k \mathbf{W}_k \mathbf{P}_k \mathbf{x}_k + \mathbf{H}_k \sum_{l=1, l \neq k}^{K} \mathbf{W}_l \mathbf{P}_l \mathbf{x}_l + \mathbf{n}_k, \quad (1)$$

where $\mathbf{n}_k$ is an i.i.d. complex Gaussian noise vector of $k$-th user with zero mean and covariance matrix $\mathbf{R}_{n,k}$.

Throughout this study, complete channel information is assumed to be perfectly known to base station, with a total power constraint $P_c$. Moreover, for simplicity, we consider the case that each mobile user only has one receive antenna; that is, $M_{R,k} = 1, k = 1, \cdots, K$. Thus, the power allocation matrices $\mathbf{P}_k$ reduce to scales $P_k$, the channel matrices $\mathbf{H}_k$ and precoding matrices $\mathbf{W}_k$ reduce to vectors $\tilde{\mathbf{h}}_k$ and $\mathbf{w}_k$, respectively, for $k = 1, 2, \cdots, K$.

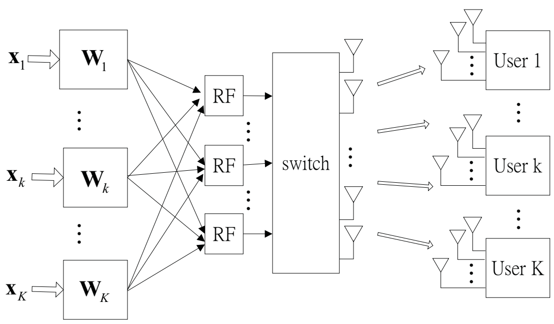


Fig. 1: A multiuser system with transmit antenna selection.

### A. Zero-forcing Precoding

In zero-forcing precoding [4], precoding vectors are determined to cancel the interference arisen from other users, *i.e.*, determine $\mathbf{w}_l$ so that $\tilde{\mathbf{h}}_k \mathbf{w}_l = 0$ for $l \neq k$. Let the selected transmit antenna set $\mathcal{S} \subset \{1, \cdots, M_T\}$ whose elements represent the indices of transmit antennas at the BS, and $|\mathcal{S}| = M_S$. Then $\mathbf{H}_S = [\tilde{\mathbf{h}}_{S,1}^T \ \tilde{\mathbf{h}}_{S,2}^T \ \cdots \tilde{\mathbf{h}}_{S,K}^T]^T$ and $\mathbf{W}_S = [\mathbf{w}_{S,1} \ \mathbf{w}_{S,2} \ \cdots \mathbf{w}_{S,K}]$ denote the channel matrices and precoding matrices of the chosen antenna set, respectively. Then we have the following mathematical expression

$$\mathbf{y} = \mathbf{H}_S \mathbf{W}_S \mathbf{P} \mathbf{x} + \mathbf{n}. \quad (2)$$

To make interference free, $\mathbf{W}_S$ can be chosen as the pseudo-inverse of $\mathbf{H}_S$ so,

$$\mathbf{W}_S = \mathbf{H}_S^{\dagger} = \mathbf{H}_S^H (\mathbf{H}_S \mathbf{H}_S^H)^{-1}, \quad (3)$$

where $\mathbf{P} = \text{diag}\{\sqrt{P_1}, \cdots, \sqrt{P_K}\}$ and $\mathbf{x} = [x_1 \ x_2 \ \cdots \ x_k]^T$. The sum rate using the zero forcing precoding is given by

$$R_{\text{ZF}} = \max_{\sum_{k=1}^{K} ||\mathbf{w}_k|| P_k \leq P_c} \sum_{k=1}^{K} \log(1 + P_k). \quad (4)$$

If the optimal power allocation is applied, the optimal $P_k$ in (4) can be obtained by waterfilling

$$P_k = (\lambda \alpha_k - 1)^+, \quad (5)$$

where $\{x\}^+$ represents $\max\{x, 0\}$, $\alpha_k = \frac{1}{||\mathbf{w}_i||^2}$ is the effective channel gain to the $k$-th user [4], and the level of waterfilling $\lambda$ is chosen to satisfy

$$\sum_{k=1}^{K} (\lambda - \frac{1}{\alpha_k})^+ = P_c. \quad (6)$$

If equal power allocation is used, $P_k = \beta_{\text{ZF}}$ for $k = 1, \cdots, K$, then the power matrix $\mathbf{P} = \sqrt{\beta_{\text{ZF}}} \cdot \mathbf{I}$, and (4) becomes

$$R_{\text{ZF}} = K \log(1 + \beta_{\text{ZF}}). \quad (7)$$

Moreover, with the power constraint $P_c$, the scaling factor $\beta_{\text{ZF}}$ can be obtained as follows:

$$\beta_{\text{ZF}} = \frac{P_c}{\text{tr}(\mathbf{W}_S^H \mathbf{W}_S)}. \quad (8)$$

### B. MMSE precoding

Zero-forcing precoding enables interference-free for all individual users and the system can thus be treated as a point-to-point MISO communication. However, if some of the channels are in bad condition; that is, channel gains $||\mathbf{h}_k||$ are small for some $k$, and the system needs large power to compensate the bad channel condition. Therefore, the scaling factor $\beta_{\text{ZF}}$ is small, and so does the channel capacity $R_{\text{ZF}}$ in this case.

Linear MMSE scheme can reach a good tradeoff between noise and interference and is suitable to be used to overcome this problem. Similar to the well-known MMSE receiver, we can apply the MMSE procedure at the transmit side [2]. In addition to the channel state information, the BS is assumed to be known as, *i.e.* the covariance matrix $\mathbf{R}_n = \mathbb{E}\{\mathbf{n}\mathbf{n}^H\}$. The MMSE precoder is then given as follows:

$$\mathbf{W}_S = (\mathbf{H}_S^H \mathbf{H}_S + \frac{\text{tr}(\mathbf{R}_n)}{P_c} \cdot \mathbf{I})^{-1} \mathbf{H}_S^H. \quad (9)$$

In BC channels, it may be difficult for each user to obtain the channel state information from other users. Again, with zero-forcing precoding, each user can be treated as independent point-to-point communication so that waterfilling power allocation can be applied without jointly decoding in receive terminals. However, with MMSE precoding, jointly receive decoding design may not be practical because all user signals are jointly coupled and interfered with each others. The waterfilling power allocation is complicated to be applied for MMSE precoding. Hence, equal power is allocated to all users in MMSE precoding, *i.e.* $\mathbf{P} = \sqrt{\beta_{\text{MMSE}}} \cdot \mathbf{I}$, where $\beta_{MMSE}$ is a scaling factor determined by power constraint $P_c$ given by

$$\beta_{\text{MMSE}} = \frac{P_c}{\text{tr}((\mathbf{T}_S^{-1} \mathbf{H}_S^H)(\mathbf{T}_S^{-1} \mathbf{H}_S^H)^H)}, \quad (10)$$

where $\mathbf{T}_S$ is defined by

$$\mathbf{T}_S = \mathbf{H}_S^H \mathbf{H}_S + \frac{\text{tr}(\mathbf{R}_n)}{P_c} \cdot \mathbf{I}. \quad (11)$$

Thus, the channel capacity using the MMSE precoding with equal power allocation can be expressed by (see [5])

$$R_{\text{MMSE}} = \sum_{k=1}^{K} \log(1 + \text{SINR}_k), \quad (12)$$

where $\text{SINR}_k$ represents the signal to interference plus noise ratio of the $k$-th user, and can be expressed as

$$\text{SINR}_k = \frac{P_k \cdot ||\tilde{\mathbf{h}}_k \mathbf{w}_{Sk}||^2}{\sum_{j=1, j\neq k}^{K} P_j \cdot ||\tilde{\mathbf{h}}_k \mathbf{w}_{Sj}||^2 + \mathbf{R}_n^{(k,k)}}, \quad (13)$$

where $\mathbf{R}_n^{(k,k)}$ is $k$-th diagonal term of $\mathbf{R}_n$ and $P_k = \beta_{\text{MMSE}}$ for all $k = 1, \cdots, K$, since equal power allocation is used.

## III. Proposed Transmit antenna selection

The proposed transmit antenna selections are introduced in this section. Let $\mathbf{H} = [\mathbf{h}_1 \;\; \mathbf{h}_2 \cdots \mathbf{h}_{M_T}]$, where $\mathbf{H}$ represents the $K \times M_T$ channel matrix using all $M_T$ transmit antennas and $\mathbf{h}_l$ represents the effective channel vector of $l$-th transmit antenna, $l = 1, 2 \cdots, M_T$.

### A. Optimal antenna selection with linear precoding

To obtain the the antenna set with optimal data rate, all the possible combinations need to be calculated and then choose the best one. That is,

$$\begin{aligned} S &= \arg\max_{S'} R_{S'} \\ s.t.\;\; &\text{tr}(\mathbf{P}_{S'}) \leq P_C, \end{aligned} \quad (14)$$

where $R_{S'}$ is the sum rate of transmit antenna set $S'$, and $|S'| = M_S$. From (14), as $M_T$ grows, the whole search space is $C_{M_S}^{M_T}$ which is prohibitively large. For example, with $M_T = 100$ and $M_S = 4$, there are $C_4^{100} \approx 3.5 \times 10^6$ possible sets. Therefore, the search procedure becomes complicated and may somewhat be impractical.

### B. Proposed antenna selection with linear precoding

Instead of using the optimal solution which demands a huge computations when $M_T$ is large, we propose a suboptimal search strategy which has low complexity but can achieve the optimal performance very similarly. The algorithm is described as follows:

**Algorithm 1:** The proposed algorithm

**Initialization**: Let $S = \{1, 2, \cdots, M_T\}$, $|S| = M_C$ **while** $|S| \geq M_S$ **do**

1) $S_k = S - \{k\}$
2) $m = \arg\max_k R_{S_k}$, s.t. $\text{tr}(\mathbf{P}_{S_k}) \leq P_C$
3) $S = S - \{m\}$

The set $S$ is the proposed transmit antenna set.

In step 1, the $k$-th transmit antenna is removed and the remaining antenna set is $\mathcal{S}_k$. Then, calculate the corresponding achievable sum rate using the using the effective channel matrix $\mathbf{H}_{S_k}$; compare the achievable sum rate and determine which antenna should be removed so as to maximize the sum rate in step 2. In step 3, the determined antenna is removed from set $S$. Steps 1-3 are repeated until the number of element of $S$ is equal to $M_S$. With this algorithm, the computational complexity can be greatly reduced compared to the exhaustive search. However, the computational effort is still high since we need to calculate the precoding matrices and the corresponding sum rate operation in each iteration. These operations involve matrix inverse and calculation of matrix determinant, which are complicated operations especially when the matrices size are large. To overcome this problem, an equivalent (without losing performance compared to Algorithm 1) but much simplified algorithm is proposed in the following subsection.

### C. Proposed antenna selection with zero-forcing precoding

In Section II-A, a zero-forcing precoder mentioned in (3) gives $\mathbf{H}_\mathcal{S} \mathbf{W}_\mathcal{S} = \mathbf{I}$. Thanks to the interference-free property of zero-forcing precoding, the matrix operations reduce to the summation of all users' rates. Therefore, by applying Algorithm 1, the criterion in step 2) can be rewritten as

$$\begin{aligned} &\arg\max_k \sum_{l=1}^{K} \log(1 + P_{\mathcal{S}_k, l}) \\ &\text{s.t.} \sum_{l=1}^{K} ||\mathbf{w}_{\mathcal{S}_k}|| P_{\mathcal{S}_k, l} \leq P_c \end{aligned} \quad (15)$$

However, calculating (15) still needs to determine the precoding matrix $\mathbf{W}_{S_k}$ that contains matrix inverse and matrix multiplications. The computation effort can actually be greatly reduced as follows: consider equal power allocation rather than optimal water-filling, the sum of transmission rate is given by

$$\begin{aligned} R_{\mathcal{S}_k} &= K \log(1 + \frac{P_c}{\text{tr}(\mathbf{W}_{\mathcal{S}_k} \mathbf{W}_{\mathcal{S}_k}^H)}) \\ &= K \log(1 + \frac{P_c}{\text{tr}((\mathbf{H}_{\mathcal{S}_k} \mathbf{H}_{\mathcal{S}_k}^H)^{-1})}). \end{aligned} \quad (16)$$

The computation of sum rate becomes a relatively simple form as in (16), which only needs to calculate the diagonal terms of $(\mathbf{H}_{\mathcal{S}_k} \mathbf{H}_{\mathcal{S}_k}^H)^{-1})$ rather than the precoding matrix $\mathbf{W}_\mathcal{S}$ in each iteration. Therefore, the selection problem becomes

$$\arg\max_k \log(1 + \frac{P_c}{\text{tr}((\mathbf{H}_{\mathcal{S}_k} \mathbf{H}_{\mathcal{S}_k}^H)^{-1})})$$

$$\equiv \arg\max_k \frac{1}{\text{tr}((\mathbf{H}_{\mathcal{S}_k} \mathbf{H}_{\mathcal{S}_k}^H)^{-1})} \quad (17a)$$

$$\equiv \arg\min_k \text{tr}((\mathbf{H}_{\mathcal{S}_k} \mathbf{H}_{\mathcal{S}_k}^H)^{-1}), \quad (17b)$$

(17a) is because that logarithm is a monotonic function and $P_c$ is a constant, and maximizing the inversion of a number is equivalent to minimizing it leads to (17b).

We see that successive application of selection rule (17b) demands matrix inverses. Again, by applying some matrix properties and equalities, the matrix inverse with antenna selection processing can be manipulated to a relatively simple form. Let $\overline{\mathbf{H}}_{\mathcal{S}}$ be a $K \times (M_T - M_S)$ submatrix of $\mathbf{H}$ that is not in set $\mathcal{S}$. The following equation holds:

$$\mathbf{H}\mathbf{H}^H = \mathbf{H}_{\mathcal{S}}\mathbf{H}_{\mathcal{S}}^H + \overline{\mathbf{H}}_{\mathcal{S}}\overline{\mathbf{H}}_{\mathcal{S}}^H. \quad (18)$$

By transposing (18), taking matrix inverse, and using the following matrix inversion property:

$$(\mathbf{Z} + \mathbf{U}\mathbf{V}^H)^{-1} = \mathbf{Z}^{-1} + \mathbf{Z}^{-1}\mathbf{U}(\mathbf{I} + \mathbf{V}^H\mathbf{Z}^{-1}\mathbf{U})^{-1}\mathbf{V}^H\mathbf{Z}^{-1}. \quad (19)$$

The inverse of $(\mathbf{H}_{\mathcal{S}}\mathbf{H}_{\mathcal{S}}^H)$ in (18) can be rewritten as

$$\begin{aligned}(\mathbf{H}_{\mathcal{S}}\mathbf{H}_{\mathcal{S}}^H)^{-1} &= (\mathbf{H}\mathbf{H}^H - \overline{\mathbf{H}}_{\mathcal{S}}\overline{\mathbf{H}}_{\mathcal{S}}^H)^{-1} \\ &= \mathbf{A} + \mathbf{A}\overline{\mathbf{H}}_{\mathcal{S}}(\mathbf{I} - \overline{\mathbf{H}}_{\mathcal{S}}^H\mathbf{A}\overline{\mathbf{H}}_{\mathcal{S}})^{-1}\overline{\mathbf{H}}_{\mathcal{S}}^H\mathbf{A} \quad (20)\end{aligned}$$

where $\mathbf{A} = (\mathbf{H}\mathbf{H}^H)^{-1}$. Suppose the initial information of trace of $\mathbf{A}$ is available, (17b) becomes

$$\arg\min_k \operatorname{tr}(\mathbf{A}\overline{\mathbf{H}}_{\mathcal{S}_k}(\mathbf{I} - \overline{\mathbf{H}}_{\mathcal{S}_k}^H\mathbf{A}\overline{\mathbf{H}}_{\mathcal{S}_k})^{-1}\overline{\mathbf{H}}_{\mathcal{S}_k}^H\mathbf{A}). \quad (21)$$

We make use of the antenna elimination rule in (21) for the proposed greedy method, and then update (20) so that we can obtain the sum rate easily. Moreover, since one transmit antenna is removed per iteration, $K \times (M_T - M_S)$ matrix $\overline{\mathbf{H}}_{\mathcal{S}}$ reduces to a $K \times 1$ column vector $\overline{\mathbf{h}}_{\mathcal{S}}$. Therefore, (21) can be further simplified to

$$\begin{aligned}&\arg\min_k \operatorname{tr}\left(\frac{\mathbf{A}\overline{\mathbf{h}}_k\overline{\mathbf{h}}_k^H\mathbf{A}}{1 - \overline{\mathbf{h}}_k^H\mathbf{A}\overline{\mathbf{h}}_k}\right) \\ &= \arg\min_k \frac{\operatorname{tr}(\mathbf{A}\overline{\mathbf{h}}_{\mathcal{S}_k}\overline{\mathbf{h}}_k^H\mathbf{A})}{1 - \overline{\mathbf{h}}_k^H\mathbf{A}\overline{\mathbf{h}}_k} \\ &= \arg\min_k \frac{\operatorname{tr}(\overline{\mathbf{h}}_k^H\mathbf{A}\mathbf{A}\overline{\mathbf{h}}_k)}{1 - \overline{\mathbf{h}}_k^H\mathbf{A}\overline{\mathbf{h}}_k} \\ &= \arg\min_k \frac{||\overline{\mathbf{h}}_k^H\mathbf{A}||^2}{1 - \overline{\mathbf{h}}_{\mathcal{S}_k}^H\mathbf{A}\overline{\mathbf{h}}_{\mathcal{S}_k}}. \quad (22)\end{aligned}$$

Algorithm 2 gives the revised search procedure with zero-forcing beamforming.

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**Algorithm 2:** The modified algorithm with ZFBF

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**Initialization**: Let $\mathcal{S} = \{1, 2, \cdots, M_T\}$, $|\mathcal{S}| = M_C$, then calculus and define $\mathbf{A}_{\mathcal{S}} = (\mathbf{H}\mathbf{H}^H)^{-1}$
**while** $|\mathcal{S}| \geq M_S$ **do**
  1) $\mathcal{S}_k = \mathcal{S} - \{k\}$
  2) $m = \arg\min_k \frac{||\overline{\mathbf{h}}_k^H\mathbf{A}_{\mathcal{S}}||^2}{1 - \overline{\mathbf{h}}_k^H\mathbf{A}_{\mathcal{S}}\overline{\mathbf{h}}_k}$
  3) $\mathbf{A}_{\mathcal{S}} := \mathbf{A}_{\mathcal{S}} + \frac{\mathbf{A}_{\mathcal{S}}\overline{\mathbf{h}}_m\overline{\mathbf{h}}_m^H\mathbf{A}_{\mathcal{S}}}{1 - \overline{\mathbf{h}}_m^H\mathbf{A}_{\mathcal{S}}\overline{\mathbf{h}}_m}$
  4) $\mathcal{S} = \mathcal{S} - \{m\}$
$R_{\mathcal{S}} = K\log(1 + \frac{P_c}{\operatorname{tr}(\mathbf{A}_{\mathcal{S}})})$
The set $\mathcal{S}$ is the proposed transmit antenna set

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*D. Proposed antenna selection with MMSE precoder*

In this subsection, the proposed antenna selection procedure in Algorithm 1 is applied in the MMSE precoding. Similar to the zero-forcing beamforming, to reduce the complicated computation for MMSE precoding matrix, matrix computations such as matrix inverse should be avoided. This can be achieved by applying (19) to (11), *i.e.*

$$\begin{aligned}(\mathbf{T}_{\mathcal{S}})^{-1} &= (\mathbf{H}_{\mathcal{S}}^H\mathbf{H}_{\mathcal{S}} + \alpha\mathbf{I})^{-1} \\ &= (\alpha\mathbf{I})^{-1} + (\alpha\mathbf{I})^{-1}\mathbf{H}_{\mathcal{S}}^H(\mathbf{I} + \mathbf{H}_{\mathcal{S}}(\alpha\mathbf{I})^{-1}\mathbf{H}_{\mathcal{S}}^H)^{-1}\mathbf{H}_{\mathcal{S}} \\ &= \frac{\mathbf{I}}{\alpha} + \frac{1}{\alpha}\mathbf{H}_{\mathcal{S}}^H(\alpha\mathbf{I} + \mathbf{H}_{\mathcal{S}}\mathbf{H}_{\mathcal{S}}^H)^{-1}\mathbf{H}_{\mathcal{S}}, \quad (23)\end{aligned}$$

where $\alpha = \frac{\operatorname{tr}(\mathbf{R}_n)}{P_c}$. Moreover, together with (18), the matrix inverse property in (19) can be applied to the inverse term in (23) again to simplify the computations, *i.e.*

$$\begin{aligned}(\alpha\mathbf{I} + \mathbf{H}_{\mathcal{S}}\mathbf{H}_{\mathcal{S}}^H)^{-1} &= (\alpha\mathbf{I} + \mathbf{H}\mathbf{H}^H - \overline{\mathbf{H}}_{\mathcal{S}}\overline{\mathbf{H}}_{\mathcal{S}}^H)^{-1} \\ &= \mathbf{A} + \mathbf{A}\overline{\mathbf{H}}_{\mathcal{S}}(\mathbf{I} + \overline{\mathbf{H}}_{\mathcal{S}}^H\mathbf{A}\overline{\mathbf{H}}_{\mathcal{S}})^{-1}\mathbf{A}, \quad (24)\end{aligned}$$

where we define $\mathbf{A} = (\alpha\mathbf{I} + \mathbf{H}\mathbf{H}^H)^{-1}$.

Furthermore, the removed matrix block $\overline{\mathbf{H}}_{\mathcal{S}}$ becomes one dimensional vector $\overline{\mathbf{h}}_{\mathcal{S}}$ when the proposed greedy search method in Algorithm 1 is used. Therefore, (24) can be rewritten as

$$(\alpha\mathbf{I} + \mathbf{H}_{\mathcal{S}}\mathbf{H}_{\mathcal{S}}^H)^{-1} = \mathbf{A} + \frac{\mathbf{A}\overline{\mathbf{h}}_{\mathcal{S}}\overline{\mathbf{h}}_{\mathcal{S}}^H\mathbf{A}}{1 - \overline{\mathbf{h}}_{\mathcal{S}}^H\mathbf{A}\overline{\mathbf{h}}_{\mathcal{S}}}. \quad (25)$$

Using (25), we can simply update the inverse term in (23) and then obtain inverse of $\mathbf{T}_{\mathcal{S}}$. The proposed search procedure for MMSE precoding is summarized in Algorithm 3.

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**Algorithm 3:** The modified algorithm with MMSE precoder

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**Initialization**: Let $S = \{1, 2, \cdots, M_T\}$, $|S| = M_C$, then calculus and define $\mathbf{A}_S = (\alpha\mathbf{I} + \mathbf{H}\mathbf{H}^H)^{-1}$
**while** $|S| \geq M_S$ **do**
  1) $S_k = S - \{k\}$
  2) $\mathbf{W}_{S_k} = (\mathbf{T}_{S_k})^{-1}\mathbf{H}_{S_k}^H$, where $(\mathbf{T}_{S_k})^{-1} = \frac{\mathbf{I}}{\alpha} + \frac{1}{\alpha} \cdot \mathbf{H}_{S_k}^H\mathbf{A}_S\mathbf{H}_{S_k}$
  3) $m = \arg\max_k \sum_{l=1}^K \log(1 + \mathrm{SINR}_{S_k,l})$, where $\mathrm{SINR}_{S_k,l}$ is given in (13)
  4) $\mathbf{A}_S := \mathbf{A}_S + \frac{\mathbf{A}_S\overline{\mathbf{h}}_m\overline{\mathbf{h}}_m^H\mathbf{A}_S}{1 - \overline{\mathbf{h}}_m^H\mathbf{A}_S\overline{\mathbf{h}}_m}$
  5) $S = S - \{m\}$
$R_S = \sum_{l=1}^K \log(1 + \mathrm{SINR}_{S,l})$
The set $S$ is the proposed transmit antenna set

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## IV. Simulation results

In the section, simulation results are provided to demonstrate the performance of the proposed algorithms. We compare the sum throughput of the proposed strategies and that of the optimal schemes, *i.e.* ZF precoding and MMSE precoding using exhaustive search. Furthermore, the comparison of zero-forcing with water-filling procedure and equal power allocation is also provided. More than 5000 different channel realizations are used in all simulations.

**Example 1. Sum rate comparison of ZF precoding:** The sum throughput as a function of total number of transmit antennas $M_T$ with equal power ZF precoding is shown in Fig. 2, for $M_S = 6$, $K = 6$ and various SNR values 0, 5, 10 and 15 dB. The proposed ZF scheme can achieve about 98% sum rate of the optimal ZF scheme, *i.e.*, ZF with the exhaustive search, with significant lower complexity.

**Example 2. Sum rate comparison of MMSE precoding:** Let the parameter settings be the same as that in **Example 1.** except that the MMSE precoding is used instead of the ZF precoding. Fig. 3 shows the sum throughput of equal-power MMSE precoding as a function of number of transmit antennas. Again, the achievable sum rate using the proposed low-complexity antenna selection is very close to that of the optimal exhaustive-search scheme.

**Example 3. Simplified ZF precoding with water-filling:** Algorithm 2 with ZF precoding is based on equal power assumption. If Algorithm 1 is to be used in the transmitter with the optimal power allocation, the water-filling procedure is needed to determine the power allocation matrix for every sum throughput evaluation. Let us call it Strategy 1 for convenience. However, Strategy 1 may lead to high computational complexity; also, (22) cannot be used to simplify the rule of removing antennas in Strategy 1. Hence, we just apply one water-filling procedure in the determined antenna subset by Algorithm 2. Let us call it Strategy 2. Fig. 4 shows the sum rate comparison for Strategies 1 and 2. With lower complexity, Strategy 2 can achieve nearly 98% of the sum throughput of Strategy 1.

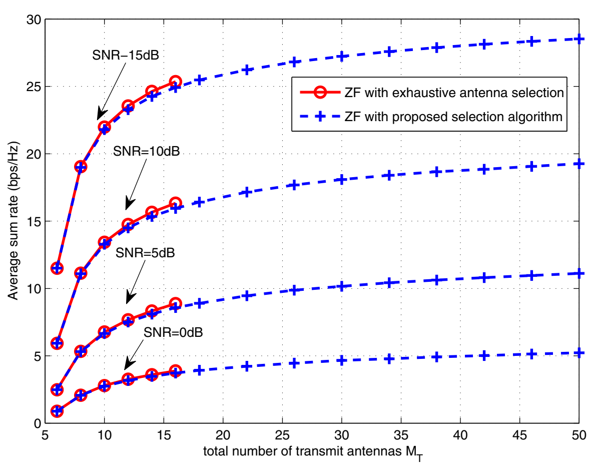


Fig. 2: Sum throughput of antenna selection for $K = M_S = 6$ using equal power ZF precoding.

## V. Conclusion

This paper proposes a low-complexity transmit antenna selection algorithm for multiuser MIMO systems in downlink direction. Simulation results demonstrated that the proposed algorithm can achieve over 98% of the optimal throughput obtained by the exhaustive search. Furthermore, when equal power is used for transmit streams, the proposed algorithm can

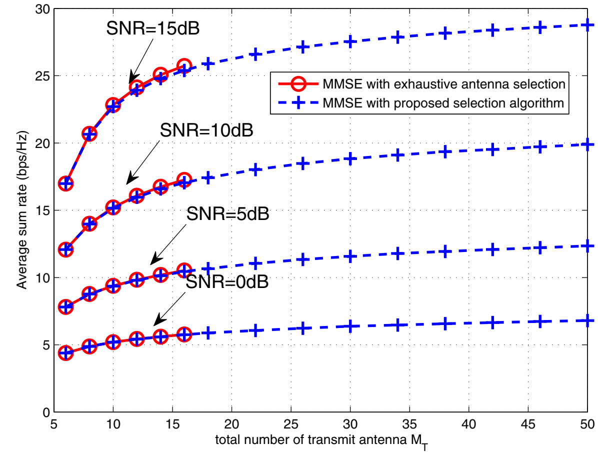


Fig. 3: Sum throughput of antenna selection for $K = M_S = 6$ using equal power MMSE precoding.

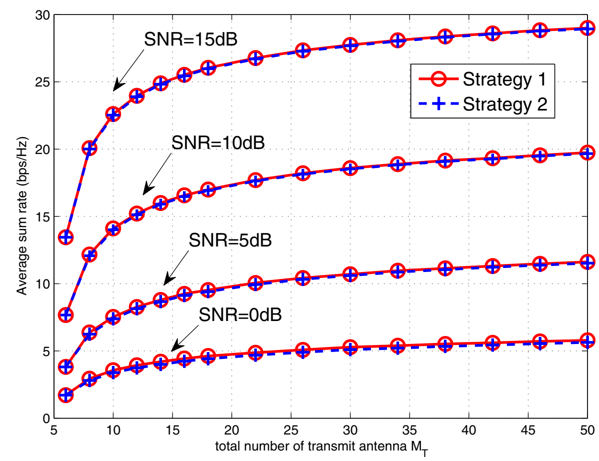


Fig. 4: Sum throughput comparison of equal power and water-filling with ZF precoding, $K = M_S = 6$.

further simplify the computational complexity for ZF precoding and MMSE precoding without performance degradation. We conclude that the proposed algorithm can achieve nearly optimal performance but with much simplified implementational complexity.

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